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International
Journal
Of Modern Engineering Research (IJMER)

Analytic Features of Inventory Problems of Deteriorating Items Under Different Environments

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ABSTRACT: *This paper presents analytic features of classical inventory problems in Crisp, Fuzzy and Stochastic environments with deteriorating items. The aim of this paper is to provide a comprehensive study of different practical classical inventory problem in different conditions under different environments. To determine the optimal order quantity and minimum total cost, a relative numerical comparative methodology is developed with the numerical example and sensitive analysis with the help of LINGO software.*
keywords: *Inventory problem, Fuzzy demand, stochastic demand, Deterioration, optimal order quantity, Global Criteria Method.*

I. INTRODUCTION

The traditional inventory model considers in a static environment, where the demand for the item under consideration is assumed to be constant, for the sake of simplicity. The classical EOQ model is based on the assumption that the demand rate for the product is constant and that it is independent of the price of the product. This is not true in many situations, in the case of a monopolistic firm. A monopolist always influences the demand of the product by manipulating its price. In such cases the demand rate should be treated as a decision variable However, it is observed that in practical situations, constant demand can be justified only for the maturity phase of the product.

Deterioration is defined as decay, damage or spoilage. Many products, such as clothes, fashion accessories, mobile phones, need to prove their worth before they are generally accepted. Hence, it is justifiable to approximate the demand for a product to be represented by a time dependent function during its growth stage. In real-life situations, the inventory lost by deterioration. Owing to this fact, how to control and maintain inventories of deteriorating items becomes an important problem for decision makers in modern organization. Inventory models in which unit deteriorate while in storage, have drawn attention of various researchers in recent years. During the past few decades, many researchers have studied inventory models for deteriorating items such as volatile liquids, blood banks, medicines, electronic items and fashion goods. Inventory models in which unit deteriorate while in storage, have drawn attention of various researchers in recent years. Decay in radioactive elements, spoilage in food grain storage, pilferages from on hand inventory etc. are continuous in time and roughly proportionally to the on hand inventory. Therefore, deterioration is usually a function of the total amount of inventory on hand. Ghare and Schrader (1963) were the first proponents to establish a model for an exponentially decaying inventory. Covert and Philip (1973) extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution. Later, there are several interesting papers related to deterioration with and without shortage such as Shah and Jaiswal (1977), Goyal and Giri (2001), Bhunia and Maiti (1999), Chung and Tsai (2001), Sana et al. (2004).

Shortages are allowed here and are completely backlogged. Aggarwal S. P. and Jain (1978) consider shortages. But in Chakraborty, T., Giri, B. C., Chaudhuri, K. S. (2001), Dave, U(2003), Goswami, A. and Chaudhuri, K. S.(1991), Jalan, A. Giri, R. R., Chaudhuri, K. S. (2000), Kundu. A., Chakraborty.(2009). They have considered different deterioration and Shortages.

In most models, holding cost and deterioration cost are known and constant. But holding cost and deterioration cost are may not always be constant. In generalization of EOQ models, various functions describing holding cost and deterioration cost were considered by several researchers like Naddor (1966), Vander Veen (, Muhlemann and 967) Valtis Spanopoulos (1980), Weiss (1982), and Goh (1994) etc.

In practice, suppliers offer their customers a fixed period without interest during the permissible delay time period. This fixed period is said to be credit period. Allowing a delay in payment to the supplier is a form of price-discount. Such a convenience is to motivate customers to order more quantities because paying later reduces the purchase cost indirectly. Buzacott (1977) developed the first EOQ model taking inflation into account. Then Bierman (1977) and many authors have studied later on different aspect of that.

In the crisp inventory models, all the parameters in the total cost are known and have definite values. But in the practical situation it is not possible. Hence fuzzy inventory models fulfill that gap. Different fuzzy inventory models occur due to various fuzzy cost parameters in the total cost. Researchers related to this area are: Bellman and Zadeh (1965), Zimmermann (1978), (1991), Chen and Ouyang (2006), Mahata and Goswami (2009a), (2009b). Yao and Chiang (2003) presented an inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. Dutta et al. (2005) developed a single-period inventory model with fuzzy random variable demand. In this study, they have applied graded mean integration representation method to find the optimum order quantity. Chen and Ouyang (2006) developed a fuzzy inventory model with permissible delay in payment. Chen and Hsieh (1999, 2000), Lin and Yao (2000), Yao and Chiang (2003), Chen and Ouyang (2006), Chen and Chang (2008) presented an optimization of fuzzy production inventory model. In this study, they have considered a fuzzy opportunity cost and trapezoidal fuzzy costs under crisp production quantity or fuzzy production quantity in order to extend the traditional production inventory model to the fuzzy environment. They have use function principle as arithmetical operations of fuzzy total production inventory cost and also use the graded mean integration representation method to defuzzify the fuzzy total production and inventory cost.

