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A New Four Dimensional Chaotic Attractor Based on Two-Degrees of Freedom Nonlinear Mechanical System

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ABSTRACT: *In this study, a four dimensional chaotic attractor based on two-degrees of freedom nonlinear mechanical system is generated by exploiting the chaotic behavior of nonlinear systems. Chaotic behavior is obtained by transforming the system's model which contains second order non-linear term into the state space and with appropriate parameter's values. Then, the analyses are done to prove the system shows chaotic behavior and the properties of the chaotic attractor are presented. Moreover, the new attractor is modelled via MATLAB Simulink and the simulation results are obtained. As a result, with this study a chaotic attractor generated from a system's model which can be encountered in real world is added in the literature.*
Keywords: *Chaos, chaotic attractors, dynamical analysis, mechanical systems, nonlinear systems*

I. INTRODUCTION

The phenomenon chaos, the chaotic systems and their application have been topical subjects in the recent years. In addition to this, the studies about chaotic systems and attractors are usually employ the common systems like Lorenz, Chua, Rössler or the modified versions of these systems[1-3]. The other chaotic systems in the literature are often expressed only mathematically and cannot be used in applications such as modelling of real systems [4-6].

The phenomenon chaos is, in fact, a behavior of nonlinear system and there are studies which employ the models of non-linear systems. Especially in the studies about mechanical systems, to analysis of the chaotic behavior of the nonlinear system is usually realized with non-linear input functions [7-9].

In this study, addition of a new chaotic attractor the literature is aimed The chaotic attractor is four dimensional and generated by applying linear input to a two-degrees of freedom nonlinear mechanical system. Moreover, the dynamic properties of the new chaotic attractor is presented by employing many different analysis methods (Lyapunov exponents, bifurcation diagram, time series, phase portraits) as a difference from other similar studies in the literature.

In the next part of the article, the transformation of a two-degrees of freedom nonlinear mechanical system's model into state space form and determination of parameters of the new chaotic attractors are realized. In the third part, chaotic behavior of the attractor is presented by realizing the new chaotic attractor's analysis. In the third part, the simulation results of MATLAB Simulink are presented. The last part covers evaluations and conclusions.

II. THE NEW CHAOTIC ATTRACTOR BASED ON TWO-DEGREES OF FREEDOM NONLINEAR MECHANICAL SYSTEM

A two-degrees of freedom nonlinear mechanical system is given in Fig. 1. In this system k1 is a spring that has property of second order nonlinearity as in Equation (1). This system can be modelled with two equations as in Equation (2) due to the it is a two-degrees of freedom system [10].

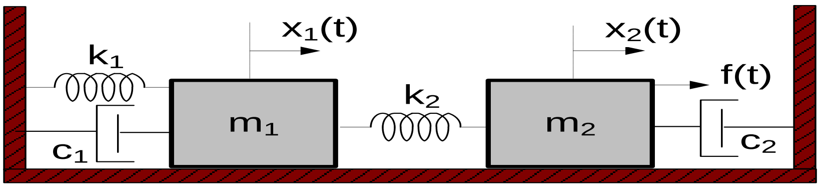


Figure 1: A two-degrees of freedom nonlinear mechanical system

$$f_{k_2}(t) = k_2(x(t))^2 \tag{1}$$

$$\begin{aligned} m_1\ddot{x}_1(t) + c_1\dot{x}_1(t) + k_1x_1(t) + k_2(x_1(t) - x_2(t))^2 &= 0 \\ m_2\ddot{x}_2(t) + c_2\dot{x}_2(t) + k_2(x_2(t) - x_1(t))^2 &= f(t) \end{aligned} \tag{2}$$

The *f*(t) in Fig. 1 and Equation (2) is input function and applied as a step function in this study. Equation (2) is a nonlinear second order differential equation system. By reducing order of this differential equation into the first order differential equation, a four dimensional differential equation system is obtained. To reduce order, the highest ordered term is left alone as in Equation (3).

$$\begin{aligned} \ddot{x}_1(t) &= \frac{-\left[c_1\dot{x}_1(t) + k_1x_1(t) + k_2(x_1(t) - x_2(t))^2\right]}{m_1} \\ \ddot{x}_2(t) &= \frac{f(t) - \left[c_2\dot{x}_2(t) + k_2(x_2(t) - x_1(t))^2\right]}{m_2} \end{aligned} \tag{3}$$

