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Mhd Flow of a Non-Newtonian Fluid Through An Inclined Channel

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ABSTRACT:- In this paper we study an unsteady flow of a dusty fluid through an inclined channel with influence of pulsatile pressure gradient by the effect of uniform magnetic field . we solve the equations by using variable separable method and Fourier transform techniques. The graphs drawn for the velocities of both fluid and dust phase give us the relationship between the variables which is increasing or decreasing.

Keyword: pulsatile pressure gradient, an inclined channel, separable method

I. INTRODUCTION

Many authors in the past few years have studied the flow of immiscible viscous electrically conducting fluids and their different transport phenomena. This fluid is also called dusty Rivlin-Ericksen second order fluid. The influence of dust particles on visco-elastic fluid flow has its importance in many applications such as extrusion of plastic in the manufacture of rayon and Nylon. Saffman et al., (1962) studied the stability of a laminar flow of dusty gas with uniform distribution of dust particles. Michel (1965) considered the Kelvin-Helmholtz instability of the dusty gas. Michael and Miller (1965) discussed the motion of the dusty gas enclosed in the same infinite space above a rigid plane boundary.

Liu (1966) discussed the differential equation describing the relaxation phenomenon for the flow induced in an incompressible dusty gas by an infinite plate performing oscillations in its own plane . Reddy (1972) examined unsteady laminar flow of a fluid with uniform distribution of dust particles through a rectangular channel .Sastry and Seetharmaswamy (1982) studied the MHD dusty viscous flow through a circular pipe. Khare and Singh (2010) investigated the MHD flow of a dusty viscous incompressible fluid confined between two vertical walls with volume fraction of dust .we have studied the unsteady dusty visco-elastic liquid in a channel bounded by two parallel plate

II. FORMULATION AND SOLUTION OF THE PROBLEM

We have studied the unsteady dusty visco-elastic liquid in a channel bounded by two parallel plates. the change in velocity profiles for dust and liquid particles has been depicted graphically .the X-axis is taken along the plate and the Y-axis is normal to it . The basic equations of hydromagnetic flow are

$$\frac{\partial u_1'}{\partial t'} + (u'_1.\nabla)u_1' = -\frac{1}{\rho}\nabla p' + (\gamma' + \beta\nabla)\nabla^2 u_1' + \frac{k\,N_0}{\rho'(1+c^2)}(u_2' - u_1') \qquad (1)$$

$$\frac{\partial u_1'}{\partial t'} + (u_2'.\nabla)u_2' = \frac{K}{m}(u_1' - u_2') \qquad (2)$$

$$\mathrm{div}u_1' = 0 \quad , \ \mathrm{div}u_2' = 0 \qquad (3)$$

where $u_1', u_2'$is the velocity vector of fluid and dust particles respectively,p' the pressure ρ is the density of the fluid,$\gamma'$is kinematic coefficient of viscosity , t' is the time, m is the mass of dust particles, N_0 is the number density of dust particles ,k the stokes resistance coefficient c is the non-newtonian factor, On the following equations we take the following assumptions

1) The flow is fully developed.
2) The fluid is electrically neutral.

3) Fluid properties are invariable.
4) The dust particles are spherical in shape and are uniformly distributed.
5) Chemical reaction, mass transfer and radiation between the particles and fluid are not considered.
6) The temperature is uniform within a particle.
7) Interaction between particles themselves is not considered.
8) The displacement current is zero .
9) The Hall effects are negligible .
10) Only the electromagnetic body forces are present.
11) Viscous dissipation is neglected.
12) The buoyancy force is neglected.
13) The number density of the dust particles is constant through the motion.