Goyal (1985) established a single-item inventory model under permissible delay in payments. Mandal and Phaujdar (1989) studied the above mentioned author model by including interest earned from the sales revenue on the stock remaining beyond the settlement period. Aggarwal and Jaggi (1995) extended Goyal's model to the case of deterioration. Jamal et al. (1997) extended Aggarwal and Jaggi (1995) with allowable shortages. De et al. (2003) developed an economic production quantity inventory model involving fuzzy demand rate and fuzzy deterioration rate. Chung et al. (2005) presented optimal inventory policies under permissible delay in payments depending on the ordering quantity**.** Chen and Ouyang (2006) extended Jamal (1997) by fuzzifying the carrying cost rate, interest paid rate and interest earned rate simultaneously, based on the interval-valued fuzzy numbers and triangular fuzzy numbers to fit the real world. Recently, Chung et al. (2009) developed a model to determine an optimal ordering policy under conditions of allowable shortages and permissible delay in payment

However, the behavior of customer for a particular product is hardly known and remains constant. Thus the assumption of probabilistic demand is more appropriate to capture uncertain demand. Shah and Shah (1993) formulated a probabilistic inventory model when delay in payments is permissible. Later on, it was extended by Shah (1993) to study the effect of deterioration on the optimal solution. Shah and Shah (1998) developed a probabilistic inventory model for deteriorating items under trade credit policy by considering discrete time. Shah (1997), (2004) allowed shortages to analyze the results when demand is stochastic and trade credit is offered. De and Goswami (2005) analyzed an economic order quantity model for deteriorating items under scenario of credit financing, to minimize the joint relevant costs for two players in the supply chain, viz. retailer and customer. Shah et. al. (2013) considering deteriorating inventory model with finite production rate and two-level of credit financing for stochastic demand. The concept of fuzzy random variable and its fuzzy expectation was introduced by Kwakernaak (1978) and later developed by Puri and Ralescu (1986). Recently, fuzzy random variable demand has also been defined by Gil et al. (1998). Since the annual demand, the lead-time demand and lead-time plus one period's demand are all fuzzy random variables, so the total associated cost is also a fuzzy random variable and its expectation is a unique fuzzy number. For the ease of computation, the fuzzy expected cost has been defuzzified by its posibilistic mean (2).

In this present paper, we have developed a classical EOQ model for deteriorating items in different conditions under different environments. Shortages are allowed here and are completely backlogged. Permissible delay in payments is also considered here. Demand is considered as constant in Crisp environment and then it is to be considered as fuzzy, stochastic and combinations of them. Thus, the aim of this paper is to develop a comparative inventory model (literature) for which the organization gets better profit relative to different environment. A methodology is developed such that the total cost in that sense is minimized and, in the process, the optimal period of review and the optimal target inventory level is determined.

The organization of the paper is as follows: in Section 2, the preliminary concept of fuzzy numbers, possibilistic mean value of a fuzzy number and fuzzy random variable has been discussed. Basic assumptions and notation are given in section 3. Section 4 deals with the methodology where the model has been developed and the optimality conditions have been deduced. A numerical example for different cases has been given in Section 5 to illustrate the methodology discussed then provide sensitivity analysis regarding that. Section 6 observation regarding whole model the comparison is given and finally the results obtained have been summarized in Section 7.

II. Preliminary Concepts

2.1 Triangular Fuzzy Number (TFN):

A TFN $\tilde{a}$ is specified by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function $\mu_{\tilde{a}}(x) : F \rightarrow [0,1]$ as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \le x \le a_3 \\ 0 & \text{, Otherwise} \end{cases}$$

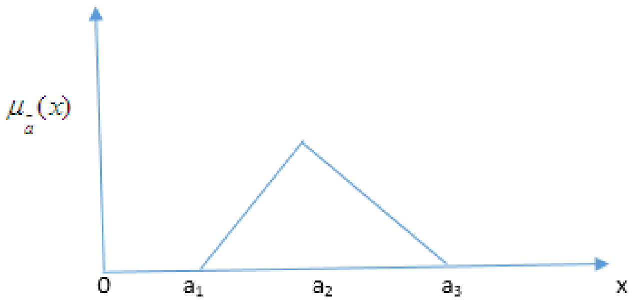

Fig 1. Membership function of a TFN

2.2 Trapezoidal Fuzzy Number (TrFN):

A TrFN $\tilde{a}$ is specified by the four parameters (a_1, a_2, a_3, a_4) and is defined by its continuous membership function $\mu_{\tilde{a}}(x) : F \rightarrow [0,1]$ as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{Otherwise} \end{cases}$$

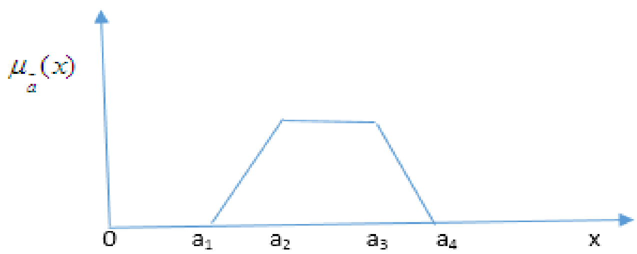

Fig 2. Membership function of a TrFN

2.3 Alpha-cut:

A α-cut of a fuzzy number $\tilde{a}$ in universal set X, is denoted by the symbol ${}^{\alpha}\mu_A(x)$ and is defined as following crisp set

$^{\alpha}\mu_A(x) = \{x : \mu_{\tilde{A}}(x) \geq \alpha, \ x \in X\}$ *Where* $\alpha \in [0,1]$, $^{\alpha}\mu_A(x)$ is a non-empty bounded closed interval contained in X and it can be denoted by $^{\alpha}\mu_A(x)$ = $[a^-(\alpha), \ a^+(\alpha)]$, where $a^-(\alpha)=a_1+\alpha(a_2-a_1)$ and $a^+(\alpha)=a_3-\alpha(a_3-a_2)$ are known as lower and upper bounds of the closed interval respectively.

