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Optimal Selective Maintenance Strategy for Continuum Flow Transmission System

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ABSTRACT: *In many real-world flow transmission systems, certain maintenance actions should be performed during the interval to complete the next mission with higher reliability. However, due to the limitation of time and available resources, not all maintenance actions can be performed. In this paper, the selective maintenance strategy and the Beta distribution is adopted to model the behaviour of capacity. Four maintenance options, namely strategy without actions, capacity recovery, performance improvement and replacement are introduced. An optimization model is proposed with the objective of maximizing the average expected capacity, under the time and cost constraints. In the experiment, a parallel system is studied and a more general model is introduced afterwards. Experimental result shows that the proposed model is able to generate the optimized maintenance strategy for the given system configuration.*

Keywords: *Selective maintenance, Flow transmission system, Beta distribution, GA*

I. INTRODUCTION

For continuum flow transmission system, Lisnianski [1] proposed a discrete approximation approach based on the universal generating function technique to estimate boundary points of several reliability measures for continuum state systems. Levitin [12] gives one approach for reliability evaluation of flow transmission system for series-parallel systems and bridge system . Ga ́miz et .al [4] propose a method to build a structure function for a continuum flow transmission system by using multivariate nonparametric regression techniques, which took certain analytical restrictions on the variable of interest into account

For discrete flow transmission system, there are has been numerous studies in this field. Since the discrete case can always be approached by enumeration method, the research focus on developing efficient Algorithm for large flow transmission systems, which is also known as multistate network or capacitated flow network. Assume that all MPs and MCs are known in advance, the study focus on developing efficient algorithm to find d-MPs or d-MCs, which representing the system state vectors with capacity of . Then the reliability can be calculated using Inclusion-Exclusion method, Sum of Disjoint method and etc. More related work could be found in Yeh [17-19], Lin [22], Coit et.al [8], Kapur et.al [15], Jane and Laih [6][7].

Few study for multistate flow transmission system conduct the maintenance issue. Yeh [20] first introduced maintenance problem into multistate flow system in terms of maintenance cost, which is defined as the overall cost of restoring a system from its failed state back to its original state. Lin [23] studied the maintenance issue in cloud computing network (CCN), which extended Yeh's work, that the components in the CCN should be capacity recovered to their highest capacities when only units of data can be sent. Todinov [10] proposed a framework for analysis and optimization of repairable flow network, where the repairable network is that a renewal of failed component is taking place after a certain delay for repair. Each edge is characterized by a cumulative time to failure distribution and a cumulative time to repair distribution.

The first study on mathematical modeling of selective maintenance can refer to Rice et.al [21]. Chen et.al [2] first studied the selective maintenance optimization for multi-state systems. Cassady et.al [3] considered that the lifetime of the components follow Weibull distribution, and several maintenance actions can be select consist of capacity recovery, and corrective or preventive replacement. Rajagopalan et.al [13] proposed a series of four improvements on the enumerative solution procedure for selective maintenance problem.

Lust et.al [16] developed a heuristics and exact method based on a branch and bound procedure to study the optimal selective maintenance problem, to pick out a subset of actions to undertake whose total execution duration fits in the time window and that yields maximum reliability when the system is restarted after the maintenance period. Maillart et.al [9] considered a corrective selective maintenance model that identifies which components to replace in the finitely long period of time between missions performed by a series-parallel system. A stochastic dynamic program is formed and suggestion had been made that future work should concentrate on single-mission problem. Liu and Huang [24] formulate the multi-state system using universal generating function method and proposed a Generic Algorithm to solve this complicated optimization problem where both multi-state systems and imperfect maintenance models are taken into account. Zhu et.al [5] studied a cost-based selective maintenance decision-making method for manufacturing system, in which machines are connected in series or in parallel. An algorithm combining the heuristic rules and Tabu search is proposed to solve the presented selective maintenance model.

II. SYSTEM DESCRIPTION AND ASSUMPTIONS

Considering a flow transmission system under multi-phased missions, due to the random failures and other unexpected factors, the capacity of each component in such a system can be regarded as statistically independent and continuum valued. For example, the transportation line of military logistic system may experience random damage, when attacked by its enemy during the mission. Thus, the capacity of delivering supply for each transportation line is a random variable at an arbitrary time t, with a certain distribution. The performance of the system at an arbitrary time t is defined as the expected flow transmitting capacity of the system at time t.

However, as time goes, the component will experience a gradual deterioration caused by the changing of its physical property. We take the military logistic system as an example again; the transportation line will deteriorate over a period of time caused by the environment. The continuous operation and loading also deteriorate the roadmap as well. Thus, the performance of each component in flow transmission system is characterized by capacity and time. The capacity represents the current state of the component, while the time, which affects the distribution of the capacity, represents the deterioration of the entire component. The performance of the system is defined as the expected flow transmitting capacity during the mission time period.

After each mission, the system is inspected and the current capacity of each component can be evaluated. Then the distribution of capacity is updated. In order to complete the next mission with higher reliability, certain maintenance actions will be performed during the interval to improve either the capacity, or the distribution of capacity, or both. However, due to the limitation of time and available resources, not all the maintenance actions can be performed. Therefore, the selective maintenance strategy is necessary.