Maxwell's equations, together with Ohm's law and the law of electromagnetic conservation are written in the case of zero –displacement and hall current as:

$$\nabla \times B = J \quad (4)$$

$$\nabla \times E = \frac{-\partial B}{\partial t} \quad (5)$$

$$J = \sigma_1 (E + V \times B) \quad (6)$$

$$\nabla . B = 0 \quad (7)$$

$$\nabla . E = 0 \quad (8)$$

$$\frac{\partial u_1'}{\partial t'} = -\frac{1}{\rho}\frac{\partial P'}{\partial x'} + (\gamma' + \beta \frac{\partial}{\partial t'})\frac{\partial^2 u_1'}{\partial y^2} u_1' + \frac{k\, N_0}{\rho'(1+c^2)}(u_2' - u_1') \quad (9)$$

$$\frac{\partial u_2'}{\partial t'} = \frac{K}{m}(u_1' - u_2') \quad (10)$$

Which are to be solved subject to the boundary conditions

$$t' = 0 \quad , \quad u_1' = u_2' = 0$$

$$t' > 0 \quad , \quad -\frac{1}{\rho}\frac{\partial P'}{\partial x'} = c_1 \quad \text{(constant)} \quad (11)$$

$$y' = \pm h \quad , \quad u_1' = u_2' = 0$$

Changing it into non dimensional form by putting

$y' = yh$, $x' = xh$, $t' = \frac{t\, h^2}{\gamma'}$, $u_1' = \frac{u\, \gamma'}{h}$, $u_2' = \frac{v\, \gamma'}{h}$, $P' = \frac{\rho' p \gamma'^2}{h^2}$, $k = \frac{m\gamma'}{\sigma h^2}$, $N_0 = \frac{l\rho}{m}$, we get

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \left(1 - \lambda \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \frac{l}{\sigma(1+c^2)}(u - v) \quad (12)$$

$$\frac{\sigma}{m}\frac{\partial v}{\partial t} = u - v \quad (13)$$

$$t = 0 \quad : \quad u = v = 0$$

$$t > 0 \quad : \quad u = 0 \quad at\ y = \pm 1 \quad (14)$$

Where $\lambda = \frac{-\beta}{h^2}$, $c_1 = -\frac{\partial p}{\partial x}$

$$\frac{\partial u}{\partial t} = c_1 + \left(1 - \lambda \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} + \frac{l}{\sigma(1+c^2)}(u - v) \quad (15)$$

Applying the laplace Transform ,we have (14)&(16)

$$S\bar{u} = \frac{c_1}{S} + (1 - \lambda S)\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{l}{\sigma(1+c^2)}(\bar{u} - \bar{v}) \quad (16)$$

$$\frac{\sigma}{m} S\bar{\bar{v}} = \bar{u} - \bar{v} \quad (17)$$

Where $\bar{u} = \int_0^\infty u e^{-St}\, dt$, $\bar{v} = \int_0^\infty v e^{-St}\, dt$

The boundary condition (15) are transformed to

$$\bar{u} = \bar{v} = 0 \quad at \quad y = \pm 1 \quad (18)$$

Solving equation (17)& (18) with (19)

$$\frac{d^2 \bar{u}}{dy^2} - M^2 \bar{u} + \frac{c_1}{S(1 - \lambda S)} = 0 \quad (19)$$

where $$M^2 = \frac{S(1+c^2)(m+\sigma S) + lS}{(1-\lambda S)(1+c^2)(m+\sigma S)} \quad (20)$$

By solving the above differential equation by special solution and general solution

$$\frac{d^2 \bar{u}}{dy^2} - M^2 \bar{u} = -\frac{C_1}{S(1 - \lambda S)}$$

At first we find the solution for

$$.\frac{d^2 \bar{u}}{dy^2} - M^2 \bar{u} = 0$$

Then the solution is $\bar{u}(y) = c_1 e^{My} + c_2 e^{-My}$

Now for this term $-\frac{C_1}{S(1-\lambda S)}$ suppose equal to C^*

then $\frac{d^2\bar{u}}{dy^2} - M^2\bar{u} = C^*$

Substitute $y = A$ in above equation

$\frac{dy}{dx} = \frac{d^2y}{dx^2} = 0 \quad that\ means \quad M^2 A = -C^*$

$$M^2 A = \frac{-C_1}{S(1-\lambda S)}$$

Then

$$A = \frac{C_1}{S(1-\lambda S)M^2}$$

now the solution is $u(y) = c_1 e^{My} + c_2 e^{-My} + \frac{c}{S(1-\lambda S)M^2}$

