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International Journal Of Modern Engineering Research (IJMER)

# Semi-Star-Regular Open Sets and Associated Functions

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***ABSTRACT:*** *The aim of this paper is to introduce various functions associated with semi*regular open sets. Here semi*r-continuous, semi*r-irresolute, contra-semi*r-continuous and contra-semi*r-irresolute functions are defined. Characterizations for these functions are given. Further their fundamental properties are investigated. Many other functions associated with semi*regular open sets and their contra versions are introduced and their properties are studied. In addition strongly semi*r-irresolute functions, contra-strongly semi*r-irresolute functions, semi*r-totally continuous, totally semi*r-continuous functions and semi*r-homeomorphisms are introduced and their properties are investigated.*

***Keywords:*** *semi*r-continuous, semi*r-irresolute, semi*regular open, semi*regular closed, pre-semi*regular open function, pre-semi*regular closed function.*

## I. INTRODUCTION

In 1963, Levine [1] introduced the concept of semi-continuity in topological spaces. Dontchev [2] introduced contra-continuous functions. Crossely and Hildebrand [3] defined pre-semi-open functions. Noiri defined and studied semi-closed functions. In 1997, Contra-open and Contra-closed functions were introduced by Baker. Dontchev and Noiri [4] introduced and studied contra-semi-continuous functions in topological spaces. Caldas [5] defined Contra-pre-semi-closed functions and investigated their properties. S.Pasunkili Pandian [6] defined semi*-pre-continuous and semi*-pre-irresolute functions and their contra versions and investigated their properties. Quite recently, the authors [7] introduced some new concepts, namely semi*regular open sets, semi*regular closed sets, semi*r-Interior, semi*r-Closure of a subset. In this paper various functions associated with semi*regular open sets are introduced and their properties are investigated.

**Preliminaries**

Throughout this paper $X$, $Y$ and $Z$ will always denote topological spaces on which no separation axioms are assumed.

**Definition 2.1**[8]**:** A subset $A$ of a topological space $(X, \tau)$ is called (i) generalized closed (briefly g-closed) if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open .

(ii) generalized open (briefly g-open) if $X\backslash A$ is g-closed in $X$.

**Definition 2.2:** Let $A$ be a subset of $X$. Then (i) generalized closure[9] of $A$ is defined as the intersection of all g-closed sets containing $A$ and is denoted by $Cl^*(A)$.

(ii) generalized interior of $A$ is defined as the union of all g-open subsets of $A$ and is denoted by $Int^*(A)$.

**Definition 2.3:** A subset $A$ of a topological space $(X, \tau)$ is (i) semi-open [1] (resp. α-open[6], semi α-open[6], semi-preopen[12], semi*open, semi*α-open[6], semi*-preopen[12]) if $A\subseteq Cl(Int(A))$ (resp. $A\subseteq Int(Cl(Int(A))$, $A\subseteq Cl(Int(Cl(Int(A)))$, $A\subseteq Cl(Int(Cl(A)))$, $A\subseteq Cl^*(Int(A))$, $A\subseteq Cl^*(\alpha Int(A)$, $A\subseteq Cl^*(pInt(A))$, ) (ii) semi-closed (resp. α-closed[6], semi α-closed[6],

semi-preclosed[12], semi*-closed, semi*α-closed[8], semi*-preclosed[6]) if $Int(Cl(A))\subseteq A$

(resp. $Cl(Int(Cl(A)))\subseteq A$, $Int(Cl(Int(Cl(A)) \subseteq A$, $Int^*(Cl(A))\subseteq A$, $Int^*(\alpha Cl(A))\subseteq A$, $Int^*(pCl(A))\subseteq A$).

(iii) semi*regular open [7] if $A=Cl^*(rInt(A)$ and semi*regular closed if $Int^*(rcl(A))=A$

**Definition 2.4:** Let $A$ be a subset of $X$. Then

(i)The semi*r-interior [7] of $A$ is defined as the union of all semi*regular open subsets of $A$ and is denoted by $s^*rInt(A)$.

(ii) The semi*r-closure [7] of $A$ is defined as the intersection of all semi*α-closed sets containing $A$ and is denoted by $s^*rCl(A)$.

**Definition 2.5:** A function $f : X\longrightarrow Y$ is said to be semi-continuous [1] (resp. contra-semi-continuous [4], semi*-continuous, contra-semi*-continuous, semi α-continuous [6] ) if $f^{-1}(V)$ is semi-open (resp. semi-closed, semi*-open, semi*-closed, semi α-open) in $X$ for every open set $V$ in $Y$.

**Definition 2.6:** A function $f:X\longrightarrow Y$ is said to be r-continuous (resp. r*-continuous, semi-pre-continuous [14], semi*-pre-continuous [6]) if $f^{1}(V)$ is regular open(resp. r*-open, semi-preopen, semi*-preopen) in $X$ for every open set $V$ in $Y$.
**Definition 2.7:** A topological space $X$ is said to be
(i) $T_{1/2}$ if every g-closed set in $X$ is closed.
(ii) locally indiscrete if every open set is closed.
(iii)Extremely disconnected if closure of an open set is open.
**Theorem 2.8:**[7]
**(i)** Every Semi*regular open set is Semi*$\alpha$-open.
**(ii)** Every Semi*regular open set is Semi*pre-open.
**(iii)** Every Semi*regular open set is Semi*open.
**(iv)** Every Semi* regular open set is Semi open.
**(v)** Every Semi*regular open set is Semi $\alpha$-open.
**(vi)** Every Semi*regular open set is Semi pre-open.
**(vii)** Every Semi*regular open set is regular generalized open set.
**(viii)**Every Semi*regular open set is generalized pre regular open set.
**(ix)** Every Semi*regular open is regular weakly generalized open set.

**Remark 2.9**:[7] Similar results for semi*regular closed sets are also true.
**Theorem 2.10:** [7] (i) Arbitrary union of semi*regular open sets is also semi*regular open.
**(ii)** If $A$ is semi*regular open in $X$ and $B$ is open in $X$, then $A\cup B$ is semi*regular open in $X$.
**(iii)**A subset $A$ of a space X is semi*regular open if and only if $s^*rInt(A)=A$.