Equation (3) can be rearranged as in Equation (4) to observe the system parameters more clearly.

$$\begin{aligned} \ddot{x}_1(t) &= -\frac{c_1}{m_1}\dot{x}_1(t) - \frac{k_1}{m_1}x_1(t) - \frac{k_2}{m_1}(x_1(t) - x_2(t))^2 \\ \ddot{x}_2(t) &= \frac{f(t)}{m_2} - \frac{c_2}{m_2}\dot{x}_2(t) - \frac{k_2}{m_2}(x_2(t) - x_1(t))^2 \end{aligned} \tag{4}$$

State variables in Equation (4) can be determined as in Equation (5).

$$x_1(t) = x, \qquad x_2(t) = y, \qquad \dot{x}_1(t) = z, \qquad \dot{x}_2(t) = w, \tag{5}$$

With determined state variables, Equation (4) can be rearranged as in Equation (6).

$$\begin{aligned} \dot{x} &= z \\ \dot{y} &= w \\ \dot{z} &= -\frac{c_1}{m_1}z - \frac{k_1}{m_1}x - \frac{k_2}{m_1}(x-y)^2 \\ \dot{w} &= \frac{f(t)}{m_2} - \frac{c_2}{m_2}w - \frac{k_2}{m_2}(y-x)^2 \end{aligned} \tag{6}$$

It is possible determine the system parameter in Equation (6) separately as in Equation (7).

$$\frac{c_1}{m_1} = \alpha, \qquad \frac{k_1}{m_1} = \beta, \qquad \frac{k_2}{m_1} = \gamma, \qquad \frac{c_2}{m_2} = \delta, \qquad \frac{k_2}{m_2} = \lambda \tag{7}$$

Since f(t) in Equation (6) is a step function, it can be considered as a separate parameter as in Equation (8).

$$\frac{f(t)}{m_2} = \sigma \tag{8}$$

When the parameters in Equation (6) are substituted with the parameters in Equation (7) and 8, the new chaotic attractor generated from a two-degrees of freedom nonlinear mechanical system can be written as in Equation (9).

$$\begin{aligned} \dot{x} &= z \\ \dot{y} &= w \\ \dot{z} &= -\alpha z - \beta x - \gamma(x-y)^2 \\ \dot{w} &= \sigma - \delta w - \lambda(y-x)^2 \end{aligned} \tag{9}$$

After this stage, the system shows chaotic behavior when the parameters' and the initial values of the system in Equation (9) are determined appropriately. Thus, a new chaotic attractor is obtained from a two-degrees of freedom nonlinear mechanical system.

III. DYNAMIC ANALYSIS OF THE NEW CHAOTIC ATTRACTOR

For a system to show chaotic behavior, results of some analysis must meet certain criteria. In this part, the chaotic behavior of the generated system in the previous section (chaotic attractor in Equation (9)) is examined with five different analysis methods namely Lyapunov exponents, bifurcation diagram, phase portraits, time series and sensitivity analysis. In the analyses the values of parameter α is between 1.15 and 2, $\beta = 10$, $\gamma = 20$, $\sigma = 1.25$, $\delta = 10$, $\lambda = 20$ and the initial values x(0), y(0), z(0) and w(0) equal to zero (0,0,0,0).

In the Lyapunov Exponents analysis, for a four dimensional system to be chaotic the exponent values must be negative, negative, 0 and positive (-,-,0,+). The Lyapunov exponent analysis graph for the new chaotic attractor with parameters and initial values given above is shown in Fig. 2. According to Fig. 2 the system in Equation (9) has generally two negative, a zero and a positive Lyapunov exponents when the parameter α's value is between 1.15 and 2. When the graph in Fig. 2 is examined, there are some ranges in which the system shows non-chaotic behavior and the system exit chaos just after the parameter α's value exceeds 1.9. Thus, the system show chaotic behavior for the parameter α's value is between 1.15 and 1.9.

The chaotic behavior of the system can be examined with Bifurcation analysis too. In the bifurcation diagram, if the trajectories can be observed clearly the system exhibits non-chaotic behavior. The bifurcation diagram of the new chaotic attractor is given Fig. 3 and it can be seen in the Fig. the trajectories cannot be observed clearly; hence the new system exhibits chaotic behavior. However, the trajectories can be observed clearly when the parameter α's value exceeds 1.9 and after this value system shows non-chaotic behavior just in the Lyapunov Exponents analysis. According to Fig. 3 the system show chaotic behavior for the parameter α's value is between
1.15 and 1.9.