we have $u = 0 \quad at \quad y = \pm 1$

$0 = c_1 e^{My} + c_2 e^{-My} + \frac{c}{S(1-\lambda S)M^2}$

$0 = -c_1 e^{-My} + c_2 e^{My} + \frac{c}{S(1-\lambda S)M^2}$

By solving the last equations $c_1 = c_2$ then the solution is

$u(y) = c_1(e^{My} + e^{-My}) + \frac{c}{S(1-\lambda S)M^2}$

Now again substitute $u = 0 \quad at \quad y = 1$ to find relation between c and c_1

$$0 = c_1(e^M + e^{-M}) + \frac{c}{S(1-\lambda S)M^2}$$

$$c = -c_1(e^M + e^{-M})S(1-\lambda S)M^2$$

$$c_1 = \frac{-c}{(e^M + e^{-M})S(1-\lambda S)M^2}$$

$$u(y) = c_1(e^{My} + e^{-My}) - c_1(e^M + e^{-M})$$

$$u(y) = c_1[e^{My} + e^{-My} - (e^M + e^{-M})]$$

$$u(y) = c_1(e^M + e^{-M})\left[\frac{e^{My} + e^{-My}}{e^M + e^{-M}} - 1\right]$$

$$u(y) = \frac{-c}{S(1-\lambda S)M^2}\left[\frac{\frac{e^{My} + e^{-My}}{2}}{\frac{e^M + e^{-M}}{2}} - 1\right]$$

$$u(y) = \frac{c}{S(1-\lambda S)M^2}\left[1 - \frac{\frac{e^{My} + e^{-My}}{2}}{\frac{e^M + e^{-M}}{2}}\right]$$

$$u(y) = \frac{c}{S(1-\lambda S)M^2}\left[1 - \frac{\cosh My}{\cosh M}\right]$$

$$\bar{u} = \frac{c_1}{M^2 S(1-\lambda S)}\left(1 - \frac{\cosh My}{\cosh y}\right) \quad (21)$$

$$\bar{v} = \frac{m c_1}{M^2 S(1-\lambda S)(m+\sigma S)}\left(1 - \frac{\cosh My}{\cosh y}\right) \quad (22)$$

Applying Laplace inversion formula

$$u = \frac{1}{2\pi i}\int_{\delta - i\infty}^{\delta + i\infty} \bar{u}\, e^{st}\, dt \quad (23)$$

Here δ is the greatest then the real part of all the singularity of $\bar{u}$

$$u = \frac{1}{2\pi i}\int_{\delta - i\infty}^{\delta + i\infty} \frac{c_1}{M^2 S(1-\lambda S)}\left(1 - \frac{\cosh My}{\cosh y}\right) e^{st}\, dt$$

Similarly for v and taking inversion laplace transform and with the help of calculus 0f residues, the above equation (22)& (23) yields.

$$u = \frac{c_1}{R^2}\left(1 - \frac{\cosh\ Ry}{\cosh\ R}\right) + \frac{4c_1}{\pi}\sum_{r=1}^{\infty}\frac{(-1)^r \cos\left(\frac{2r+1}{2}\right)\pi(1-\sigma S_1)^2(1+\lambda S_1)e^{-s_1 t}}{(2r+1)c_{11}^{'}} + \frac{4c_1}{\pi}\sum_{r=1}^{\infty}\frac{(-1)^r \cos\left(\frac{2r+1}{2}\right)\pi(1-\sigma S_2)^2(1+\lambda S_2)e^{-s_2 t}}{(2r+1)c_{11}^{''}} \qquad (24)$$

And

$$v = \frac{c_1}{R^2}\left(1 - \frac{\cosh Ry}{\cosh R}\right) + \frac{4c_1}{\pi}\sum_{r=1}^{\infty}\frac{(-1)^r \cos\left(\frac{2r+1}{2}\right)\pi(1-\sigma S_1)\ \ (1+\lambda S_1)e^{-s_1 t}}{(2r+1)c_{11}^{'}} +$$

$$\frac{4c_1}{\pi}\sum_{r=1}^{\infty}\frac{(-1)^r \cos\left(\frac{2r+1}{2}\right)\pi(1-\sigma S_2)\ \ (1+\lambda S_2)e^{-s_2 t}}{(2r+1)c_{11}^{''}} \qquad (25)$$