2.1. Single component and assumptions

For a single component, the performance index is capacity. The Beta distribution is adopted to describe the capacity of the component. Beta distribution is a continuous probability distribution on the interval $[0,1]$. Without loss of generosity, assume that the capacity of the component i follows a Beta distribution with parameter a and b. Both a and b are positive shape parameters. The probability distribution of the capacity is given as:

$$f_i(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1-x)^{\beta-1}x^{\alpha-1}, x \in [0,1] \qquad (1)$$

where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^x dx$, and $\Gamma(\beta) = \int_0^\infty x^{\beta-1}e^x dx$. $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$ is normalization constant to ensure that probability integrates to unity.

In real applications, it is reasonable to assume that the capacity of a component will be its maximum value at the beginning and deteriorate as time goes. Thus, the capacity at earlier age is more likely to remain in high value and goes down as component ages. In order to capture this character, further assume that the parameter a is fixed and parameter b is a function of time t, which follows distribution with parameter l. According to characters of Bata distribution, if a is fixed to be 1, the PDF has an increasing trend while b is below 1 and decreasing; the PDF has a decreasing trend while b is above 1 and increasing. To illustrate this idea, consider the parameter b as a function of time t given below:

$$\beta(t) = \lambda t, t \in [0,\infty) \qquad (2)$$

Thus, the probability distribution function is:

$$f_i(x) = \frac{\Gamma(\alpha+\lambda t)}{\Gamma(\alpha)\Gamma(\lambda t)}(1-x)^{\lambda t-1}x^{\alpha-1}, x \in [0,1], t \in [0,\infty) \qquad (3)$$

As it can be seen (Fig. 1) that when $\alpha = 1$ and $\lambda = 0.1$, the distribution of capacity changes as time t increases.

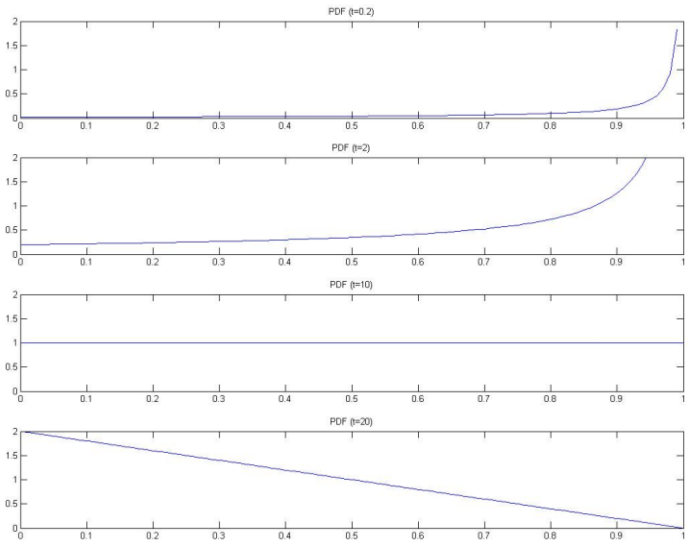

Fig.1. *Probability distribution of Beta distribution when* $\alpha = 1$ *and* $\beta(t) = 0.1 \times t$

In reality, the Beta distribution needs to be modified to fit the distribution of capacity. The parameter a and b should be estimated based on availble data. Since it is not the focus of this project, the a is assumed to be deterministic as 1 and b , as function of time t, is given as follows:

$$\beta(t) = 1 - e^{-\lambda t}, t \in [0, \infty)$$

Thus, the Beta distribution is given as:

$$f_i(x) = \frac{\Gamma(\alpha + 1 - e^{-\lambda t})}{\Gamma(\alpha)\Gamma(1 - e^{-\lambda t})}(1-x)^{-e^{-\lambda t}} x^{\alpha-1}, x \in [0,1], t \in [0, \infty)$$

2.2.System description and assumptions

A flow transmission system consist of n components, the capacity of component i at time t is a random variable, denote as $X_i(t), i \in \{1, 2, ..., n\}$. Without loss of generosity, $X_i(t) \in [0,1]$, 0 means the capacity of the component is 0 and 1 means the maximal capacity of component. Let $f_i(x,t)$ denote the probability distribution function for capacity of component i at time t, the output expectation is used. For single component i, the capacity expectation at any arbitrary time t is defined as

$$E(X_i, t) = \int_0^1 x_i \times f_i(x_i, t) dx_i$$

For the system, let $\phi(X,t)$ denote the capacity of the system at time t. The capacity expectation of the system at any arbitrary time t is defined as $E[\phi(X,t)]$.

However, for any time period $T = [0, t_0]$, the component experienced a gradual deterioration, which affects the probability distribution of the capacity. Thus, the average capacity expectation during the time period should be adopt to express the performance of the system, which is given as follows[14]:

$$\frac{\int_0^{t_0} E[\phi(X,t)]}{t_0}$$

Assume that the system has been put into operation for time period $T_0 = [0, t_0]$, which also known as the effective age of the system. And the system is shut down for maintenance before the next mission (with time period of $T_1 = [0, t_1]$).

Assume that the capacity of each component at time t_0 has been inspected, denoted by x_i, $x_i \in [0,1)$; the capacity of each component can only take any number between 0 and the current capacity in the future, denote as $X_i(t) \in [0, x_i]$, $x_i \in [0,1]$. The probability density function of capacity is also updated. However, the deterioration of the component, which is known as the distribution of capacity, cannot be inspected due to limitation of time and cost. Take the military logistic system as an example; it might be easy to detect the current capacity of the transportation line, such as lower capacity due to damage on the road. However, the deterioration of the road is hard to inspect.