**Theorem 2.11:** [7] For a subset $A$ of a space X the following are equivalent:
**(i)** $A$ is semi*regular open in X.
**(ii)**$A=Cl^*(\mathrm{r}Int(A))$.
**(iii)**$Cl^*(\mathrm{r}Int(A))=Cl^*(A)$.

**Theorem 2.12:** For a subset $A$ of a space X the following are equivalent:
**(i)**$A$ is semi*regular closed in X.
**(ii)**$Int^*(\mathrm{r}Cl(A))=A$.
**(iii)**$Int^*(\mathrm{r}Cl(A))=Int^*(A)$.

**Theorem 2.13:** [7] **(i)** A subset $A$ of a space X is semi*regular closed if and only if $s^*rCl(A)=A$.
**(ii)**Let $A\subseteq X$ and let $x\in X$. Then $x\in s^*rCl(A)$ if and only if every semi*regular open set in $X$ containing $x$ intersects $A$.

**Definition 2.14:** If $A$ is a subset of $X$, the semi*r-Frontier of $A$ is defined by
$\mathrm{s^*rFr}(A)=s^*rCl(A)\setminus s^*rInt(A)$.

**Result 2.15:** If $A$ is a subset of $X$, then $\mathrm{s^*rFr}(A)=s^*rCl(A)\cap s^*rCl(X\setminus A)$.

## II. SEMI*r-CONTINUOUS FUNCTIONS

In this section we define the semi*r-continuous and contra-semi*r-continuous functions and investigate their fundamental properties.
**Definition 3.1:** A function $f:X\longrightarrow Y$ is said to be semi*r-continuous at $x\in X$ if for each open set $V$ of $Y$ containing $f(x)$, there is a semi*regular open set $U$ in $X$ such that $x\in U$ and $f(U)\subseteq V$.
**Definition 3.2:** A function $f:X\longrightarrow Y$ is said to be semi*r-continuous if $f^{-1}(V)$ is semi*regular open in $X$ for every open set $V$ in $Y$.
**Theorem 3.3:** Let $f:X\longrightarrow Y$ be a function. Then the following statements are equivalent:
**(i)** $f$ is semi*r-continuous.
**(ii)** $f$ is semi*r-continuous at each point $x\in X$.
**(iii)** $f^{-1}(F)$ is semi*regular closed in $X$ for every closed set $F$ in $Y$.
**(iv)** $f(s^*rCl(A))\subseteq Cl(f(A))$ for every subset $A$ of $X$.
**(v)** $s^*rCl(f^{-1}(B))\subseteq f^{-1}(Cl(B))$ for every subset $B$ of $Y$.
**(vi)** $f^{-1}(Int(B))\subseteq s^*rInt(f^{-1}(B))$ for every subset $B$ of $Y$.

**(vii)** $Int^*(rCl(f^{-1}(F)))=Int^*(f^{-1}(F))$ for every closed set $F$ in $Y$.
(viii) $Cl^*(rInt(f^{-1}(V)))=Cl^*(f^{-1}(V))$ for every open set $V$ in $Y$.
**Proof: (i)⇒(ii):** Let $f:X\longrightarrow Y$ be semi*r-continuous. Let $x\in X$ and $V$ be an open set in $Y$ containing $f(x)$. Then $x\in f^{-1}(V)$. Since $f$ is semi*r-continuous, $U=f^{-1}(V)$ is a semi*regular open set in $X$ containing $x$ such that $f(U)\subseteq V$.
**(ii)⇒(i):** Let $f:X\longrightarrow Y$ be semi*r-continuous at each point of $X$. Let V be an open set in Y.
Let $x\in f^{-1}(V)$. Then V is an open set in Y containing $f(x)$. By (ii), there is a semi*regular open set $Ux$ in $X$ containing $x$ such that $f(x)\in f(Ux)\subseteq V$. (ie) $Ux\subseteq f^{-1}(V)$. Hence $f^{-1}(V)=\cup\{Ux : x\in f^{-1}(V)\}$.
By Theorem 2.10(i), $f^{-1}(V)$ is semi*regular open in $X$.
**(i)⇒(iii):** Let $F$ be a closed set in $Y$. Then $V=Y\backslash F$ is open in $Y$. Then $f^{-1}(V)$ is semi*regular open in X. Therefore $f^{-1}(F)=f^{-1}(Y\backslash V)=X\setminus f^{-1}(V)$ is semi*regular closed.
**(iii)⇒(i):** Let V be an open set in Y. Then F=Y\V is closed. By (iii), $f^{-1}(F)$ is semi*regular closed. Hence $f^{-1}(V)=f^{-1}(Y\backslash F)=X\backslash f^{-1}(F)$ is semi*regular open in $X$.
**(iii)⇒(iv):** Let $A\subseteq X$. Let $F$ be a closed set containing $f(A)$. Then by (iii), $f^{-1}(F)$ is a semi*regular closed set containing $A$. This implies that $s^*rCl(A)\subseteq f^{-1}(F)$ and hence $f(s^*rCl(A))\subseteq F$.
**(iv)⇒(v):** Let $B\subseteq Y$ and let $A=f^{-1}(B)$. By assumption, $f(s^*rCl(A))\subseteq Cl(f(A))\subseteq Cl(B)$. This implies that $s^*rCl(A)\subseteq f^{-1}(Cl(B))$.
**(v)⇒(iii):** Let B be closed in Y. Then $Cl(B)=B$. Therefore (v) implies $s^*rCl(f^{-1}(B))\subseteq f^{-1}(B)$. Hence $s^*rCl(f^{-1}(B))=f^{-1}(B)$. By Theorem 2.13(i), $f^{-1}(B)$ is semi*regular closed.
**(v)⇔(vi):** The equivalence of (v) and (vi) can be proved by taking the complements.
**(vii)⇔(iii):** Follows from Theorem 2.12.
**(viii)⇔(i):** Follows from Theorem 2.11.