Where

$$R = M^2 \quad \text{at } S \to 0 \quad c_{11}^{'} = \frac{1+l-2S_2\sigma+\sigma^2 S_1{}^2+\lambda S_1{}^2\sigma l}{1-s_1\sigma+\lambda s_1-\lambda s_1^2\sigma}$$

$$c_{11}^{''} = \frac{1+l-2S_2\sigma+\sigma^2 S_2{}^2+\lambda S_2{}^2\sigma l}{1-s_2\sigma+\lambda s_2-\lambda s_2^2\sigma}$$

$$S_1 = \frac{-Y_1+Y}{2\sigma(1-\pi^2\lambda(\frac{2r+1}{2})^2)},\quad S_2 = \frac{-Y_1-Y}{2\sigma(1-\pi^2\lambda(\frac{2r+1}{2})^2)}$$

$$Y_1 = 1+l+\pi^2\sigma(\frac{2r+1}{2})^2 - \pi^2\lambda(\frac{2r+1}{2})^2$$

$$Y = \sqrt{Y_1{}^2 - 4\sigma\left[1-\pi^2\lambda\left(\frac{2r+1}{2}\right)^2\right]\times\pi^2\left(\frac{2r+1}{2}\right)^2} \qquad (26)$$

Now we apply some values for non-newtonian factor in above relation for u *and* v which represented fluid and dust velocity respectively . We make tables with change values of other variables start by visco-elastic parametr λ then mass m and density ρ as following

S. No.	Non-newtonian factor c	Dust velocity at $\lambda = 1.1$	Dust velocity at $\lambda = 1.2$	Dust velocity at $\lambda = 1.25$	Dust velocity at $\lambda = 1.3$
1	0.1	0.4581	0.4631	0.4737	0.4931
2	0.2	0.4591	0.4642	0.4747	0.4941
3	0.3	0.4606	0.4658	0.4762	0.4956
4	0.4	0.4625	0.4677	0.4782	0.4974
5	0.5	0.4646	0.4698	0.4803	0.4994
6	0.6	0.4667	0.4720	0.4824	0.5015
7	0.7	0.4687	0.4741	0.4845	0.5035
8	0.8	0.4706	0.4761	0.4864	0.5054
9	0.9	0.4724	0.4779	0.4882	0.5071
10	1	0.4739	0.4795	0.4898	0.5086

Table No. (1)

(m=1 , N=1 , K=1 , $C_1 = 1,\ \rho = 1$, t=2 , $\sigma = 1$)