Fig. 4 and 5 is obtained for the parameter α's value equals to 1.5. If the 2-D and 3-D phase portraits in these Figures are examined, there are relations between the state variables. Moreover, all the phase portraits are continuous without intersecting each other. This shows the suggested system shows chaotic behavior.

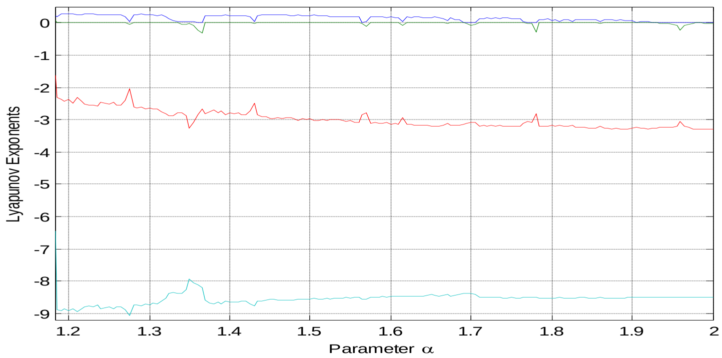


Figure 2: Lyapunov exponents analysis of the new chaotic attractor in Equation (9)

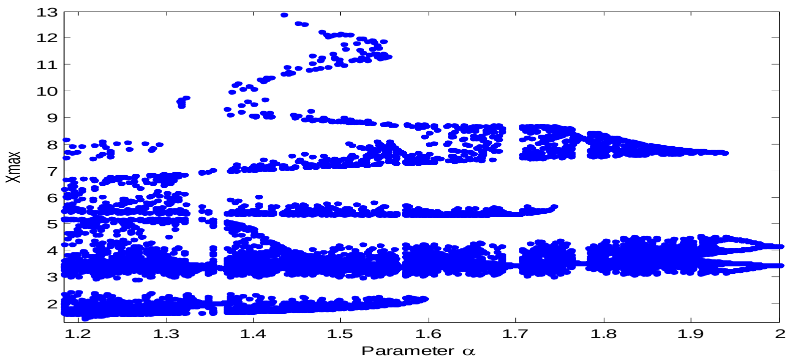


Figure 3: Bifurcation diagram of the new chaotic attractor in Equation (9)