**Theorem 3.4: (i)** Every Semi*r-continuous function is semi*$\alpha$-continuous function.
**(ii)** Every Semi*r-continuous function is semi*pre-continuous function.
**(iii)** Every Semi*r-continuous function is semi*continuous function.
**(iv)** Every Semi*r-continuous function is semi$\alpha$-continuous function.
**(v)** Every Semi*r-continuous function is semi pre-continuous function.
**Proof:** Follows from Theorem 2.8
**Remark 3.5:** In general the converse of each of the statements in Theorem 3.4 is not true.

**Theorem 3.6:** If the topology of the space $Y$ is given by a basis B, then a function $f:X\longrightarrow Y$ is semi*r-continuous if and only if the inverse image of every basic open set in $Y$ under $f$ is semi*regular open in $X$.
**Proof:** Suppose $f:X\longrightarrow Y$ is semi*r-continuous. Then inverse image of every open set in $Y$ is semi*regular open in $X$. In particular, inverse image of every basic open set in $Y$ is semi*regular open in $X$. Conversely, let $V$ be an open set in $Y$. Then $V=\cup B_i$ where $B_i\in$B.
Now $f^{-1}(V)=f^{-1}(\cup B_i)=\cup f^{-1}(B_i)$. By hypothesis, $f^{-1}(B_i)$ is semi*regular open for each i.
By Theorem 2.10(i), $f^{-1}(V)=\cup f^{-1}(B_i)$ is semi*regular open. Hence $f$ is semi*r-continuous.

**Theorem 3.7:** A function $f:X\longrightarrow Y$ is not semi*r-continuous at point $x\in X$ if and only if $x$ belongs to the semi*r-frontier of the inverse image of some open set in $Y$ containing $f(x)$.
**Proof:** Suppose $f$ is not semi*r-continuous at $x$. Then by Definition 3.1, there is an open set $V$ in $Y$ containing $f(x)$ such that $f(U)$ is not a subset of $V$ for every semi*regular open set $U$ in $X$ containing $x$. Hence $U\cap(X\backslash f^{-1}(V))\neq\emptyset$ for every semi*regular open set $U$ containing $x$. By Theorem 2.13(ii), we get $x\in s^*rCl(X\backslash f^{-1}(V))$. Also $x\in f^{-1}(V)\subseteq s^*rCl(f^{-1}(V))$.
Hence $x\in s^*rCl(f^{-1}(V))\cap s^*rCl(X\backslash f^{-1}(V))$. By the Result 2.15, $x\in s^*rFr(f^{-1}(V))$. On the other hand, let $f$ be semi*r-continuous at $x\in X$. Let $V$ be any open set in $Y$ containing $f(x)$. Then there exists a semi*regular open set $U$ in $X$ containing $x$ such that $f(U)\subseteq V$. That is, $U$ is a semi*regular open set in $X$ containing $x$ such that $U\subseteq f^{-1}(V)$. Hence $x\in s^*rInt(f^{-1}(V))$. Therefore by Definition 2.14, $x\notin s^*rFr(f^{-1}(V))$.

**Theorem 3.8:** Let $f:X\longrightarrow\Pi X_\alpha$ be semi*r-continuous where $\Pi X_\alpha$ is given the product topology and $f(x)=(f_\alpha(x))$. Then each co-ordinate function $f_\alpha:X\longrightarrow X_\alpha$ is semi*r-continuous.
**Proof:** Let $V$ be an open set in $X\alpha$. Then $f_\alpha^{-1}(V)=(\pi_\alpha\circ f)^{-1}(V)=f^{-1}(\pi_\alpha^{-1}(V))$, where
$\pi_\alpha:\Pi X_\alpha\longrightarrow X_\alpha$ is the projection map. Since each $\pi_\alpha$ is continuous, $\pi_\alpha^{-1}(V)$ is open in $\Pi X_\alpha$.
By the semi*r-continuity of $f$, $f_\alpha^{-1}(V)=f^{-1}(\pi_\alpha^{-1}(V))$ is semi*regular open in $X$. Therefore $f_\alpha$ is semi*r- continuous.

**Theorem 3.9:** Let $f:X\longrightarrow\Pi X_\alpha$ be defined by $f(x)=(f_\alpha(x))$ and $\Pi X_\alpha$ be given the product topology. Suppose S*RO($X$) is closed under finite intersection. Then $f$ is semi*r-continuous if each coordinate function $f_\alpha:X\longrightarrow X_\alpha$ is semi*r-continuous.

**Proof:** Let $V$ be a basic open set in $\Pi X\alpha$. Then $V=\cap\pi_\alpha^{-1}(V\alpha)$ where each $V_\alpha$ is open in $X_\alpha$, the intersection being taken over finitely many α's. Now $f^{-1}(V)= f^{-1}(\cap\pi_\alpha^{-1}(V_\alpha)) =\cap(f^{-1}(\pi_\alpha^{-1}(V\alpha)))= \cap(\pi_\alpha\circ f)^{-1}(V)=\cap f_\alpha^{-1}(V)$ is semi*regular open, by hypothesis. Hence by Theorem 3.6, $f$ is semi*r-continuous.

**Theorem 3.10:** Let $f:X\longrightarrow Y$ be continuous and $g:X\longrightarrow Z$ be semi*r-continuous. Let $h:X\longrightarrow Y\times Z$ be defined by $h(x)=(f(x),g(x))$ and $Y\times Z$ be given the product topology. Then $h$ is semi*r-continuous.

**Proof:** By virtue of Theorem 3.6, it is sufficient to show that inverse image under $h$ of every basic open set in $Y\times Z$ is semi*regular open in $X$. Let $U\times V$ be a basic open set in $Y\times Z$. Then
$h^{-1}(U\times V) = f^{-1}(U)\cap g^{-1}(V)$. By continuity of $f$, $f^{-1}(U)$ is open in $X$ and by semi*r-continuity of $g$, $g^{-1}(V)$ is semi*regular open in $X$. By Theorem 2.10(ii), we get $h^{-1}(U\times V) = f^{-1}(U)\cap g^{-1}(V)$ is semi*regular open.