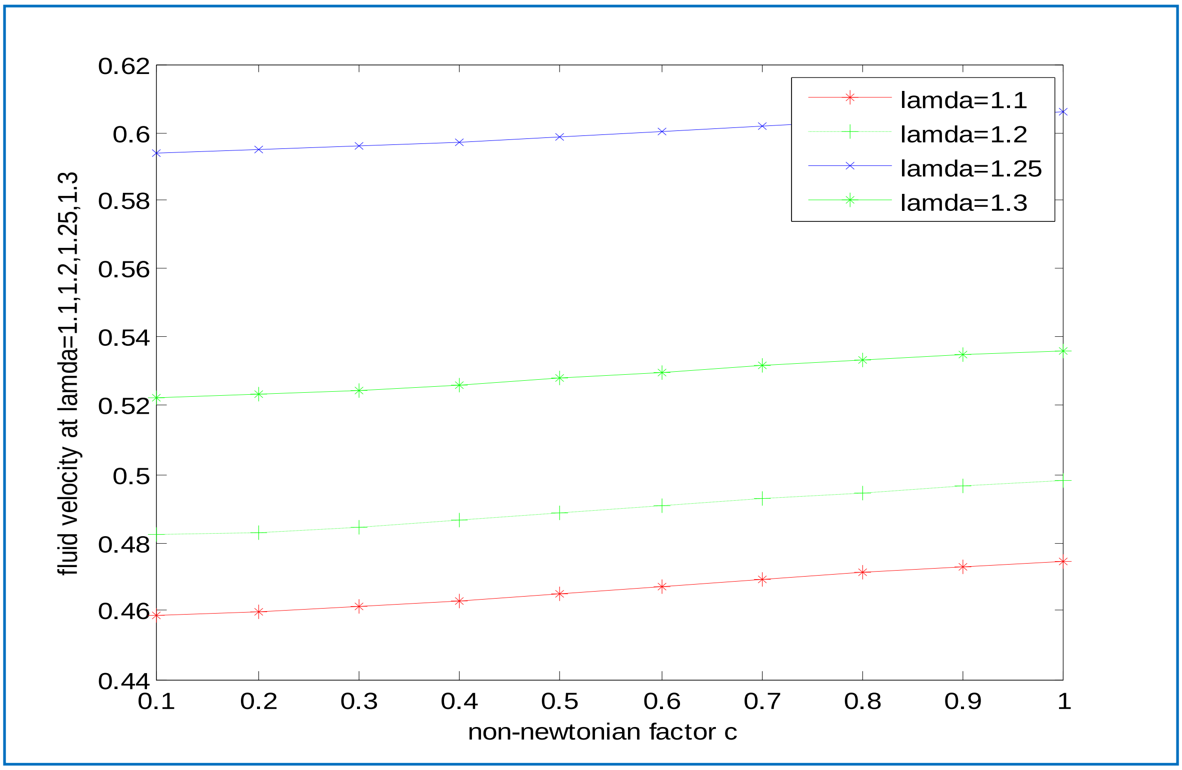

Table (1) reveals as the non-newtonian factor of fluid increases, the velocity of fluid phase increases for every visco-elastic parameter (λ) .it is evident from the relation (24) that the non-newtonian factor of fluid is present in denominator of term R^2 which is in the denominator of the expression of the velocity fluid ,then the non-newtnian factor (c) is represented in the numerator but the term R which in the cosh is small effective comparing with R^2 in the denominator of that relation . the above graph supported our data in the table.

S. No.	Non-newtonian factor c	Dust velocity at $\lambda = 1.1$	Dust velocity at $\lambda = 1.2$	Dust velocity at $\lambda = 1.25$	Dust velocity at $\lambda = 1.3$
1	0.1	0.4581	0.4631	0.4737	0.4931
2	0.2	0.4591	0.4642	0.4747	0.4941
3	0.3	0.4606	0.4658	0.4762	0.4956
4	0.4	0.4625	0.4677	0.4782	0.4974
5	0.5	0.4646	0.4698	0.4803	0.4994
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8	0.8	0.4706	0.4761	0.4864	0.5054
9	0.9	0.4724	0.4779	0.4882	0.5071
10	1	0.4739	0.4795	0.4898	0.5086

Table No. (2)

(m=1 , N=1 , K=1 , $C_1 = 1$, $\rho = 1$, t=2 , $\sigma = 1$)