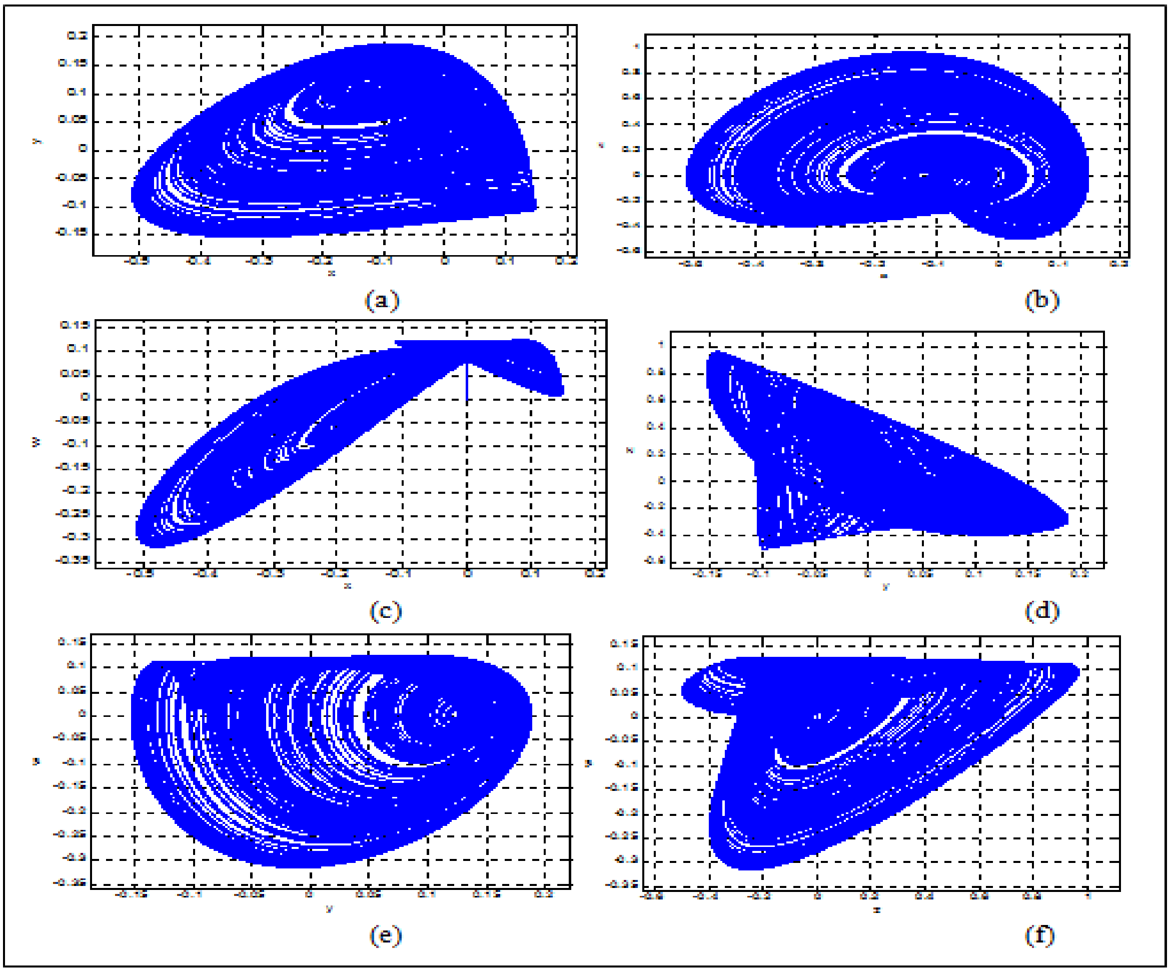


Figure 4: Two dimension phase portraits of the new chaotic attractor in Equation (9): (a) x-y, (b) x-z, (c) x-w, (d) y-z, (e) y-w, (f) z-w

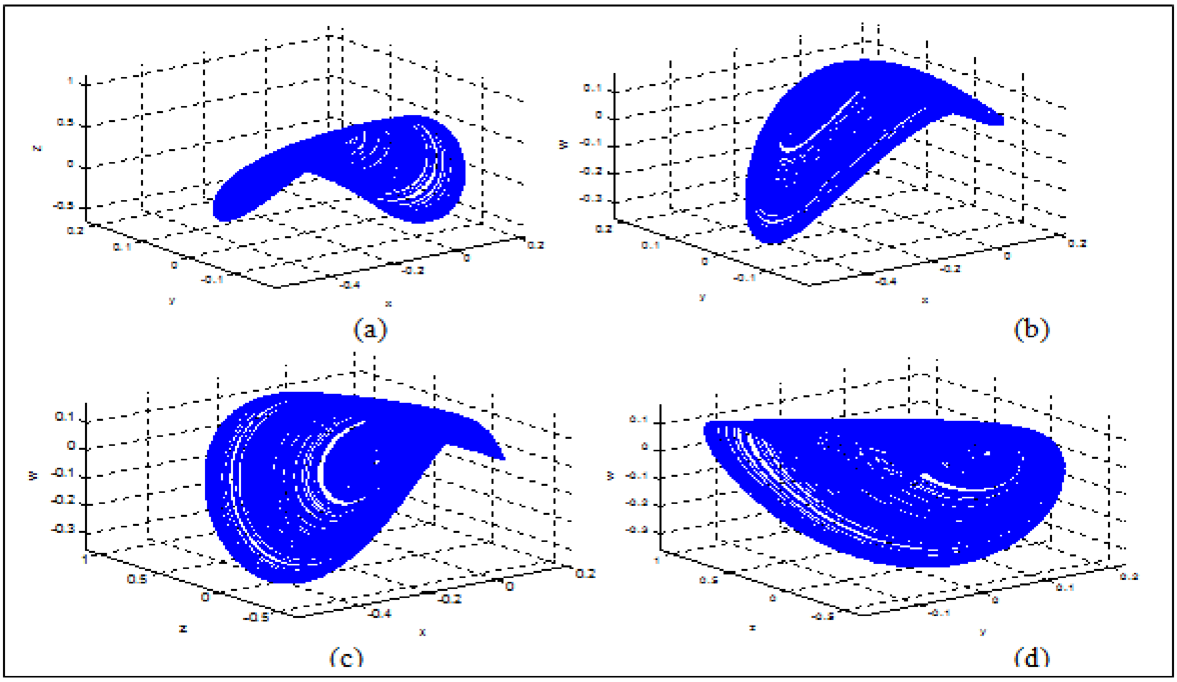


Figure 5: Three dimension phase portraits of the new chaotic attractor in Equation (9): (a) x-y-z, (b) x-y-w, (c) x-z-w, (d) y-z-w

Another analysis method for examining the chaotic behavior of the systems is time series of the state variables. All the time series must vary randomly with time and be aperiodic. Fig. 6 is obtained for the parameter α's value equals to 1.5. As it can be seen in Fig. 6, all the time series vary randomly with time and are aperiodic. According to this analysis system shows chaotic behavior.