**Remark 3.11:** The above theorem is true even if $f$ is semi*r-continuous and $g$ is continuous.

**Theorem 3.12:** Let $f: X\longrightarrow Y$ be semi*r-continuous and $g: Y\longrightarrow Z$ be continuous.
Then $g\circ f: X\longrightarrow Z$ is semi*r-continuous.

**Proof:** Let $V$ be an open set in $Z$. Since $g$ is continuous, $g^{-1}(V)$ is open in $Y$. By semi*r-continuity of $f$, $(g\circ f)^{-1}(V)= f^{-1}(g^{-1}(V))$ is semi*regular open in $X$. Hence $g\circ f$ is semi*r-continuous.

**Remark 3.13:** Composition of two semi*r-continuous functions need not be semi*r-continuous.

**Definition 3.14:** A function $f:X\longrightarrow Y$ is called contra-semi*r-continuous if $f^{-1}(V)$ is semi*regular closed in $X$ for every open set $V$ in $Y$.

**Theorem 3.15:** For a function $f:X\longrightarrow Y$, the following are equivalent:
(i) $f$ is contra-semi*r-continuous.
(ii) For each $x\in X$ and each closed set $F$ in $Y$ containing $f(x)$, there exists a semi*regular open set $U$ in $X$ containing $x$ such that $f(U)\subseteq F$.
(iii)The inverse image of each closed set in $Y$ is semi*regular open in $X$.
(iv)$Cl^*(rInt(f^{-1}(F)))=Cl^*(f^{-1}(F))$ for every closed set $F$ in $Y$.
(v)$Int^*(rCl(f^{-1}(V)))=Int^*(f^{-1}(V))$ for every open set $V$ in $Y$.

**Proof: (i)⇒(ii):** Let $f:X\longrightarrow Y$ be contra-semi*r-continuous. Let $x\in X$ and $F$ be a closed set in $Y$ containing $f(x)$. Then $V=Y\backslash F$ is an open set in $Y$ not containing $f(x)$. Since $f$ is contra-semi*r-continuous, $f^{-1}(V)$ is a semi*regular closed set in $X$ not containing $x$. That is, $f^{-1}(V)= X\backslash f^{-1}(F)$ is a semi*regular closed set in $X$ not containing $x$. Therefore $U=f^{-1}(F)$ is a semi*regular open set in $X$ containing $x$ such that $f(U)\subseteq F$.

**(ii)⇒(iii):** Let $F$ be a closed set in $Y$. Let $x\in f^{-1}(F)$, then $f(x)\in F$. By (ii), there is a semi*regular open set $Ux$ in $X$ containing $x$ such that $f(x)\in f(Ux)\subseteq F$. That is, $x\in Ux\subseteq f^{-1}(F)$. Therefore $f^{-1}(F)=\cup\{Ux\subseteq f^{-1}(F)\}$. By Theorem 2.10(i), $f^{-1}(F)$ is semi*regular open in $X$.

**(iii)⇒(iv):** Let $F$ be a closed set in $Y$. By (iii), $f^{-1}(F)$ is a semi*regular open set in $X$. By Theorem 2.11, $Cl^*(rInt(f^{-1}(F)))= Cl^*(f^{-1}(F))$.

**(iv)⇒(v):** If $V$ is any open set in $Y$, then $Y\backslash V$ is closed in $Y$. By (iv), we have $Cl^*(rInt(f^{-1}(Y\backslash V))) =Cl^*(f^{-1}(Y\backslash V))$. Taking the complements, we get $Int^*(rCl(f^{-1}(V)))=Int^*(f^{-1}(V))$.

**(v)⇒(i):** Let $V$ be any open set in $Y$. Then by assumption, $Int^*(rCl(f^{-1}(V)))=Int^*(f^{-1}(V))$. By Theorem 2.12, $f^{-1}(V)$ is semi*regular closed .

**Theorem 3.16:** Every contra-semi*r-continuous function is contra-semi r-continuous.

**Proof:** Let $f:X\longrightarrow Y$ be contra-semi*r-continuous. Let $V$ be an open set in $Y$. Since $f$ is contra-semi*r-continuous, $f^{-1}(V)$ is semi*regular closed in $X$. By Remark 2.9, $f^{-1}(V)$ is semi*regular closed in $X$. Hence $f$ is contra-semi-r-continuous.

**Remark 3.17:** It can be easily seen that the converse of the above theorem is not true.

## III. SEMI*r-IRRESOLUTE FUNCTIONS

In this section we define the semi*r-irresolute and contra-semi*r-irresolute functions and investigate their fundamental properties.

**Definition 4.1:** A function $f:X\longrightarrow Y$ is said to be semi*r-irresolute at $x\in X$ if for each semi*regular open set $V$ of $Y$ containing $f(x)$, there is a semi*regular open set $U$ of $X$ such that $x\in U$ and $f(U)\subseteq V$.

**Definition 4.2:** A function $f:X\longrightarrow Y$ is said to be semi*r-irresolute if $f^{-1}(V)$ is semi*regular open in $X$ for every semi*regular open set $V$ in $Y$.

**Definition 4.3:** A function $f:X\longrightarrow Y$ is said to be contra-semi*r-irresolute if $f^{-1}(V)$ is semi*regular closed in $X$ for every semi*regular open set $V$ in $Y$.
**Definition 4.4:** A function $f:X\longrightarrow Y$ is said to be strongly semi*r-irresolute if $f^{-1}(V)$ is open in $X$ for every semi*regular open set $V$ in $Y$.
**Definition 4.5:** A function $f:X\longrightarrow Y$ is said to be contra-strongly semi*r-irresolute if $f^{-1}(V)$ is closed in $X$ for every semi*regular open set $V$ in $Y$.