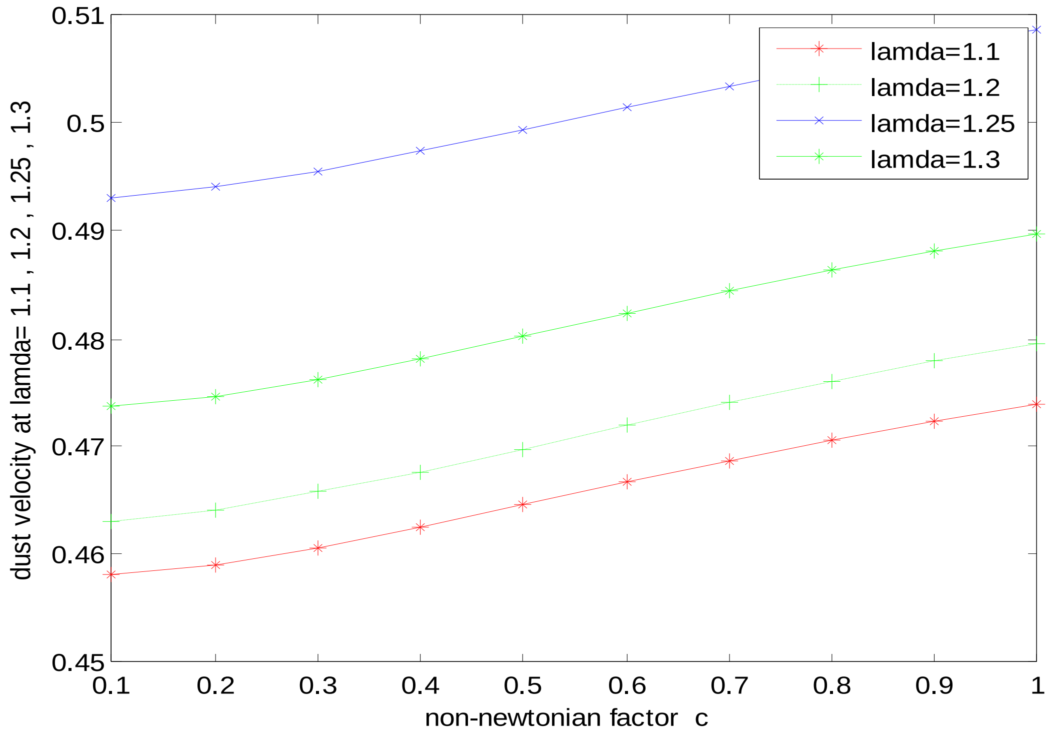


Table (2) reveals as the non-newtonian factor of dust increases , the velocity of dust phase increases for every visco-elastic parameter (λ) . it is evident from the relation (25) that the non-newtonian factor of dust is present in denominator of term R^2 which is in the denominator of the expression of the velocity dust ,then the non-newtonian factor (c) is represented in the numerator but the term R which in the cosh is small effective comparing with R^2 in the denominator of that relation .the above graph supported our data in the table (2).

S. No.	Non-newtonian factor c	Fluid velocity $m = 1$	Fluid velocity $m = 1.5$	Fluid velocity at $m = 2$	Fluid velocity $m = 2.5$
1	0.1	0.6288	0.5658	0.5288	0.5052
2	0.2	0.6294	0.5666	0.5297	0.5062
3	0.3	0.6303	0.5677	0.5311	0.5077
4	0.4	0.6314	0.5691	0.5328	0.5096
5	0.5	0.6326	0.5707	0.5346	0.5117
6	0.6	0.6339	0.5723	0.5365	0.5138
7	0.7	0.6351	0.5739	0.5383	0.5158
8	0.8	0.6362	0.5753	0.5400	0.5177
9	0.9	0.6372	0.5766	0.5415	0.5195
10	1	0.6382	0.5778	0.5429	0.5210

Table No.(3)

($\lambda = 1$, N=1 , K=1 , $C_1 = 1$, $\rho = 1$, t=2 , $\sigma = 0.8$)