2.4 Expected Value Of A Fuzzy Random Variable:

Let $\tilde{a}$ be a Trapezoidal Fuzzy number (trFN) with the membership function $\mu_{\tilde{a}}(x)$ continuous mapping: $\mu_{\tilde{a}}(x) : F \to [0,1]$

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & if\ a_1 \leq x \leq a_2 \\ 1 & if\ a_2 \leq x \leq a_3 \\ \dfrac{a_4-x}{a_4-a_3} & if\ a_3 \leq x \leq a_4 \\ 0 & Otherwise \end{cases}$$

According to Bellman et. al. (1966), Zadeh (1978), Dubois and Prade (1983), Liu and Iwamura (1998a, 1998b) and Liu and Liu (2002), the Possibility, Necessity and Credibility measure of a TrFN can be defining as:

$$Pos(\tilde{A} \geq r) = \begin{cases} 1 & if\ r \leq a_3 \\ \dfrac{a_4-r}{a_4-a_3} & if\ a_3 \leq r \leq a_4 \\ 0 & if\ r \geq a_4 \end{cases} \text{ and } Nes(\tilde{A} \geq r) = \begin{cases} 1 & if\ r \leq a_1 \\ \dfrac{a_2-r}{a_2-a_1} & if\ a_1 \leq r \leq a_2 \\ 0 & if\ r \geq a_2 \end{cases}$$

And,

$$Cr(\tilde{A} \geq r) = \begin{cases} 1 & if\ r \leq a_1 \\ \dfrac{a_2-\rho a_1}{a_2-a_1} - \dfrac{(1-\rho)r}{a_2-a_1} & if\ a_1 \leq r \leq a_2 \\ \rho & if\ a_2 \leq r \leq a_3 \\ \dfrac{\rho(a_4-r)}{a_4-a_3} & if\ a_3 \leq r \leq a_4 \\ 0 & if\ r \geq a_4 \end{cases} \text{ and } Cr(\tilde{A} \leq r) = \begin{cases} 0 & if\ r \leq a_1 \\ \rho\dfrac{r-a_1}{a_2-a_1} & if\ a_1 \leq r \leq a_2 \\ \rho & if\ a_2 \leq r \leq a_3 \\ \dfrac{\rho a_4 - a_3 + r(1-\rho)}{a_4-a_3} & if\ a_3 \leq r \leq a_4 \\ 1 & if\ r \geq a_4 \end{cases}$$

Based on the credibility Measure, Liu and Liu (2002) presented the expected value operator of a fuzzy variable as follows:

Definition:

Let $\tilde{X}$ be a normalized fuzzy variable, the expected value of the fuzzy variable $\tilde{X}$ is defined by

$$E[\tilde{X}] = \int_0^{\infty} Cr(\tilde{X} \geq r)dr - \int_{-\infty}^{0} Cr(\tilde{X} \leq r)dr \qquad \ldots\ldots\ldots\ldots. (1)$$

When the right side of (1) is of form ∞, -∞, and the expected value is not defined. Also, the expected value operation has been proved to be linear for bounded fuzzy variables, i.e. for any two bounded fuzzy variables $\tilde{X}$ and $\tilde{Y}$, we have

$$E[a\tilde{X} + b\tilde{Y}] = aE[\tilde{X}] + bE[\tilde{Y}]$$ For any real numbers a and b.

The expected value of fuzzy variable $\tilde{X} = [a_1, a_2, a_3, a_4],\ 0 < \rho < 1$ is defined as

$$E[\tilde{X}] = \frac{1}{2}[(1-\rho)(a_1 + a_2) + \rho(a_3 + a_4)] \qquad \text{…………… (2)}$$

2.5 Global Criteria Method:

The fuzzy model is a multi-objective model which is solved by Global Criteria (GC) Method with the help of Generalized Reduction Gradient technique. The Multi-Objective Non Linear Integer Programming (MONLIP) problems are solved by Global Criteria Method converting it into a single objective optimization problem.

The solution procedure is as follows:

Step-1: Solve the multi-objective programming problem as a single objective problem using one objective at a time ignoring the other.