**Theorem 4.6:** Every semi*r-irresolute function is semi*r-continuous.
**Proof:** Let $f:X\longrightarrow Y$ be semi*r-irresolute. Let $V$ be open in $Y$. Then by Theorem 2.8(ii), $V$ is semi*regular open. Since $f$ is semi*r-irresolute, $f^{-1}(V)$ is semi*regular open in $X$. Thus $f$ is semi*r-continuous.
**Theorem 4.7:** Every constant function is semi*r-irresolute.
**Proof:** Let $f:X\longrightarrow Y$ be a constant function defined by $f(x)=y_0$ for all $x$ in $X$, where $y_0$ is a fixed point in $Y$. Let $V$ be a semi*regular open set in $Y$. Then $f^{-1}(V)=X$ or $\phi$ according as $y_0\in V$or $y_0\notin V$. Thus $f^{-1}(V)$ is semi*regular open in $X$. Hence $f$ is semi*r-irresolute.

**Theorem 4.8:** Let $f:X\longrightarrow Y$ be a function. Then the following are equivalent:
**(i)** $f$ is semi*r-irresolute.
**(ii)** $f$ is semi*r-irresolute at each point of $X$.
**(iii)** $f^{-1}(F)$ is semi*regular closed in $X$ for every semi*regular closed set $F$ in $Y$.
**(iv)** $f(s*rCl(A))\subseteq s*rCl(f(A))$ for every subset $A$ of $X$.
**(v)** $s*rCl(f^{-1}(B))\subseteq f^{-1}(s*rCl(B))$ for every subset $B$ of $Y$.
**(vi)** $f^{-1}(s*rInt(B))\subseteq s*rInt(f^{-1}(B))$ for every subset $B$ of $Y$.
**(vii)** $Int^*(rCl(f^{-1}(F)))=Int^*(f^{-1}(F))$ for every semi*regular closed set $F$ in $Y$.
**(viii)** $Cl^*(rInt(f^{-1}(V)))=Cl^*(f^{-1}(V))$ for every semi*regular open set $V$ in $Y$.
**Proof: (i)⇒(ii):** Let $f:X\longrightarrow Y$ be semi*r-irresolute. Let $x\in X$ and $V$ be a semi*regular open set in $Y$ containing $f(x)$. Then $x\in f^{-1}(V)$. Since $f$ is semi*r-irresolute, $U=f^{-1}(V)$ is a semi*regular open set in $X$ containing $x$ such that $f(U)\subseteq V$.
**(ii)⇒(i):** Let $f:X\longrightarrow Y$ be semi*r-irresolute at each point of $X$. Let V be a semi*regular open set in Y. Let $x\in f^{-1}(V)$. Then V is a semi*regular open set in Y containing $f(x)$. By (ii), there is a semi*regular open set $Ux$ in $X$ containing $x$ such that $f(x)\in f(U_x)\subseteq V$. Therefore $U_x\subseteq f^{-1}(V)$. Hence $f^{-1}(V)=\cup\{U_x : x\in f^{-1}(V)\}$. By Theorem 2.10(i), $f^{-1}(V)$ is semi*regular open in $X$.
**(i)⇒(iii):** Let $F$ be a semi*regular closed set in $Y$. Then $V=Y\backslash F$ is semi*regular open in $Y$. Then $f^{-1}(V)$ is semi*regular open in X. Therefore $f^{-1}(F)=f^{-1}(Y\backslash V)=X\backslash f^{-1}(V)$ is semi*regular closed.
**(iii)⇒(i):** Let V be a semi*regular open set in Y. Then F=Y\V is semi*regular closed. By (iii), $f^{-1}(F)$ is semi*regular closed. Hence $f^{-1}(V)=f^{-1}(Y\backslash F)=X\backslash f^{-1}(F)$ is semi*regular open in $X$.
**(iii)⇒(iv):** Let $A\subseteq X$. Let $F$ be a semi*regular closed set containing $f(A)$. Then by (iii), $f^{-1}(F)$ is a semi*regular closed set containing $A$. This implies that $s*rCl(A)\subseteq f^{-1}(F)$ and hence $f(s*rCl(A))\subseteq F$. Therefore $f(s*rCl(A))\subseteq s*rCl(f(A))$.
**(iv)⇒(v):** Let $B\subseteq Y$ and let $A=f^{-1}(B)$. By assumption, $f(s*rCl(A))\subseteq s*rCl(f(A))\subseteq s*rCl(B)$. This implies that $s*rCl(A)\subseteq f^{-1}(s*rCl(B))$. Hence $s*rCl(f^{-1}(B))\subseteq f^{-1}(s*rCl(B))$.
**(v)⇒(iii):** Let F be semi*regular closed set in Y. Then $s*rCl(F)=F$. Therefore (v) implies $s*rCl(f^{-1}(F))\subseteq f^{-1}(s*rCl(F))$. Hence $s*rCl(f^{-1}(F))=f^{-1}(F)$. By Theorem 2.13(i), $f^{-1}(F)$ is semi*regular closed.
**(v)⇔(vi):** The equivalence of (v) and (vi) can be proved by taking the complements.
**(vii)⇔(iii):** Follows from Theorem 2.12.
**(viii)⇔(i):** Follows from Theorem 2.11.

**Theorem 4.9:** Let $f:X\longrightarrow Y$ be a function. Then $f$ is not semi*r-irresolute at a point $x$ in $X$ if and only if $x$ belongs to the semi*r-frontier of the inverse image of some semi*regular open set in $Y$ containing $f(x)$.
**Proof:** Suppose $f$ is not semi*r-irresolute at $x$. Then by Definition 4.1, there is a semi*regular open set $V$ in $Y$ containing $f(x)$ such that $f(U)$ is not a subset of $V$ for every semi*regular open set $U$ in $X$ containing $x$. Hence $U\cap(X\backslash f^{-1}(V))\neq\phi$ for every semi*regular open set $U$ containing $x$. Thus $x\in s*rCl(X\backslash f^{-1}(V))$.Since $x\in f^{-1}(V)\subseteq s*rCl(f^{-1}(V))$, we have $x\in s*rCl(f^{-1}(V))\cap s*rCl(X\backslash f^{-1}(V))$. Hence by Theorem 2.15, $x\in s*rFr(f^{-1}(V))$. On the other hand, let $f$ be semi*r-irresolute at $x$. Let $V$ be a semi*regular open set in $Y$ containing $f(x)$. Then there is a semi*regular open set $U$ in $X$ containing $x$ such that $f(x)\in f(U)\subseteq V$. Therefore $U\subseteq f^{-1}(V)$. Hence $x\in s*rInt(f^{-1}(V))$. Therefore by Definition 2.14, $x\notin s*rFr(f^{-1}(V))$ for every semi*regular open set $V$ containing $f(x)$.