III. MAINTENANCE OPTIONS AND CORRESPONDING RESOURCES CONSUMED

Four maintenance options are available in the proposed method: Strategy without actions; capacity recovery, which restores the capacity of the component to maximum and leave the distribution of the capacity at current stage; performance improvement, which restores the capacity of the component to maximum and improve the distribution of the capacity with specified degree and replacement. Further assume that all four-decision variables are binary $\{0,1\}$ and only one action can be made on each component.

3.1 Strategy without actions

Under this alternative, the capacity of the current component remains, which is denoted as x_i, $x_i \in [0,1)$. The distribution of capacity is therefore updated.

We take the Beta distribution as an example, the PDF of the capacity of the component is

$$f_i(x,t) = \frac{\frac{\Gamma[\alpha+\beta(t)]}{\Gamma(\alpha)\Gamma[\beta(t)]}\left(1-\left(\frac{x}{x_i}\right)\right)^{\beta(t)-1}\left(\frac{x}{x_i}\right)^{\alpha-1}}{x_i}, x \in [0, x_i],$$

The distribution of capacity is updated as

$$\beta(t) = 1 - e^{-\lambda(t+t_0)}, t \in [0, t_1]$$

The corresponding cost and time is 0.

3.2 Capacity recovery

Take the Beta distribution as an example, the PDF of the capacity of the component after this scenario is

$$f_i(x,t) = \frac{\Gamma[\alpha+\beta(t)]}{\Gamma(\alpha)\Gamma[\beta(t)]}(1-x)^{\beta(t)-1}x^{\alpha-1}, x \in [0,1]$$

The distribution of capacity remains at:

$$\beta(t) = 1 - e^{-\lambda(t+t_0)}, t \in [0, t_1]$$

A binary decision variable U_i is defined to denote whether to perform the capacity recovery on the component i.

$$U_i = \begin{cases} 1, \text{ Perfrom minimal repair on component } i; \\ 0, \text{ Otherwise;} \end{cases}$$

The total time for component i under 'capacity recovery' include the fixed time t_i^f and capacity restoring time t_i^r (To be simplified, this value is known based on difference between the current capacity and maximal capacity for component i), which is $T_i^M = t_i^f + t_i^r$.

The total cost for component i under 'capacity recovery' include the fixed cost c_i^f and capacity restoring cost c_i^r (To be simplified, this value is known based on difference between the current capacity and maximal capacity for component i), which is $C_i^M = c_i^f + c_i^r$.

3.3 Performance improvement

Under this alternative, the capacity of component is restored to previous maximum capacity. Meanwhile, the distribution of capacity for the component is improved by specified degree. The idea of age reduction is used.

Take the Beta distribution as an example, the PDF of the capacity of the component after this scenario is

$$f_i(x,t) = \frac{\Gamma[\alpha + \beta(t)]}{\Gamma[\alpha]\Gamma[\beta(t)]}(1-x)^{\beta(t)-1}x^{\alpha-1}, x \in [0,1]$$

The distribution of capacity for the component is improved by age reduction factor b

$$\beta(t) = 1 - e^{-\lambda(t+b\times t_0)}, t \in [0,t_1], b \in (0,1)$$

Since the age reduction model is adopted for 'performance improvement', the age reduction factor b can be any value between 0 and 1, where 0 indicate the same situation as capacity recovery and 1 indicate the same situation as replacement. Without loss of generosity, the age reduction factor is assumed to be deterministic (b_0) in the analytical example and different levels in numerical example.

A binary decision variable V_i is defined to denote whether to perform performance improvement on the component i .

$$V_i = \begin{cases} 1, \text{ Perfrom imperfect repair on component } i; \\ 0, \text{ Otherwise;} \end{cases}$$

The total time for component i under 'performance improvement' includes the fixed time t_i^f , capacity time t_i^r (To be simplified, this value is pre calculated based on difference between the current capacity and maximal capacity for component i) and the performance improvement time , which is $T_i^I = t_i^f + t_i^r + t_i^b$.

The total cost for component under 'performance improvement' includes the fixed cost c_i^f , capacity restoring cost c_i^r (To be simplified, this value is pre calculated based on difference between the current capacity and maximal capacity for component i) and the performance improvement cost c_i^b which is $C_i^I = c_i^r + c_i^f + c_i^b$.

3.4 Replacement

Under this alternative, a brand new component is taking place of the current component. The capacity of component is updated to previous maximum capacity. Meanwhile, the distribution of capacity for the component is updated with effective age of 0.

Take the Beta distribution as an example, the PDF of the capacity of the component after this scenario is

$$f_i(x,t) = \frac{\Gamma[\alpha + \beta(t)]}{\Gamma(\alpha)\Gamma[\beta(t)]}(1-x)^{\beta(t)-1}x^{\alpha-1}, x \in [0,1]$$

The distribution of capacity for the component is updated as

$$\beta(t) = 1 - e^{-\lambda t}, t \in [0,t_1]$$

A binary decision variable W_i is defined to denote whether to perform replacement on the component i .

$$W_i = \begin{cases} 1, \text{ Perfrom replacement on component } i; \\ 0, \text{ Otherwise;} \end{cases}$$

The total time for component i under replacement include the fixed time t_i^f and replacement time t_i^p, which is $T_i^R = t_i^f + t_i^p$.