Step-2: From the result of Step-1, determine the ideal objective vector, (say $TC^{+\min}, TC^{-\min}$) and (say $TC^{+\max}, TC^{-\max}$). Here the ideal objective vector is use as a reference point. The problem is then to solve the auxiliary problem:

$$\textbf{Min (GC) =Minimize } \left\{ \left(\frac{TC^{+} - TC^{+\max}}{TC^{+\max} - TC^{+\min}} \right)^{p} + \left(\frac{TC^{-} - TC^{-\min}}{TC^{-\max} - TC^{-\min}} \right)^{p} \right\}^{\frac{1}{p}} \qquad \text{…….. (a)}$$

Where $1 \le p < \infty$. As usual value of p is 2. The method is also sometimes called compromise Programming. Hence

III. Assumptions And Notations

The following notations are used in our study:

Q(t)	Inventory level at any time t,
θ(t)	Deterioration rate of the inventory,
D(t)	Demand rate of the inventory,
c	Purchasing cost per unit item,
C_h	Inventory holding cost per unit,
C_d	Inventory deterioration cost per unit,
C_s	Shortage cost per unit,
C_0	Setup cost per cycle,
I_e	interest earned per dollar in stocks per unit time by the supplier
I_c	interest charged per dollar in stocks per unit time by the supplier, $I_c \geq I_e$
m	the permissible delay in settling account
t_1	Time to zero inventories
TC	Total cost
ETC	Expected total cost

The basic assumptions are:

- The demand rate is D(t) = D, a constant rate of demand first then consider it as Fuzzy, Stochastic and combination of them.
- Deterioration rate θ(t) = θ, a constant rate of deterioration.
- There is no repair or replenishment of deteriorating items.
- Single item inventory is considered.
- Replenishment is instantaneous.
- Shortages are allowed and fully backlogged.

IV. DEVELOPMENT AND ANALYSIS OF MODEL

We assume that q is the total amount of inventory purchased or procured at the beginning of each period and after fulfilling backorders. The inventory level Q(t) gradually diminishes due to demand and deterioration during the period [0, t_1] and ultimately falls to zero at $t=t_1$. Now, during the period [t_1, T] shortages occur which is completely backlogged.

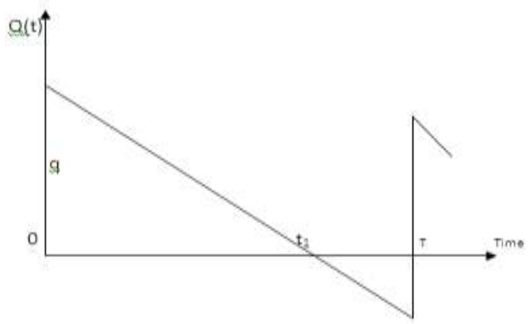

Fig. 2 Inventory model of the system

Hence, the variation of the inventory with respect to time can be described by the differential equation:

$$\frac{dQ(t)}{dt} + \theta Q(t) = -D, 0 \le t \le t_1 \qquad \text{…………… (3)}$$

$$\frac{dQ(t)}{dt} = -D, t_1 \le t \le T \qquad \text{…………... (4)}$$

With the boundary conditions Q(0)=q and $Q(t_1)=0$.

Solution of (1) is
$$Q(t) = q e^{-\theta t} - \frac{D}{\theta}\left(1 - e^{-\theta t}\right), 0 \le t \le t_1 \qquad \text{…… (5)}$$

At $t=t_1$, $Q(t_1)=0$ then from (5), we have

$$\frac{D}{\theta} - \left(q + \frac{D}{\theta}\right) e^{-\theta t_1} = 0 \qquad \text{…………. (6)}$$

This expression clearly shows the initial inventory level of the above model.

Solution of (4) is
$$Q(t) = D(t_1 - t), t_1 \le t \le T \qquad \text{…………. (7)}$$

Now, the different costs regarding the model are as follows:

Set-up cost:

For every cycle, the set up cost is = C_0

Purchase cost:

The Purchasing cost for the q^{th} level of inventory is

$$PC = cq \qquad \text{…………… (8)}$$

Holding cost:

Inventory is available in the system during the time interval [0, t_1]. Hence the cost for holding inventory in stock is computed for this time period only is as:

$$HC = C_h \int_0^{t_1} Q(t)\, dt$$

$$= C_h \int_0^{t_1} \left\{ q e^{-\theta t} - \frac{D}{\theta}\left(1 - e^{-\theta t}\right) \right\} dt$$

$$= \frac{C_h}{\theta} \left\{ q\left(1 - e^{-\theta t_1}\right) - \frac{D}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right) \right\} \qquad \text{…………… (9)}$$

Deterioration cost:

Inventory is available in the system during the time interval [0, t_1]. Hence the cost for Deterioration is computed for this time period only is as:

$$DC = C_d \int_0^{t_1} \theta(t)\, Q(t)\, dt$$

$$= C_d \int_0^{t_1} \theta \left\{ q e^{-\theta t} - \frac{D}{\theta}\left(1-e^{-\theta t}\right)\right\} dt$$

$$= C_d \left\{ q \left(1-e^{-\theta t_1}\right) - \frac{D}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right)\right\} \qquad \text{…………… (10)}$$

Shortage cost:
Shortages due to stock-out are accumulated in the system during the interval [t_1, T]. The shortages cost during this time is as:

$$\text{SC} = C_s \int_{t_1}^{T} Q(t)\, dt$$

$$= C_s \int_{t_1}^{T} D\left(t_1 - t\right) dt$$

$$= C_s\, D \left\{ T t_1 - \frac{1}{2}\left(T^2 + t_1^2\right)\right\} \qquad \text{…………… (11)}$$