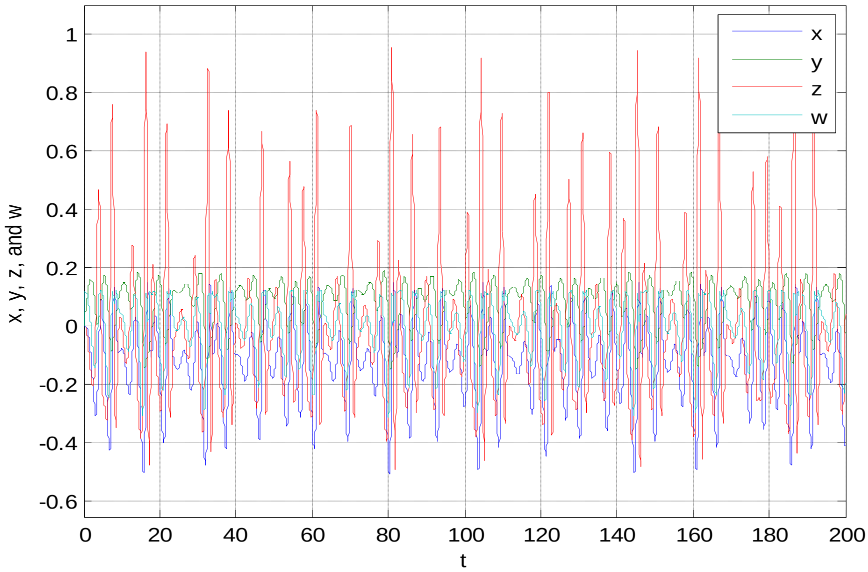


Figure 6: Time series of state variables of the new chaotic attractor in Equation (9)

In this study, the last realized analysis is sensitivity to the initial values. Chaotic systems are very sensitive to the initial values. In Fig. 7, the time series of z state variable of the chaotic system is given. As it can be seen in the Fig., 0.0001 change in the initial value z(0) leads to a huge difference after certain time elapsed. Thus, this analysis too proves that the new system shows chaotic behavior.

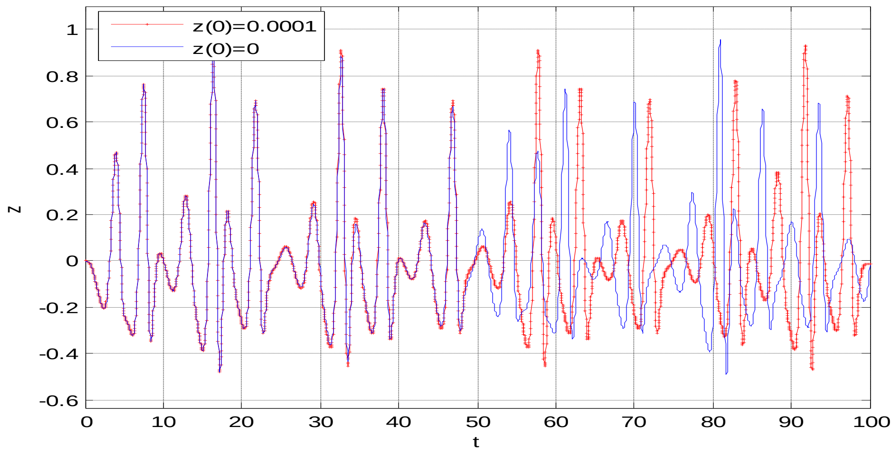


Figure 7: Sensitivity of state variable z of the new chaotic attractor in Equation (9)

IV. MATLAB SIMULINK MODEL AND SIMULATION RESULTS OF THE NEW CHAOTIC ATTRACTOR

In this section, simulation results of the new chaotic attractor are obtained in MATLAB Simulink and the analysis results obtained in the previous section are verified in the simulation. The Simulink model of the chaotic attractor in Equation (9) is shown in Fig. 8. In this model, for every state variable there is an integrator block. Moreover, the entire state variable's time series are obtained from scope components. In addition to this, x-y and w-z phase portraits are obtained by using Graph components. In the Simulink application, the parameters taken as $\alpha = 1.5$, $\beta = 10$, $\gamma = 20$, $\sigma = 1.25$, $\delta = 10$ and $\lambda = 20$ and all the initial values x(0), y(0), z(0) and w(0) equals to 0 to verify the analysis result in the previous section.