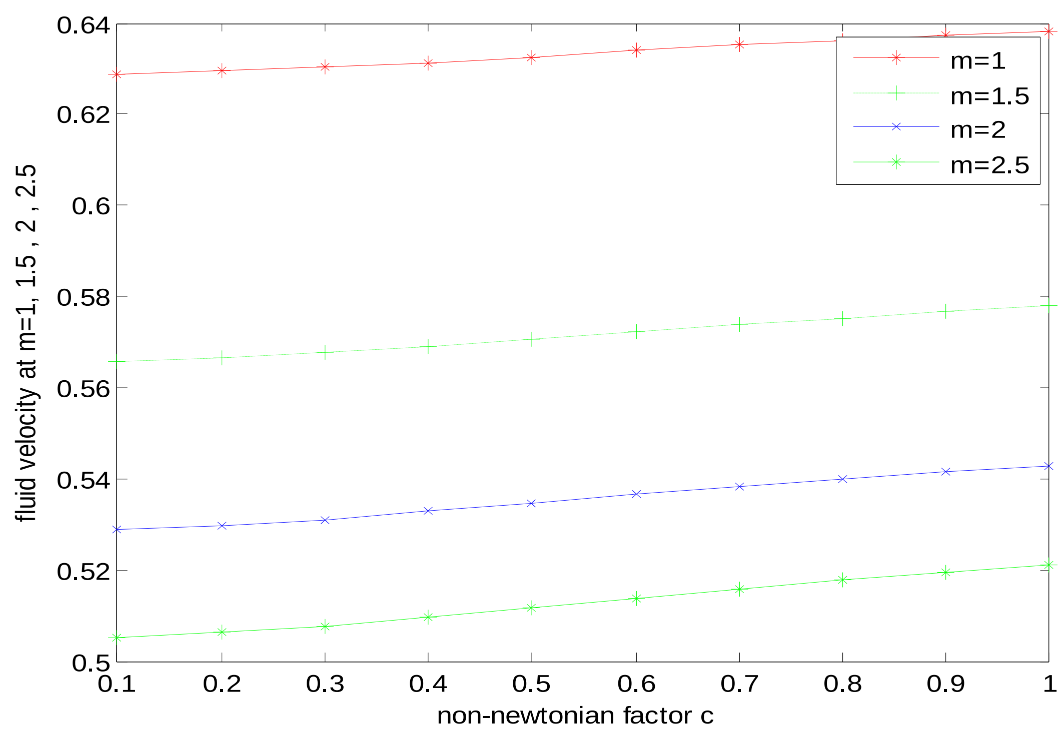
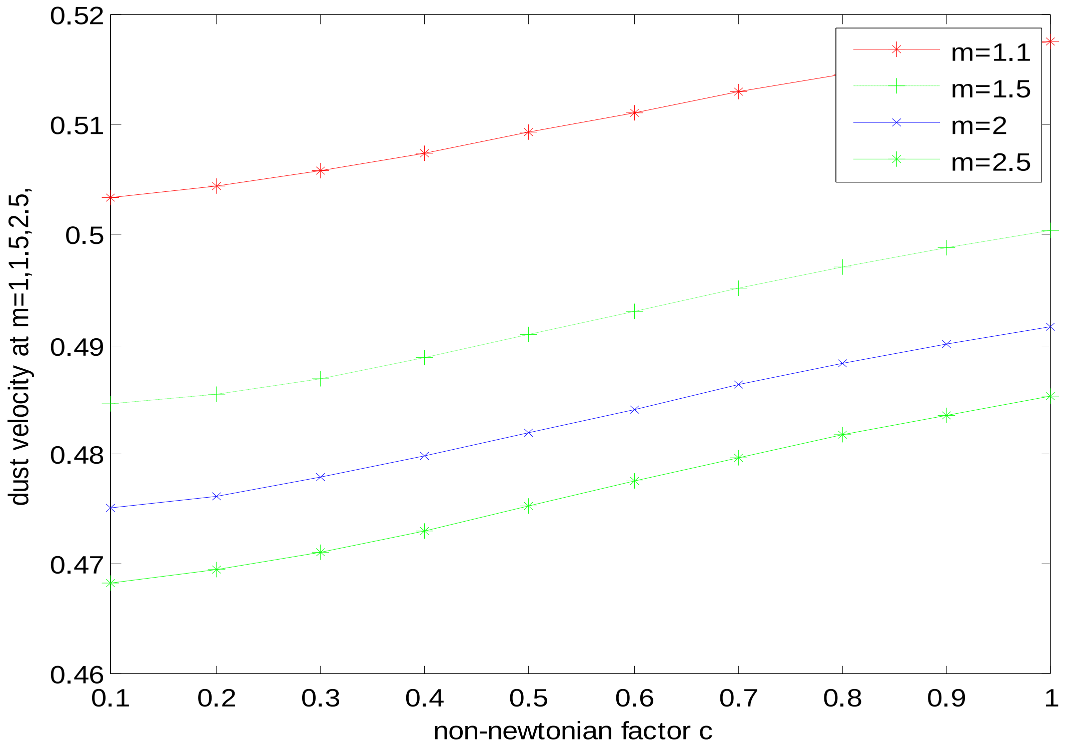


Table (3) reveals as the non-newtonian factor of fluid increases , the velocity of fluid phase increases for every visco-elastic parameter (m) . it is evident from the relation (24) that the non-newtonian factor of fluid is present in denominator of term R^2 which is in the denominator of the expression of the velocity fluid ,then the non-newtonian factor (c) is represented in the numerator but the term R which in the cosh is small effective comparing with R^2 in the denominator of that relation. The above graph supported our data in the table (3).