**Theorem 4.10:** Every contra-semi*r-irresolute function is contra-semi*r-continuous.
**Proof:** Let $f:X \longrightarrow Y$ be a contra-semi*r-irresolute function. Let $V$ be an open set in $Y$. Then $V$ is semi*regular open in $Y$. Since $f$ is contra-semi*r-irresolute, $f^{-1}(V)$ is semi*regular closed in $X$. Hence $f$ is contra-semi*r-continuous.
**Theorem 4.11:** For a function $f:X \longrightarrow Y$, the following are equivalent:
(i) $f$ is contra-semi*r-irresolute.
(ii) The inverse image of each semi*regular closed set in $Y$ is semi*regular open in $X$.
(iii) For each $x \in X$ and each semi*regular closed set $F$ in $Y$ with $f(x) \in F$, there exists a semi*regular open set $U$ in $X$ such that $x \in U$ and $f(U) \subseteq F$.
(iv) $Cl^*(\mathrm{r}Int(f^{-1}(F)))=Cl^*(f^{-1}(F))$ for every semi*regular closed set $F$ in $Y$.
(v) $Int^*(\mathrm{r}Cl(f^{-1}(V)))=Int^*(f^{-1}(V))$ for every semi*regular open set $V$ in $Y$.
**Proof:(i)⇒(ii):** Let $F$ be a semi*regular closed set in $Y$. Then $Y \setminus F$ is semi*regular open in $Y$. Since $f$ is contra-semi*r-irresolute, $f^{-1}(Y \setminus F)= X \setminus f^{-1}(F)$ is semi*regular closed in $X$.
**(ii)⇒(iii):** Let $F$ be a semi*regular closed set in $Y$ containing $f(x)$.Then $U= f^{-1}(F)$ is a semi*regular open set containing $x$ such that $f(U) \subseteq F$.
**(iii)⇒(iv):** Let $F$ be a semi*regular closed set in $Y$ and $x \in f^{-1}(F)$, then $f(x) \in F$. By assumption, there is a semi*regular open set $U_x$ in $X$ containing $x$ such that $f(x) \in f(U_x) \subseteq F$ which implies that $x \in U_x \subseteq f^{-1}(F)$. This follows that $f^{-1}(F)= \cup\{U_x : x \in f^{-1}(F)\}$. By Theorem 2.10(i), $f^{-1}(F)$ is semi*regular open in $X$. By Theorem 2.11, $Cl^*(\mathrm{r}Int(f^{-1}(F)))=Cl^*(f^{-1}(F))$.
**(iv)⇒(v):** Let $V$ be a semi*regular open set in $Y$. Then $Y \setminus V$ is semi*regular closed in $Y$. By assumption, $Cl^*(\mathrm{r}Int(f^{-1}(Y \setminus V)))= Cl^*(f^{-1}(Y \setminus V))$. Taking the complements we get,
$Int^*(\mathrm{r}Cl(f^{-1}(V)))=Int^*(f^{-1}(V))$.
**(v)⇒(i):**Let $V$ be any semi*regular open set in $Y$. Then by assumption,
$Int^*(\mathrm{r}Cl(f^{-1}(V)))=Int^*( f^{-1}(V))$. By Theorem 2.12, $f^{-1}(V)$ is semi*regular closed in $X$.

**Theorem 4.12:** (i) Every strongly semi*r-irresolute function is semi*r-irresolute and hence semi*r-continuous.
**Proof:** Let $f:X \longrightarrow Y$ be strongly semi*r-irresolute. Let $V$ be semi*regular open in $Y$. Since $f$ is strongly semi*r-irresolute, $f^{-1}(V)$ is open in $X$. Then $f^{-1}(V)$ is semi*regular open. Therefore $f$ is semi*r-irresolute. Hence by Theorem 4.6, $f$ is semi*r-continuous.
**Theorem 4.13:** Every constant function is strongly semi*r-irresolute.
**Proof:** Let $f:X \longrightarrow Y$ be a constant function defined by $f(x)=y_0$ for all $x$ in $X$, where $y_0$ is a fixed point in $Y$. Let $V$ be a semi*regular open set in $Y$. Then $f^{-1}(V)=X$ or $\phi$ according as $y_0 \in V$or $y_0 \notin V$. Thus $f^{-1}(V)$ is open in $X$. Hence $f$ is strongly semi*r-irresolute.

**Theorem 4.14:** Let $f:X \longrightarrow Y$ be a function. Then the following are equivalent:
(i) $f$ is strongly semi*r-irresolute.
(ii) $f^{-1}(F)$ is closed in $X$ for every semi*regular closed set $F$ in $Y$.
(iii) $f(Cl(A)) \subseteq s^*rCl(f(A))$ for every subset $A$ of $X$.
(iv)$Cl(f^{-1}(B)) \subseteq f^{-1}(s^*rCl(B))$ for every subset $B$ of $Y$.
(v) $f^{-1}(s^*rInt(B)) \subseteq Int(f^{-1}(B))$ for every subset $B$ of $Y$.
**Proof: (i)⇒(ii):** Let $F$ be a semi*regular closed set in $Y$. Then $V=Y \setminus F$ is semi*regular open in $Y$. Then $f^{-1}(V)$ is open in X. Therefore $f^{-1}(F)=f^{-1}(Y \setminus V)=X \setminus f^{-1}(V)$ is closed.
**(ii)⇒(i):** Let V be a semi*regular open set in Y. Then F=Y\V is semi*regular closed.
By (ii), $f^{-1}(F)$ is closed in X. Hence $f^{-1}(V)=f^{-1}(Y \setminus F)=X \setminus f^{-1}(F)$ is open in $X$. Therefore $f$ is strongly semi*r-irresolute.
**(ii)⇒(iii):** Let $A \subseteq X$. Let $F$ be a semi*regular closed set containing $f(A)$. Then by (ii),
$f^{-1}(F)$ is a closed set containing $A$. This implies that $Cl(A) \subseteq f^{-1}(F)$ and hence $f(Cl(A)) \subseteq F$. Therefore $f(Cl(A)) \subseteq s^*\mathrm{r}Cl(f(A))$.
**(iii)⇒(iv):** Let $B \subseteq Y$ and let $A= f^{-1}(B)$. By assumption, $f(Cl(A)) \subseteq s^*\mathrm{r}Cl(f(A)) \subseteq s^*\mathrm{r}Cl(B)$. This implies that $Cl(A) \subseteq f^{-1}(s^*\mathrm{r}Cl(B))$.
**(iv)⇒(ii):** Let F be semi*regular closed set in Y. Then by Theorem2.13(i), $s^*\mathrm{r}Cl(F)= F$. Therefore (iv) implies $Cl(f^{-1}(F)) \subseteq f^{-1}(F)$. Hence $Cl(f^{-1}(F))=f^{-1}(F)$. Therefore $f^{-1}(F)$ is closed.
**(iv)⇔(v):** The equivalence of (iv) and (v) follows from taking the complements.