The total cost for component i under replacement include the fixed cost c_i^f and replacement cost c_i^p, which is $C_i^R = c_i^f + c_i^p$.

IV. MODELING OF SELECTIVE MAINTENANCE

For each component i, the total cost is:

$$C_i = U_i \times C_i^M + V_i \times C_i^I + W_i \times C_i^R$$

The total cost for system is

$$C = \sum_{i=1}^{n} C_i$$

.

For each component i, the total time is:

$$T_i = U_i \times T_i^M + V_i \times T_i^I + W_i \times T_i^R$$

The total time spend on the system is

$$T = \sum_{i=1}^{n} T_i$$

.

Assume the cost limit is C_0, and the time limit is T_0, the nonlinear programming model for the selective maintenance strategy to maximize the average expected capacity for next mission is given as:

$$Max: \frac{\int_0^{t_1} E[\phi(X,t)]}{t_1}$$

$s.t.$

$$\begin{cases} U_i + V_i + W_i \le 1, \ i = \{0,1,2,....,n\} \\ C \le C_0 \\ T \le T_0 \\ U_i, V_i, W_i \text{ are binary} \end{cases}$$

The first constraint ensure that each component can be perform only one maintenance action or Strategy without actions; the second and third constraints are cost and time limitation.

4.1 Analytical example

In this section, consider a simple flow transmission system composed of two independent components connected in parallel as shown in Figure 1.

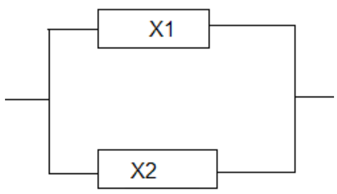

Figure 1 *flow transmission system consist of 2 components connected in parallel*

Thus, the structure function for the system is:

$$\phi(X,t) = x_1(t) + x_2(t)$$

The expected capacity of the systems is

$$E\left[\phi\left(X,t\right)\right]=\int_0^1 xf_1\left(x,t\right)dx+\int_0^1 xf_2\left(x,t\right)dx$$

The average expected capacity for next mission is

$$\frac{\int_0^{t_1} E\left[\phi\left(X,t\right)\right]}{t_1}$$

Assume that capacity of each component follows a Beta distribution with the following parameters:

$$\alpha_1=1,\alpha_2=2;\beta_1\left(t\right)=1-e^{-\frac{t}{2}},\beta_2\left(t\right)=1-e^{-\frac{t}{3}},t\in\left[0,\infty\right)$$

After the previous mission, the current capacity of each component is inspected as $x_1=0.8, x_2=0.6$. The cost related for each component is given as follows:

Fixed costs are all the same: $c_1^f=c_2^f=100$; the costs to restore the capacity are: $c_1^f=200, c_2^f=400$; the costs to improve the distribution of capacity are: $c_1^b=500$, $c_2^b=700$; the age reduction factor is $b_0=0.75$; the costs for replacement are: $c_1^p=1500$, $c_2^p=1600$.

The time related for each component is given as follows:

Fixed time is all the same: $t_1^f=t_2^f=10$; the time to restore the capacity is: $t_1^r=20, t_2^r=40$; the time to improve the distribution of capacity are: $t_1^b=50, t_2^b=70$; the time for replacement is: $t_1^p=150, t_2^p=160$.

The effective age for all the components is $c_i^f=100, i\in\left\{1,2,...,5\right\}$ $t_0=2$ year; the time period for next mission is $t_1=3$ year. The total cost constraint is $C_0=2100$ and the total time constraint is $T_0=190$. The probability distribution after each maintenance options is given:

Strategy without actions:

$$f_1\left(x,t\right)=\frac{\frac{\Gamma\left[1+1-e^{-\frac{t+t_0}{2}}\right]}{\Gamma\left(1\right)\Gamma\left[1-e^{-\frac{t+t_0}{2}}\right]}\left(1-\frac{x}{0.8}\right)^{1-e^{-\frac{t+t_0}{2}}-1}\left(\frac{x}{0.8}\right)^{1-1}}{0.8}=\frac{\frac{\Gamma\left[2-e^{-\frac{t+2}{2}}\right]}{\Gamma\left[1-e^{-\frac{t+2}{2}}\right]}\times\left(1-\frac{x}{0.8}\right)^{-e^{-\frac{t+2}{2}}}}{0.8},x\in\left[0,0.8\right],t\in\left[0,3\right]$$

$$f_2\left(x,t\right)=\frac{\frac{\Gamma\left[1+1-e^{-\frac{t+t_0}{3}}\right]}{\Gamma\left(1\right)\Gamma\left[1-e^{-\frac{t+t_0}{3}}\right]}\left(1-\frac{x}{0.6}\right)^{1-e^{-\frac{t+t_0}{3}}-1}\left(\frac{x}{0.6}\right)^{1-1}}{0.6}=\frac{\frac{\Gamma\left[2-e^{-\frac{t+2}{3}}\right]}{\Gamma\left[1-e^{-\frac{t+2}{3}}\right]}\times\left(1-\frac{x}{0.6}\right)^{-e^{-\frac{t+2}{3}}}}{0.6},x\in\left[0,0.6\right],t\in\left[0,3\right]$$