4.1 Total Cost for without and with Shortages:
Total cost of the model is accumulating all the costs that have been calculating above. Therefore, the total cost is as follows:
Case I: For model without shortages the total cost is

$$TC_I = C_0 + cq + \frac{C_h}{\theta}\left\{ q\left(1-e^{-\theta t_1}\right) - \frac{D}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right)\right\} + C_d \left\{ q \left(1-e^{-\theta t_1}\right) - \frac{D}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right)\right\} \qquad \text{…. (12)}$$

Case II: For the model with shortages the total cost is

$$TC_{II} = C_0 + cq + \frac{C_h}{\theta}\left\{ q\left(1-e^{-\theta t_1}\right) - \frac{D}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right)\right\} + C_d \left\{ q \left(1-e^{-\theta t_1}\right) - \frac{D}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right)\right\}$$
$$+ C_s\, D \left\{ T t_1 - \frac{1}{2}\left(T^2 + t_1^2\right)\right\} \qquad \text{……….…. (13)}$$

Our object is to minimize the total cost (12) & (13). Differentiating equation (14) partially with respect to q two times, we get

$$\frac{d^2 TC_I}{dq^2} > 0 \;\;\&\;\; \frac{d^2 TC_{II}}{dq^2} > 0$$

Consequently, TC_I, TC_{II} are convex functions of 'q'. Thus, there exists unique values of 'q' which minimizes TC_I, TC_{II}.

4.2 Model for time dependent Holding Cost:

Here, we assume that for some practical situation holding cost is time dependent. Let the holding cost per item be $C_h\, t^n$, for general case. In particular, when n=1 then it reduced to $C_h t$. Therefore, during the period [0, t_1] the holding cost is as:

$$HC_2 = C_h \int_0^{t_1} t\, Q(t)\, dt$$

$$= C_h \int_0^{t_1} t \left\{ q e^{-\theta t} - \frac{D}{\theta}\left(1-e^{-\theta t}\right)\right\} dt$$

$$= C_h \left\{ \frac{q t_1^2}{2} - \frac{(q\theta + D)}{3} t_1^3 \right\} \qquad \text{……….(14)}$$

And corresponding Total cost is

$$TC_2 = C_0 + cq + C_h \left\{ \frac{q t_1^2}{2} - \frac{(q\theta + D)}{3} t_1^3 \right\} + C_d \left\{ q\left(1 - e^{-\theta t_1}\right) - \frac{D}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right) \right\}$$
$$+ C_s D \left\{ T t_1 - \frac{1}{2}\left(T^2 + t_1^2\right) \right\} \qquad \text{……….(15)}$$

4.3 Model For Time Dependent Deterioration Cost:

Here, we assume that for some practical situation deterioration cost is time dependent. Let the deterioration cost per item be $C_h t^n$, for general case. In particular, when n=1 then it reduced to $C_h t$. Therefore, during the period $[0, t_1]$ the deterioration cost is as:

$$DC_3 = C_d \int_0^{t_1} t\, \theta(t)\, Q(t)\; dt \qquad = C_d \int_0^{t_1} t\; \theta \left\{ q e^{-\theta t} - \frac{D}{\theta}\left(1 - e^{-\theta t}\right) \right\} dt$$
$$= C_d\, \theta \left\{ \frac{q t_1^2}{2} - \frac{(q\theta + D)}{3} t_1^3 \right\} \qquad \text{……….(16)}$$

And corresponding Total cost is

$$TC_3 = C_0 + cq + C_h \left\{ \frac{q t_1^2}{2} - \frac{(q\theta + D)}{3} t_1^3 \right\} + C_d\, \theta \left\{ \frac{q t_1^2}{2} - \frac{(q\theta + D)}{3} t_1^3 \right\}$$
$$+ C_s D \left\{ T t_1 - \frac{1}{2}\left(T^2 + t_1^2\right) \right\} \qquad \text{……….(17)}$$

4.4 model with Permissible delay in payment:

Here, we assume that for some practical situation supplier offer a permissible delay period m (say) to retailers to buy more from supplier to make more profit. Mathematical model has been derived under two different situations, i.e., *case I:* the permissible delay period is less than or equal to replenishment cycle period for settling the account and *case II:* the permissible delay period is greater than replenishment cycle period for settling the account.

Case I: $m \leq T_1$ *(Payment at or before total depletion of inventory)*

In this case, the replenishment cycle time is longer than or equal to *m*. Therefore, the delay in payments is permitted and the total relevant cost includes both the interest charged and the interest earned.