The simulation results of the Simulink model in Fig. 8 are given in Fig. 9 and 10. As the time series of the state variable in Fig. 9 is examined, they vary with time randomly and are aperiodic. Moreover, the phase portraits in Fig. 10 are identical to the phase portraits in Fig. 4-a and 4-f respectively. Thus, it is proved with the simulation results that the system in Equation (9) is a chaotic attractor.

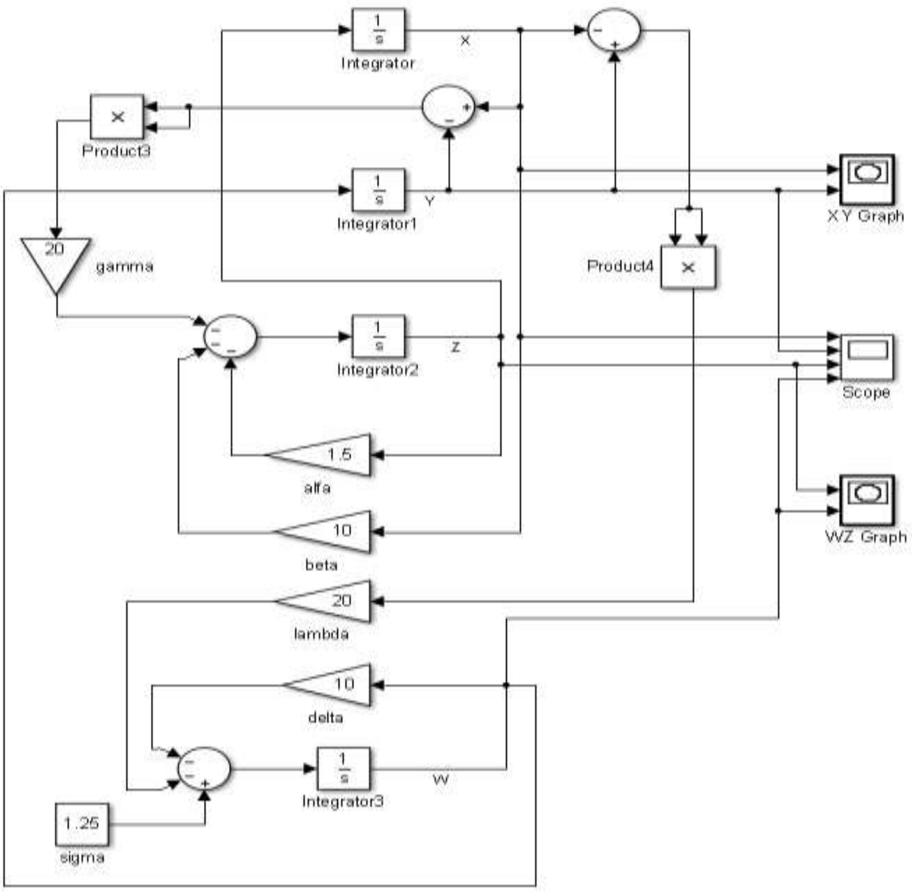


Figure 8: Simulink model of the new chaotic attractor in Equation (9)