Capacity recovery:

$$f_1\left(x,t\right)=\frac{\Gamma\left[1+1-e^{-\frac{t+t_0}{2}}\right]}{\Gamma\left(1\right)\Gamma\left[1-e^{-\frac{t+t_0}{2}}\right]}\left(1-x\right)^{1-e^{-\frac{t+t_0}{2}}-1}x^{1-1}=\frac{\Gamma\left[2-e^{-\frac{t+2}{2}}\right]}{\Gamma\left[1-e^{-\frac{t+2}{2}}\right]}\times\left(1-x\right)^{-e^{-\frac{t+2}{2}}},x\in\left[0,1\right],t\in\left[0,3\right]$$

$$f_2\left(x,t\right)=\frac{\Gamma\left[1+1-e^{-\frac{t+t_0}{3}}\right]}{\Gamma\left(1\right)\Gamma\left[1-e^{-\frac{t+t_0}{3}}\right]}\left(1-x\right)^{1-e^{-\frac{t+t_0}{3}}-1}x^{1-1}=\frac{\Gamma\left[2-e^{-\frac{t+2}{3}}\right]}{\Gamma\left[1-e^{-\frac{t+2}{3}}\right]}\times\left(1-x\right)^{-e^{-\frac{t+2}{3}}},x\in\left[0,1\right],t\in\left[0,3\right]$$

Performance improvement:

$$f_1(x,t)=\frac{\Gamma\left[1+1-e^{-\frac{t+t_0\times b_0}{2}}\right]}{\Gamma(1)\Gamma\left[1-e^{-\frac{t+t_0\times b_0}{2}}\right]}(1-x)^{1-e^{-\frac{t+t_0\times b_0}{2}}-1}x^{1-1}=\frac{\Gamma\left[2-e^{-\frac{t+1.5}{2}}\right]}{\Gamma\left[1-e^{-\frac{t+1.5}{2}}\right]}\times(1-x)^{-e^{-\frac{t+1.5}{2}}},x\in[0,1],t\in[0,3]$$

$$f_2(x,t)=\frac{\Gamma\left[1+1-e^{-\frac{t+t_0\times b_0}{3}}\right]}{\Gamma(1)\Gamma\left[1-e^{-\frac{t+t_0\times b_0}{3}}\right]}(1-x)^{1-e^{-\frac{t+t_0\times b_0}{3}}-1}x^{1-1}=\frac{\Gamma\left[2-e^{-\frac{t+1.5}{3}}\right]}{\Gamma\left[1-e^{-\frac{t+1.5}{3}}\right]}\times(1-x)^{-e^{-\frac{t+1.5}{3}}},x\in[0,1],t\in[0,3]$$

Replacement:

$$f_1(x,t)=\frac{\Gamma\left[1+1-e^{-\frac{t}{2}}\right]}{\Gamma(1)\Gamma\left[1-e^{-\frac{t}{2}}\right]}(1-x)^{1-e^{-\frac{t}{2}}-1}x^{1-1}=\frac{\Gamma\left[2-e^{-\frac{t}{2}}\right]}{\Gamma\left[1-e^{-\frac{t}{2}}\right]}\times(1-x)^{-e^{-\frac{t}{2}}},x\in[0,1],t\in[0,3]$$

$$f_2(x,t)=\frac{\Gamma\left[1+1-e^{-\frac{t}{3}}\right]}{\Gamma(1)\Gamma\left[1-e^{-\frac{t}{3}}\right]}(1-x)^{1-e^{-\frac{t}{3}}-1}x^{1-1}=\frac{\Gamma\left[2-e^{-\frac{t}{3}}\right]}{\Gamma\left[1-e^{-\frac{t}{3}}\right]}\times(1-x)^{-e^{-\frac{t}{3}}},x\in[0,1],t\in[0,3]$$

Thus the total cost and time for each maintenance option is given in Table 1

Table 1 total cost and time for each maintenance option

Component	Strategy without actions		Capacity recovery		Performance improvement		Replacement	
	Cost	Time	Cost	Time	Cost	Time	Cost	Time
1	0	0	300	30	800	80	1500	150
2	0	0	500	50	1100	120	1600	160

After calculation of average expected capacity under each maintenance strategy,
he total cost constraint is $C_0=2100$ and the total time constraint is $T_0=190$. The optimal maintenance strategy is Do 'capacity recovery' on component 1 and replacement on component 2; the Average expected capacity for next mission with time period of 3 years is 1.1951.