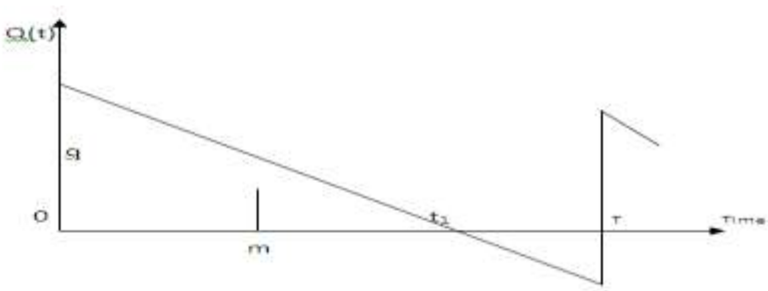

Fig. 4 Payment at or before total depletion of inventory

The interest payable during the cycle is given by

$$IC_I = c\, l_c \int_m^{t_1} Q(t)\; dt \; = c\, l_c \int_m^{t_1} \left\{ q e^{-\theta t} - \frac{D}{\theta}\left(1 - e^{-\theta t}\right) \right\} dt$$

$$= \frac{c\,I_c}{\theta}\left\{(q+D)\left(e^{-\theta m}-e^{-\theta t_1}\right)-D(t_1-m)\right\} \qquad \text{……….} (18)$$

And, the interest earned during the cycle is

$$IE_I = c\,I_e \int_0^{T_1} D\,t\;dt \quad = cI_e\,D\frac{t_1^2}{2} \qquad \text{……….} (19)$$

Therefore, the total cost over the cycle is

$$TC_{4I} = C_0 + cq + HC + DC + SC + IC_1 - IE_1 \qquad \text{……….} (20)$$

Case II $m>t_1$ *(Payment after depletion)*

In this case, the replenishment cycle time is less than *m*. The interest charged during the time period $(0, t_1)$ is equal to zero when $m>t_1$ because the supplier can be paid in full at the permissible delay, m. Therefore, the total relevant cost includes the interest earned in the cycle is the interest during the time period $(0, t_1)$ and the interest earned from the cash invested during the time period (t_1, m) after the inventory is exhausted at time t_1 and it is given by

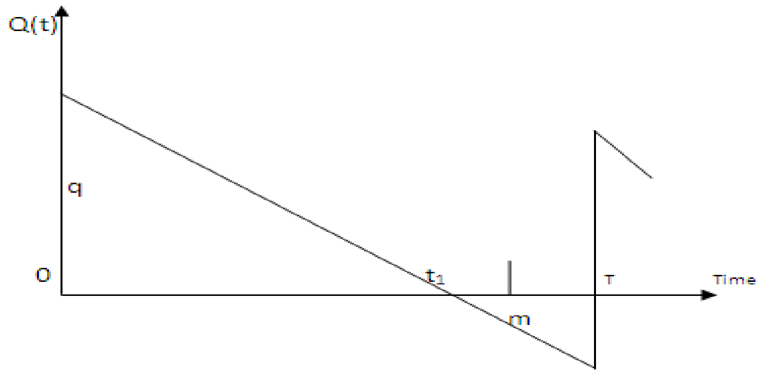

Fig. 5 Payment after depletion

$$IE_2 = c\,I_e\left\{\int_0^{t_1} D\,t\;dt \;+(m-t_1)\int_0^{T} D\,dt\right\} \quad = cI_e\,D\left\{\frac{t_1^2}{2}+(m-t_1)T\right\} \qquad \text{……….} (21)$$

Therefore, the total cost over the cycle is

$$TC_{4II} = C_0 + cq + HC + DC + SC - IE_2 \qquad \text{……….} (22)$$

For more realistic approach we consider, Case I as the optimal total cost of the model for permissible delay in payment.

Therefore, Total cost is

$$TC_4 = C_0 + cq + \frac{C_h}{\theta}\left\{q\left(1-e^{-\theta t_1}\right)-\frac{D}{\theta}\left(e^{-\theta t}+\theta t_1-1\right)\right\}+C_d\left\{q\left(1-e^{-\theta t_1}\right)-\frac{D}{\theta}\left(e^{-\theta t}+\theta t_1-1\right)\right\}$$

$$+C_s\,D\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\}+\frac{c\,I_c}{\theta}\left\{(q+D)\left(e^{-\theta m}-e^{-\theta t_1}\right)-D(t_1-m)\right\} - cI_e\,D\frac{t_1^2}{2} \qquad \text{…….} (23)$$

Our object is to minimize the total cost (23). Differentiating equation (23) partially with respect to q two times, we get

$$\frac{d^2TC_4}{dq^2} > 0$$

Consequently, TC_4 is a convex function of 'q'. Thus, there exists a unique value of 'q' which minimizes TC_4. The numerical result is given after its fuzzy model.

4.5 Model with Considering Demand as Fuzzy:

Here, we consider the model in fuzzy environment. Let the Demand D as fuzzy as a triangular fuzzy number as $\tilde{D} = [d_1, d_2, d_3]$ with the membership function $\mu_{\tilde{D}}(x)$ is defined by

$$\mu_{\tilde{D}}(x) = \begin{cases} \dfrac{x-d_1}{d_2-d_1} & if\ d_1 \le x \le d_2 \\ \dfrac{d_3-x}{d_3-d_2} & if\ d_3 \le x \le d_4 \\ 0 & Otherwise \end{cases}$$

By α-cut of the fuzzy number we have, $D^-(\alpha)$ =d_1+α(d_2-d_1) and $D^+(\alpha)$ =d_3-α(d_3-d_2) are known as lower and upper bounds respectively.

Then, the model can be express by the following differential equations:

$$\frac{dQ(t)}{dt} + \theta Q(t) = -\tilde{D},\ 0 \le t \le t_1 \qquad \text{…………… (24)}$$

$$\frac{dQ(t)}{dt} = -\tilde{D},\ t_1 \le t \le T \qquad \text{………….. (25)}$$

With the boundary conditions Q(0)=q and Q(t_1)=0.