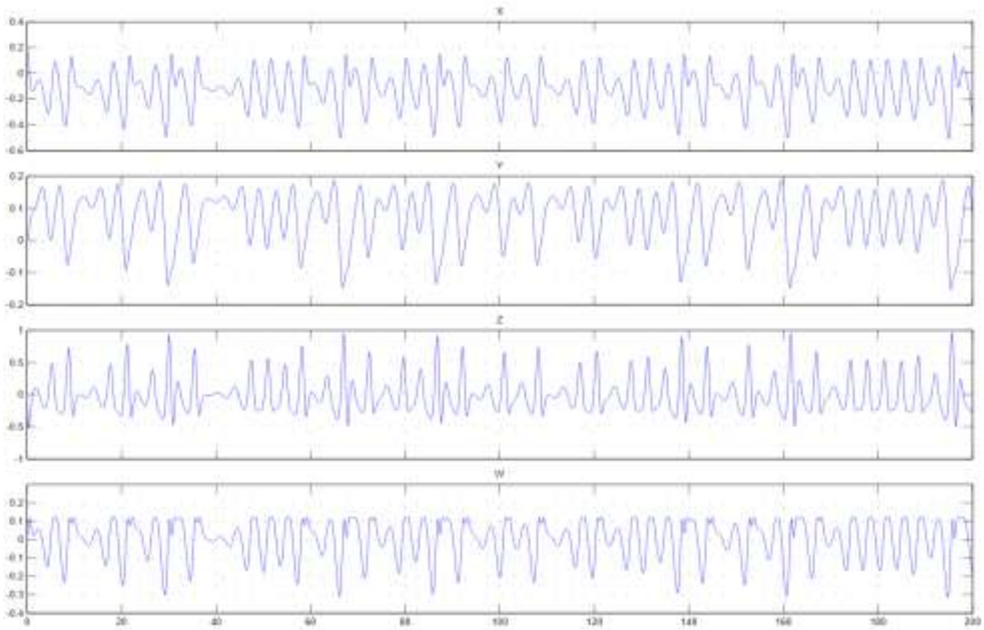
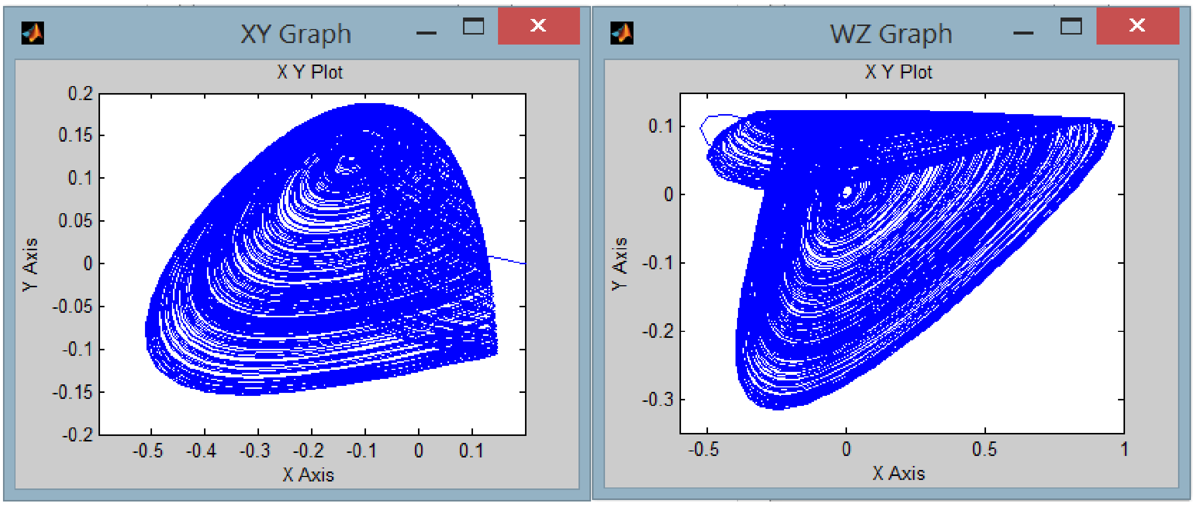


Figure 9: Simulation results of the new chaotic attractor in Equation (9)



(a) **(b)**

Figure 10: Simulink phase portraits of the new chaotic attractor in Equation (9): (a) x-y, (b) w-z

V. CONCLUSION

In this study, a new chaotic attractor is generated by exploiting the chaotic behavior of non-linear systems. It is proved that the proposed system show chaotic behavior by realizing the dynamic analysis of the generated attractor. Moreover, the realized simulation with MATLAB Simulink backs the analysis results in third section. In addition, due to the values of the state variables of the generated system are small the real life application of the proposed system can be realized easily without scaling process. Thus, the experimental modelling of the system with especially electronic component is very possible. Another advantage of the generated system is that it shows chaotic behavior without the initial conditions. The proposed chaotic attractor provides adequately long key space in the chaos based encryption applications due to there are 6 system parameters in the generated attractor. Considering all the advantages, the realized study adds value to the literature.

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