4.2More general model with more complex system

In the previous model, the imperfect maintenance was assumed with only one specific degree. Since the age reduction model was adopted in this project, the age reduction factor was assumed to be fixed, denote as b_0. But generally, b_0 can take any value between 0 and 1, which indicating different effort when doing maintenance. Thus the effective age of the component can be taken all the value from 0, which refer to replacement, to the current clock age t_0, which refer to capacity recovery. Thus, Let s_i denote different maintenance options for each component, the corresponding probability density function, related cost and time can be seen in Table 2:

Table 2 Probability density functions after each maintenance option

s_i	Maintenance Options	Failure Rate Function	Cost	Time
0	Strategy without	$f_i(x,t+t_0),x\in[0,x_i],t\in[0,t_1]$	0	0
1	Capacity recovery	$f_i(x,t+t_0),x\in[0,1],t\in[0,t_1]$	$c_i^f+c_i^r$	$t_i^f+t_i^r$
…	Performance	…	…	…

i	improvement	$f_i(x, t+b\times t_0), x\in[0,1], t\in[0,t_1]$	$c_i^f + c_i^r + (1-b)\times c_i^p$	$t_i^f + t_i^r + (1-b)\times t_i^p$
…		…	…	…
N	Replacement	$f_i(x,t), x\in[0,1], t\in[0,t_1]$	$c_i^f + c_i^r + c_i^p$	$t_i^f + t_i^r + t_i^p$

While the maintenance options increases, the size of the system can also be much larger and complex. If there are M components, each component can be performed N maintenance options, the total combinatorial maintenance strategies for the system can be as much as M^N. Thus, Meta-heuristic Algorithm is possible to search the optimal solution.

4.3 Development of GA approach

In general, the widely used Meta-heuristic Algorithms fall into four major categories, including genetic algorithm (GA), Tabu search, simulated annealing algorithm, and ant colony optimization (ACO). These algorithmes have been proved as efficient and effective approaches to search the global optimal solution (or approximate global optimal solution) of combinational, and non-linear programming problems[15]. There have been some reported works, which apply the Meta-heuristic Algorithms for selective maintenance problems. Liu and Huang [15] formulate the multi-state system using universal generating function method and proposed a Generic Algorithm to solve this complicated optimization problem where both multi-state systems, and imperfect maintenance models are taken into account. Zhu et.al [6][5] studied a cost-based selective maintenance decision-making method for manufacturing system, in which machines are connected in series or in parallel. An algorithm combining the heuristic rules and Tabu search is proposed to solve the presented selective maintenance model.In this project, The Genetic Algorithm (GA) will be selected as the approach to find the optimal solution. GA is the most widely used evolutionary for solving various optimization problems because the convergence can be controlled that the chance of falling into local optimal solution can be rare.
Applying the idea of [15], the solution, which is known as chromosome, is represented by integral string

$$S = \{s_1, s_2, ..., s_i, ..., s_M\}$$

Where s_i is a decimal digit representing the maintenance options for component i. $0 \le s_i \le N$. In reality, the number of performance improvement options N may take any finite number according to the requirement of decision maker.

4.4 Numerical example

In this section, the structure of the system used in [18] is reconsidered. Consider a flow transmission system composed of two independent subsystems connected in series, one containing three components connected in parallel and the other two connected in parallel as shown in Figure 2.

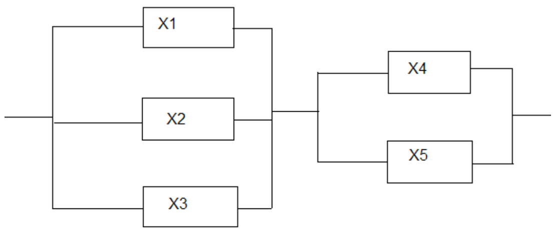

Figure 2 Flow transmission system consist of 5 components

Thus, the structure function for the system is:

$$\phi(X,t) = Min\left[x_1(t) + x_2(t) + x_3(t), x_4(t) + x_5(t)\right]$$

The expected capacity of the systems is

$$E[\phi(X,t)] = Min\left[\int_0^1 xf_1(x,t)dx + \int_0^1 xf_2(x,t)dx + \int_0^1 xf_3(x,t)dx, \int_0^1 xf_4(x,t)dx + \int_0^1 xf_5(x,t)dx\right]$$

The expected capacity for next mission is

$$\frac{\int_0^{t_1} E[\phi(X,t)]}{t_1}$$

Assume that capacity of each component follows a Beta distribution with the following parameters:

$\alpha_i = 1, i \in \{1,2,...,5\}; \beta_i(t) = 1 - e^{-\frac{t}{i}}, i \in \{1,2,...,5\}$

After the previous mission, the current capacity of each component is inspected as

$x_i = 1 - 0.2i, i \in \{1,2,...,5\}$

The cost related for each component is given as follows:

Fixed costs are all the same: $c_i^f = 100, i \in \{1,2,...,5\}$; the costs to restore the capacity are: $c_i^r = 200 \times i, i \in \{1,2,...,5\}$; For the cost to improve the overall deterioration, we assume the age reduction factor can take number from 0 to 1 with increment of 0.1. Assume the costs for replacement are $c_i^p = 1500, i \in \{1,2,...,5\}$, and then the costs are given:

$$c_i^{b_j} = (1-b_j) \times c_i^p = (1-b_j) \times 1500, i \in \{1,2,...,5\}; b_j = 0.1 \times j, j \in \{1,2,...,9\}$$

The time related for each component is given as follows:

Fixed time are all the same: $t_i^f = 10, i \in \{1,2,...,5\}$; the costs to restore the capacity are: $t_i^r = 20 \times i, i \in \{1,2,...,5\}$; For the costs to improve the overall deterioration, assume the age reduction factor can take number from 0 to 1 with increment of 0.1. Assume the time for replacement are $t_i^p = 160, i \in \{1,2,...,5\}$, then the time are given:

$$t_i^{b_j} = (1-b_j) \times t_i^p = (1-b_j) \times 160, i \in \{1,2,...,5\}; b_j = 0.1 \times j, j \in \{1,2,...,9\}$$