Solution of (24) is
$$Q(t) = q e^{-\theta t} - \frac{\tilde{D}}{\theta}\left(1-e^{-\theta t}\right),\ 0 \le t \le t_1 \qquad \text{…… (26)}$$

At t=t_1, Q(t_1)=0 then from (26), we have
$$\frac{\tilde{D}}{\theta} - \left(q + \frac{\tilde{D}}{\theta}\right) e^{-\theta t_1} = 0 \qquad \text{…………. (27)}$$

This expression clearly shows the initial inventory level of the above model.

Solution of (25) is
$$Q(t) = \tilde{D}(t_1 - t),\ t_1 \le t \le T \qquad \text{…………. (28)}$$

Corresponding costs are:
Setup cost = C_0
Purchased cost = cq
Lower and upper Holding costs are:

$$HC^- = \frac{C_h}{\theta}\left\{q\left(1-e^{-\theta t_1}\right) - \frac{D^+(\alpha)}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right)\right\}$$

$$HC^+ = \frac{C_h}{\theta}\left\{q\left(1-e^{-\theta t_1}\right) - \frac{D^-(\alpha)}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right)\right\} \qquad \text{…………. (29)}$$

Lower and upper Deterioration costs are:

$$DC^- = C_d\left\{q\left(1-e^{-\theta t_1}\right) - \frac{D^+(\alpha)}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right)\right\}$$

$$DC^+ = C_d\left\{q\left(1-e^{-\theta t_1}\right) - \frac{D^-(\alpha)}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right)\right\} \qquad \text{…………. (30)}$$

Lower and upper Shortage costs are:

$$SC^- = C_s D^+(\alpha)\left\{Tt_1 - \frac{1}{2}\left(T^2 + t_1^2\right)\right\}$$

$$SC^+ = C_s D^-(\alpha)\left\{Tt_1 - \frac{1}{2}\left(T^2 + t_1^2\right)\right\} \qquad \text{…………. (31)}$$

Therefore, the lower and upper total costs over the cycle are

$$TC^-_5 = C_0 + cq + \frac{C_h}{\theta}\left\{q\left(1-e^{-\theta t_1}\right)-\frac{D^+(\alpha)}{\theta}\left(e^{-\theta t}+\theta t_1-1\right)\right\} + C_d\left\{q\left(1-e^{-\theta t_1}\right)-\frac{D^+(\alpha)}{\theta}\left(e^{-\theta t}+\theta t_1-1\right)\right\} + C_s D^-(\alpha)\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\} \quad \ldots\ldots\ldots\ldots\ (32)$$

$$TC^+_5 = C_0 + cq + \frac{C_h}{\theta}\left\{q\left(1-e^{-\theta t_1}\right)-\frac{D^-(\alpha)}{\theta}\left(e^{-\theta t}+\theta t_1-1\right)\right\} + C_d\left\{q\left(1-e^{-\theta t_1}\right)-\frac{D^-(\alpha)}{\theta}\left(e^{-\theta t}+\theta t_1-1\right)\right\} + C_s D^-(\alpha)\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\} \quad \ldots\ldots\ldots\ldots\ (33)$$

Where $D^-(\alpha)=d_1+\alpha(d_2-d_1)$ and $D^+(\alpha)=d_3-\alpha(d_3-d_2)$ are known as lower and upper bounds respectively.
Therefore the model mathematically can be written as:

Minimize $\left\{TC_5^-,\ TC_5^+\right\}$ …………. (33a)

Subject to $0\le\alpha\le1$

Thus, the problem is a multi-objective optimization problem. To convert it as a single objective optimization problem we use Global Criteria method.
Then the above problem reduces to

Minimize GC_5 …………. (33b)

Subject to $0\le\alpha\le1$

Hence, the optimal solution is obtained by using Lingo software.

4.6 Model With Considering Deterioration And Purchasing Cost As Fuzzy:

Here, we consider the model in fuzzy environment. Let the Deterioration and purchasing cost θ & c as fuzzy as a triangular fuzzy number as $\tilde{\theta}=[\theta_1,\theta_2,\theta_3]$ and $\tilde{c}=[c_1,c_2,c_3]$ with the membership function $\mu_{\tilde{\theta}}(x)$ & $\mu_{\tilde{c}}(x)$ is defined by

$$\mu_{\tilde{\theta}}(x)=\begin{cases}\dfrac{x-\theta_1}{\theta_2-\theta_1} & if\ \theta_1\le x\le\theta_2\\ \dfrac{\theta_3-x}{\theta_3-\theta_2} & if\ \theta_3\le x\le\theta_4\\ 0 & Otherwise\end{cases} \quad \& \quad \mu_{\tilde{c}}(x)=\begin{cases}\dfrac{x-c_1}{c_2-c_1} & if\ c_1\le x\le c_2\\ \dfrac{c_3-x}{c_3-c_2} & if\ c_3\le x\le c_4\\ 0 & Otherwise\end{cases}$$

By α-cut of the fuzzy number we have, [$\theta^-(\alpha)=\theta_1+\alpha(\theta_2-\theta_1)$, $\theta^+(\alpha)=\theta_3-\alpha(\theta_3-\theta_2)$] and [$c^-(\alpha)=c_1+\alpha(c_2-c_1)$, $c^+(\alpha)=c_3-\alpha(c_3-c_2)$] are known as corresponding lower and upper bounds respectively.
Then, the model can be express by the following differential equations:

$$\frac{dQ(t)}{dt}+\tilde{\theta}\,Q(t)=-D,\ 0\le t\le t_1 \quad \ldots\ldots\ldots\ldots\ (34)$$

$$\frac{dQ(t)}{dt}=-D,\ t_1\le t\le T \quad \ldots\ldots\ldots\ldots\ (35)$$

With the boundary conditions Q(0)=q and $Q(t_1)=0$.