**Theorem 4.15:** For a function $f:X \longrightarrow Y$, the following are equivalent:
(i) $f$ is contra-strongly semi*r-irresolute.
(ii)The inverse image of each semi*regular closed set in $Y$ is open in $X$.

(iii)For each $x \in X$ and each semi*regular closed set $F$ in $Y$ with $f(x) \in F$, there exists a open set $U$ in $X$ such that $x \in U$ and $f(U) \subseteq F$.
**Proof:(i)⇒(ii):** Let $F$ be a semi*regular closed set in $Y$. Then $Y \setminus F$ is semi*regular open in $Y$. Since $f$ is contra-strongly semi*r-irresolute, $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$ is closed in $X$. Hence $f^{-1}(F)$ is open in $X$. This proves (ii).
**(ii)⇒(i):** Let $U$ be a semi*regular open set in $Y$. Then $Y \setminus U$ is semi*regular closed in $Y$. By assumption, $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is open in $X$. Hence $f^{-1}(U)$ is closed in $X$.
**(ii)⇒(iii):** Let $F$ be a semi*regular closed set in $Y$ containing $f(x)$.Then $U = f^{-1}(F)$ is an open set containing $x$ such that $f(U) \subseteq F$.
**(iii)⇒(ii):** Let $F$ be a semi*regular closed set in $Y$ and $x \in f^{-1}(F)$,then $f(x) \in F$. By assumption, there is an open set $U_x$ in $X$ containing $x$ such that $f(x) \in f(U_x) \subseteq F$ which implies $x \in U_x \subseteq f^{-1}(F)$
Hence $f^{-1}(F)$ is open in $X$.

**Theorem 4.16: (i)** Composition of semi*r-irresolute functions is semi*r-irresolute.
(ii) Inverse of a bijective semi*r-irresolute function is also semi*r-irresolute.
**Proof:** Follows from definition and set theoretic results.

## IV. MORE FUNCTIONS ASSOCIATED WITH SEMI*REGULAR OPEN SETS

**Definition 5.1:** A function f :X⟶Y is said to be semi*regular open if $f(U)$ is semi*regular open in $Y$ for every open set $U$ in $X$.
**Definition 5.2:** A function $f: X \longrightarrow Y$ is said to be contra semi*regular open if $f(U)$ is semi*regular closed in $Y$ for every open set $U$ in $X$.
**Definition 5.3:** A function $f: X \longrightarrow Y$ is said to be pre-semi*regular open if $f(U)$ is semi*regular open in $Y$ for every semi*regular open set $U$ in $X$.
**Definition 5.4:** A function $f: X \longrightarrow Y$ is said to be contra-pre semi*regular open if $f(U)$ is semi*regular closed in $Y$ for every semi*regular open set $U$ in $X$.
**Definition 5.5:** A function $f: X \longrightarrow Y$ is said to be semi*regular closed if $f(F)$ is semi*regular closed in $Y$ for every closed set $F$ in $X$.
**Definition 5.6:** A function $f: X \longrightarrow Y$ is said to be contra semi*regular closed if $f(F)$ is semi*regular open in $Y$ for every closed set $F$ in $X$.
**Definition 5.7:** A function $f: X \longrightarrow Y$ is said to be pre-semi*regular closed if $f(F)$ is semi*regular closed in $Y$ for every semi*regular closed set $F$ in $X$.
**Definition 5.8:** A function $f: X \longrightarrow Y$ is said to be contra-pre semi*regular closed if $f(F)$ is semi*regular open in $Y$ for every semi*regular closed set $F$ in $X$.
**Definition 5.9:** A bijection $f: X \longrightarrow Y$ is called a semi*regular homeomorphism if $f$ is both semi*regular irresolute and pre semi*regular open.
**Definition 5.10:** A function $f: X \longrightarrow Y$ is said to be semi*r-totally continuous if $f^{-1}(V)$ is clopen in $X$ for every semi*regular open set $V$ in $Y$.
**Definition 5.11:** A function $f: X \longrightarrow Y$ is said to be totally semi*r-continuous if $f^{-1}(V)$ is semi* regular open set in $X$ for every open set $V$ in $Y$.
**Theorem 5.12:** Let $f: X \longrightarrow Y$ and be $g: Y \longrightarrow Z$ be functions. Then (i) $g \circ f$ is pre-semi*regular open if both f and g are pre-semi*regular-open. (ii) $g \circ f$ is semi*regular open if f is semi*regular open and g is pre-semi*regular open. (iii) $g \circ f$ is pre-semi*regular closed if both f and g are pre-semi*regular closed. (iv) $g \circ f$ is semi*regular closed if both f is semi*regular closed and g is pre-semi*regular closed.
**Proof:** Follows from definitions.