The effective age for all the components is $t_0 = 5$ year; the time period for next mission is $t_1 = 4$ year. The total cost constraint is $C_0 = 4000$ and the total time constraint is $T_0 = 450$. The probability distribution after each maintenance options is given:

Strategy without actions:

$$f_i(x,t) = \frac{\dfrac{\Gamma\left[1+1-e^{-\frac{t+t_0}{i}}\right]}{\Gamma(1)\Gamma\left[1-e^{-\frac{t+t_0}{i}}\right]}\left(1-\dfrac{x}{x_i}\right)^{1-e^{-\frac{t+t_0}{i}}-1}\left(\dfrac{x}{x_i}\right)^{1-1}}{x_i} = \frac{\dfrac{\Gamma\left[2-e^{-\frac{t+5}{i}}\right]}{\Gamma\left[1-e^{-\frac{t+5}{i}}\right]}\times\left(1-\dfrac{x}{x_i}\right)^{-e^{-\frac{t+5}{i}}}}{x_i}$$

where $i \in \{1,2,...,5\}; x_i = 1-0.2i, i \in \{1,2,...,5\}; x \in [0, x_i]; t \in [0,4]$

Capacity recovery:

$$f_i(x,t) = \frac{\Gamma\left[1+1-e^{-\frac{t+t_0}{i}}\right]}{\Gamma(1)\Gamma\left[1-e^{-\frac{t+t_0}{i}}\right]}(1-x)^{1-e^{-\frac{t+t_0}{i}}-1}(x)^{1-1} = \frac{\Gamma\left[2-e^{-\frac{t+5}{i}}\right]}{\Gamma\left[1-e^{-\frac{t+5}{i}}\right]}\times(1-x)^{-e^{-\frac{t+5}{i}}}$$

where $i \in \{1,2,...,5\}; x \in [0,1]; t \in [0,4]$

Performance improvement:

$$f_i(x,t) = \frac{\Gamma\left[1+1-e^{-\frac{t+b_j\times t_0}{i}}\right]}{\Gamma(1)\Gamma\left[1-e^{-\frac{t+b_j\times t_0}{i}}\right]}(1-x)^{1-e^{-\frac{t+b_j\times t_0}{i}}-1}(x)^{1-1} = \frac{\Gamma\left[2-e^{-\frac{t+b_j\times 5}{i}}\right]}{\Gamma\left[1-e^{-\frac{t+b_j\times 5}{i}}\right]}\times(1-x)^{-e^{-\frac{t+b_j\times 5}{i}}}$$

where $i \in \{1,2,...,5\}; x \in [0,1]; b_j = 1-0.1\times j, j \in \{1,2,...,9\}; t \in [0,4]$

Replacement:

$$f_i(x,t) = \frac{\Gamma\left[1+1-e^{-\frac{t}{i}}\right]}{\Gamma(1)\Gamma\left[1-e^{-\frac{t}{i}}\right]}(1-x)^{1-e^{-\frac{t}{i}}-1}(x)^{1-1} = \frac{\Gamma\left[2-e^{-\frac{t}{i}}\right]}{\Gamma\left[1-e^{-\frac{t}{i}}\right]}\times(1-x)^{-e^{-\frac{t}{i}}}$$

where $i \in \{1,2,...,5\}; x \in [0,1]; t \in [0,4]$

The Algorithm is implemented through Matlab. Detailed code can be found in Appendix B. It consist of three parts: main function, which calls the original GA in Matlab optimization; fitness function, which calculate the average expected capacity for next 4 years mission; constraint function, which specify the cost and time limitation.

The GA gives optimal answer:

Table 3 Optimal maintenance strategy using GA

Component	Maintenance Options	Total cost	Total time	Expected Average capacity
1	Strategy without actions	4000	410	1.2492
2	Capacity recovery			
3	Strategy without actions			
4	Performance improvement with age reduction factor $b = 0.7$			
5	Performance improvement with age reduction factor $b = 0.3$			

V. CONCLUSION AND FUTURE WORK

This paper focuses on the optimal selective maintenance strategy for flow transmission system. There are many real world systems that can be modeled under this scenario, such as logistic systems during military operations; transportation systems; water distribution systems and etc. Due to many random damages, such as attack from opponents, traffic accident, bad weather or even earthquakes, the capacity of each component is a random variable at ant time point and the expected average capacity of the whole system during each mission is also stochastic. Meanwhile, the distribution of the capacity of each component will experienced a gradual deterioration due to the changing of its physical characters caused by corrosion, continuous loading of material and etc. Thus, its aging process affects the distribution of capacity for each component.

For such system in a multi-phased mission, the selective maintenance is performed under both time and cost constraints. The Beta distribution is adopted to model the behavior of capacity. The average expected capacity of the system is used as an index to evaluate the performance of the system. Four maintenance options, namely Strategy without actions, capacity recovery, performance improvement and replacement are introduced.