S. No.	Non-newtonian factor c	Dust velocity $m = 1$	Dust velocity $m = 1.5$	Dust velocity $m = 2$	Dust velocity $m = 2.5$
1	0.1	0.5035	0.4846	0.4752	0.4684
2	0.2	0.5044	0.4855	0.4763	0.4695
3	0.3	0.5058	0.4870	0.4779	0.4711
4	0.4	0.5074	0.4889	0.4799	0.4731
5	0.5	0.5093	0.4910	0.4820	0.4753
6	0.6	0.5112	0.4931	0.4842	0.4776
7	0.7	0.5130	0.4952	0.4864	0.4798
8	0.8	0.5147	0.4971	0.4883	0.4818
9	0.9	0.5162	0.4989	0.4901	0.4836
10	1	0.5176	0.5004	0.4917	0.4853

Table No.(4)

($\lambda = 1$, N=1, K=1 , $C_1 = 1$, $\rho = 1$, t=2 , $\sigma = 0.8$)



By the same method table (4) reveals as the non-newtonian factor of dust increases , the velocity of dust phase increases for every visco-elastic parameter (m) . it is evident from the relation (25) that the non-newtonian factor of dust is present in denominator of term R^2 which is in the denominator of the expression of the velocity dust ,then the non-newtonian factor (c) is represented in the numerator but the term R which in the cosh is small effective comparing with R^2 in the denominator of that relation.The above graph supported our data in the table (4).

III. CONCLUSION

Unsteady flow of a dusty visco-elastic fluid through a channel is studie . First and second figures show as the value of velocity of the fluid and dust particles increase then the value of non-newtonian factor increase. Third and forth figures show the same relation but with change values of mass for the phase of fluid and dust.

REFERENCES

[1]. C. B. Singh and P. C Ram, “Unsteady Flow of an Elec-trically Conducting Dusty Viscous Liquid Through a Channel,” indian *Journal of Pure and Applied Mathe-matics*, Vol. 8, No. 9,1977, 1022-1028.

[2]. K. K. Singh, “Unsteady Flow of Conducting Dusty Vis-cous Liquid in an Annulas,” *Acta Ciecia Indica*, Vol. 3, No. 3, 1977, p. 264.

[3]. M. L. Sharma, “MHD Flow of a Conducting Dusty Vis-cous Liquid through a Long Elliptic Duct with Pressure Gradient as function of a Time,” Ph.D Thesis Agra Uni-versity, Agra, 1980

[4]. Barret O’Nell, Elementary Differential Geometry, Academic Press, New York (1966).

[5]. D.C. Dalal, N. Datta, S.K. Mukherjea, Unsteady natural convection of a dusty fluid in an infinite rectangular channel

[6]. International Journal of Heat and Mass Transfer, 41, No. 3 (1998), 547-562. and Mass Transfer, 41, No. 3 (1998), 547-562. and Mass Transfer, 41, No. 3 (1998), 547-562.

[7]. Sa_man P. G., On the stability of laminar ow of a dusty gas, Journal of Fluid Mechanics, 13 (1962), 120{128.

[8]. Samba P., Siva Rao, Unsteady ow of a dusty viscous liquid through circular cylinder, Def. Sci. J., 19 (1969), 135{138.

[9]. Bhattacharya, T.K. (1994): Hydro magnetic flow of a dusty viscous fluid through an annulus with time dependent pressure gradient. Journal of MACT. Vol. 27, pp. 1-5.

[10]. Bjerkness, V. (1900): Meteorologiache Zeitschrift, 17, pp. 97-106.
[11]. Chamkha, Ali. J. (1996): Boundary layer laminar flow of a dusty compressible gas over a flat surface. Journal of fluid Engineering, Vol. 118, pp.179-185.
[12]. Tyn Myint-U, Partial Di_ erential Equations of Mathematic Physics, American Elsevier Publishing Company, New York, 1965.
[13]. Yang Lei and Bakhtier F.,Three-dimensional mixed convection ows in a horizontal annulus with a heated rotating
[14]. Inner circular cylinder, Int. Journal of Heat and Mass Transfer, 35(1992), 1947{1956.