**Theorem 5.14:** Let $f: X \longrightarrow Y$ be a function where $X$ is an Alexandroff space and $Y$ is any topological space. Then the following are equivalent:
(i) *f is* semi*r-totally continuous.
(ii) For each $x \in X$ and each semi*regular open set $V$ in $Y$ with $f(x) \in V$, there exists a clopen set $U$ in $X$ such that $x \in U$ and $f(U) \subseteq V$.
**Proof: (i)⇒(ii):** Suppose $f: X \longrightarrow Y$ is semi*r-totally continuous. Let $x \in X$ and let $V$ be a semi*regular open set containing $f(x)$. Then $U = f^{-1}(V)$ is a clopen set in $X$ containing $x$ and hence $f(U) \subseteq V$.
**(ii)⇒(i):** Let $V$ be a semi*regular open set in $Y$. Let $x \in f^{-1}(V)$. Then $V$ is a semi*regular open set containing $f(x)$. By hypothesis there exist a clopen set $Ux$ containing $x$ such that $f(Ux) \subseteq V$ which implies that $Ux \subseteq f^{-1}(V)$.
Therefore we have $f^{-1}(V) = \cup\{Ux : x \in f^{-1}(V)\}$.
Since each $Ux$ is open, $f^{-1}(V)$ is open. Since each $Ux$ is a closed set in the Alexandroff space $X$, $f^{-1}(V)$ is closed in $X$. Hence $f^{-1}(V)$ is clopen in $X$.

**Theorem 5.15:** A function $f : X \longrightarrow Y$ is semi*r-totally continuous if and only if $f^{-1}(F)$ is clopen in $X$ for every semi*regular closed set $F$ in $Y$.
**Proof:** Follows from definition.

**Theorem 5.16:** A function $f : X \longrightarrow Y$ is totally semi*r-continuous if and only if $f$ is both semi*r-continuous and contra-semi*r-continuous.
**Proof:** Follows from definitions.
**Theorem 5.17:** A function $f : X \longrightarrow Y$ is semi*r-totally continuous if and only if $f$ is both strongly semi*r-irresolute and contra-strongly semi*r-irresolute.
**Proof:** Follows from definitions.
**Theorem 5.18:** Let $f: X \longrightarrow Y$ be semi*r-totally continuous and $A$ is a subset of $Y$. Then the restriction $f/A : A \longrightarrow Y$ is semi*r-totally continuous.
**Proof:** Let $V$ be a semi*regular open set in $Y$. Then $f^{-1}(V)$ is clopen in $X$ and hence $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is clopen in $A$. Hence the theorem follows.

**Theorem 5.19:** Let $f : X \longrightarrow Y$ be a bijection. Then the following are equivalent:
**(i)** $f$ is semi*r-irresolute.
**(ii)** $f^{-1}$ is pre-semi*regular open.
**(iii)** $f^{-1}$ is pre-semi*regular closed.
**Proof:** Follows from definitions.
**Theorem 5.20:** A bijection $f : X \longrightarrow Y$ is a semi*r-homeomorphism if and only if $f$ and $f^{-1}$ are semi*r-irresolute.
**Proof:** Follows from definitions.

**Theorem 5.21:** (i) The composition of two semi*r-homeomorphisms is a semi*r homeomorphism
(ii) The inverse of a semi*r-homeomorphism is also a semi*r-homeomorphism.
**Proof:** (i) Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be semi*r-homeomorphisms. By Theorem 4.16 and theorem 5.13(i), $g \circ f$ is a semi*r-homeomorphism.
(ii)Let $f : X \longrightarrow Y$ be a semi*r-homeomorphism. Then by Theorem 4.16(ii) and by Theorem 5.20, $f^{-1} : Y \longrightarrow X$ is also semi*r-homeomorphism.

## ACKNOWLEDGEMENTS

The first author ,Dr.S.Pious Missier is thankful to University Grants Commission, New Delhi, for sponsoring this work under grants of Major Research Project-MRP-MATH-MAJOR-2013-30929. F.No. 43-433/2014(SR) Dt. 11.09.2015.

## REFERENCES

[1]. N. Levine, Semi-open sets and Semi-continuity in topo-logical spaces, Amer. Math.Monthly, 70(1), 1963, 36- 41.
[2]. J.Dontchev, Contra-continuous functions and strongly s-closed spaces, Internat. J. Math. & Math. Sci. 19(2), 1996, 303-310.
[3]. S.G. Crossley and S.K. Hildebrand, Semi-topological properties, Fund. Math.74, 1972, 233-254.
[4]. J. Dontchev and T. Noiri, Contra-Semi continuous func-tions, Mathematica Pannonica, 10(2), 1999, 159-168.
[5]. M. Caldas, Weak and Strong forms of irresolute maps, Internat. J. Math. & Math. Sci. 23(4) , 2000, 253-259.
[6]. A. Robert and S. Pious Missier, Semi-Star-Alpha-Open Sets and Associated Functions, International Journal of Computer Applications, 104(16), 2014, 0975 – 8887.
[7]. S. Pious Missier and R. Krishnaveni, A Stronger form of Semi Star Open Sets , IJMA- 7(7), 2016.
[8]. N. Levine, Generalized Closed Sets in Topology, Rend. Circ. Mat. Palermo, 19(2), 1970, 89-96.
[9]. W. Dunham, A new closure operator for Non-T1 topologies, Kyungpook Math. J. 22 (1), 1982, 55-60.
[10]. O. Njastad , Some Classes of Nearly Open sets, Pacific J. Math.,15 (3), 1965, 961-970.
[11]. G.B. Navalagi, Definition Bank in General Topology, Topology Atlas, 2000.
[12]. D. Andrijevic, Semi-Preopen Sets, Mat. Vesnik, 38, 1986, 24-32.
[13]. H. A. Othman, New types of α-continuous mapping, Master Degree Thesis, Al-Mustansiriya University, IRAQ, 2004.
[14]. T.M. Nour, Totally semi-continuous functions, Indian J. Pure Appl. Math., 26(7), 1995, 675-678.
[15]. S.S. Benchalli and Umadevi I Neeli., Semi-Totally Continous Functions in Topological spaces, International Mathematical Forum, 6 (10), 2011, 479-492.