As for the analytical example, a parallel system is studied, and the performance improvement scenario takes only one specific level. A more general model is introduced afterwards, where the performance improvement can take several levels from capacity recovery to replacement. A genetic Algorithm is adopted to solve the given numerical example with more complex system. Since the idea implemented in this project has not been studied maturely, there are still many concepts needs to be defined more clearly, or even more close to real application. The future work should be carried as follows: Study a few real world applications that can be modeled as flow transmission system and investigate their property and degradation process; Find a new distribution function or modify the Beta distribution to fit the behavior of real flow transmission component;Define the more realistic maintenance options for flow transmission system;Study the maintenance issue of flow transmission system with both discrete and continuous case.

REFERENCES

[1]. A. Lisnianski. Estimation of boundary points for continuum-state system reliability measures. Reliability Engineering and System Safety. 2001. 74:81-88.

[2]. C. Chen, M. Q. H. Meng, and M. J. Zuo, “Selective maintenance optimization for multi-state systems,” in IEEE Canadian Conference on Electrical and Computer Engineering. 1999. 1477–1482.

[3]. C.R.Cassady, W.P.M., Jr, and E.A.Pohl. “Selective maintenance for support equipment involving multiple maintenance actions,” European Journal of Operational Research. 2001. Vol. 129. No. 2. 252–258.

[4]. Gregory Levitin, Reliability of Multi-State Systems with Two Failure-Modes. IEEE TRANSACTIONS ON RELIABILITY. 2003. VOL. 52. NO. 3.

[5]. Haiping Zhu, Fanmao Liu, Xinyu Shao, Qiong Liu, and Yuhao Deng. A Cost-based Selective Maintenance Decision-making Method for Machining Line. Quality and Reliability Engineering International. 2011. 27:191—201

[6]. Jane, C. -C., Laih, Y. -W. A practical algorithm for computing multi-state two-terminal reliability. IEEE Transaction on Reliability. 2008. 57:295–302.
[7]. Jane, C. -C., Laih, Y. -W. A dynamic bounding algorithm for approximating multi-state two-terminal reliability. European Journal of Operational Research. European Journal of Operational Research. 2010. 205:625–637.
[8]. Jose E. Ramirez-Marquez, David W. Coit. A Monte-Carlo simulation approach for approximating multi-state two-terminal reliability. Reliability Engineering and System Safety. 2005. 87:253–264.
[9]. Lisa M. Maillart, C. Richard Cassady, Chase Rainwater, and Kellie Schneider. Selective Maintenance Decision-Making Over Extended Planning Horizons. 2009. IEEE Transactions On Reliability. Vol. 58. No. 3. 462-469.
[10]. Michael T. Todinov. Analysis and Optimization of Repairable Flow Networks With Complex Topology. IEEE Transactions On Reliability. 2011. Vol. 60, No. 1. 111-124.
[11]. Ming J. Zuo, Renyan Jiang, Richard C.M. Yam. Approaches for reliability modeling of continuous-state devices. IEEE transaction on reliability. 1999. Vol.48. No.1: 9-18.
[12]. M.L. Ga´miz, M.D., Martı´nez Miranda . Regression analysis of the structure functions for reliability evaluation of continuous-state system. Reliability Engineering and System Safety. 2010. 95:134–142.
[13]. R. Rajagopalan and C. R. Cassady. An improved selective maintenance solution approach. Journal of Quality in Maintenance Engineering. 2006. Vol. 12. No. 2. 172–185.
[14]. Russell D. Brunelle, Kailash C. Kapur. Review and classification of reliability measures for multistate and continuum models. IIE Transactions 1999. 31: 1171-1180.
[15]. Sarintip Satitsatian and Kailash C. Kapur. An algorithm for lower reliability bounds of multistate two-terminal networks. IEEE Transactions on Reliability. 2006. Vol. 55, No. 2 199-206.
[16]. T. Lust, O. Roux, F. Riane. Exact and heuristic methods for the selective maintenance problem. European Journal of Operational Research. 2009. 197:1166–1177.
[17]. Wei-Chang Yeh. A simple approach to search for all d-MCs of a limited-flow network. Reliability Engineering and System Safety. 2001. 71:15–19.
[18]. Wei-Chang Yeh. A greedy branch-and-bound inclusion-exclusion algorithm for calculating the exact multi-state network reliability. IEEE transactions on reliability. 2008. Vol. 57, No. 1. 88-93.
[19]. Wei-Chang Yeh, Yi-Cheng Lin, Yuk Ying Chung. Performance analysis of cellular automata Monte Carlo Simulation for estimating network reliability. Expert Systems with Applications. 2010. 37:3537–3544
[20]. Wei-Chang Yeh. Multistate network reliability evaluation under the maintenance cost constraint. International Journal of Production Economics 2004. 88:73–83.
[21]. W. F. Rice, C. R. Cassady, and J. A. Nachlas, “Optimal maintenance plans under limited maintenance time,” in Proceedings of the Seventh Industrial Engineering Research Conference, 1998.
[22]. Yi-Kuei Lin. A simple algorithm for reliability evaluation of a stochastic-flow network with node failure. Computers & Operations Research. 2001. 28:1277-1285.
[23]. Yi-Kuei Lin, Ping-Chen Chang. Maintenance reliability estimation for a cloud-computing network with nodes failure. Expert Systems with Applications 2011. 38:14185–14189.
[24]. Yu Liu and Hong-Zhong Huang. Optimal Selective Maintenance Strategy for Multi-State Systems Under Imperfect Maintenance. 2010. IEEE Transaction on Reliability. Vol.59. N0.2. 356-367.