Solution of (34) is $Q(t)=q\,e^{-\tilde{\theta}t}-\frac{D}{\tilde{\theta}}\left(1-e^{-\tilde{\theta}t}\right),\ 0\le t\le t_1$ … (36)

At $t=t_1$, $Q(t_1)=0$ then from (36), we have $\frac{D}{\tilde{\theta}}-\left(q+\frac{D}{\tilde{\theta}}\right)e^{-\tilde{\theta}t_1}=0$ ….. (37)

This expression clearly shows the initial inventory level of the above model.

Solution of (35) is $Q(t)=D(t_1-t),\ t_1\le t\le T$ ………. (38)

Corresponding costs are:
Setup cost = C_0
Lower and upper Purchased cost

$$PC^- = c^+(\alpha)q \;\&\; PC^+ = c^-(\alpha)q$$

Lower and upper Holding costs are:

$$HC^- = \frac{C_h}{\theta^+(\alpha)}\left\{q\left(1-e^{-\theta^+(\alpha)t_1}\right)-\frac{D}{\theta^+(\alpha)}\left(e^{-\theta^+(\alpha)t}+\theta^+(\alpha)t_1-1\right)\right\}$$

$$\& \; HC^+ = \frac{C_h}{\theta^-(\alpha)}\left\{q\left(1-e^{-\theta^-(\alpha)t_1}\right)-\frac{D}{\theta^-(\alpha)}\left(e^{-\theta^-(\alpha)t}+\theta^-(\alpha)t_1-1\right)\right\} \quad \text{…… (39)}$$

Lower and upper Deterioration costs are:

$$DC^- = C_d\left\{q\left(1-e^{-\theta^+(\alpha)t_1}\right)-\frac{D}{\theta^+(\alpha)}\left(e^{-\theta^+(\alpha)t}+\theta^+(\alpha)t_1-1\right)\right\}$$

$$\& \; DC^+ = C_d\left\{q\left(1-e^{-\theta^-(\alpha)t_1}\right)-\frac{D}{\theta^-(\alpha)}\left(e^{-\theta^-(\alpha)t}+\theta^-(\alpha)t_1-1\right)\right\} \quad \text{…… (40)}$$

Lower and upper Shortage costs are:

$$SC^- = C_s D\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\} \;\&\; SC^+ = C_s D\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\} \quad \text{……. (41)}$$

Therefore, the lower and upper total costs over the cycle are

$$TC^-_6 = C_0 + c^+(\alpha)q + \frac{C_h}{\theta^+(\alpha)}\left\{q\left(1-e^{-\theta^+(\alpha)t_1}\right)-\frac{D}{\theta^+(\alpha)}\left(e^{-\theta^+(\alpha)t}+\theta^+(\alpha)t_1-1\right)\right\}$$

$$+ C_d\left\{q\left(1-e^{-\theta^+(\alpha)t_1}\right)-\frac{D}{\theta^+(\alpha)}\left(e^{-\theta^+(\alpha)t}+\theta^+(\alpha)t_1-1\right)\right\}+C_s\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\} \quad \text{……. (42)}$$

$$TC^+_6 = C_0 + c^-(\alpha)q + \frac{C_h}{\theta^-(\alpha)}\left\{q\left(1-e^{-\theta^-(\alpha)t_1}\right)-\frac{D}{\theta^-(\alpha)}\left(e^{-\theta^-(\alpha)t}+\theta^-(\alpha)t_1-1\right)\right\}$$

$$+ C_d\left\{q\left(1-e^{-\theta^-(\alpha)t_1}\right)-\frac{D}{\theta^-(\alpha)}\left(e^{-\theta^-(\alpha)t}+\theta^-(\alpha)t_1-1\right)\right\}+C_s\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\} \quad \text{……. (43)}$$

Where [$\theta^-(\alpha)=\theta_1+\alpha(\theta_2-\theta_1)$, $\theta^+(\alpha)=\theta_3-\alpha(\theta_3-\theta_2)$] and [$c^-(\alpha)=c_1+\alpha(c_2-c_1)$, $c^+(\alpha)=c_3-\alpha(c_3-c_2)$] are known as corresponding lower and upper bounds respectively.

Therefore the model mathematically can be written as:

$$\text{Minimize} \quad \left\{TC_6^-,\; TC_6^+\right\} \quad \text{…………… (44a)}$$
$$\text{Subject to} \quad 0 \le \alpha \le 1$$

Thus, the problem is a multi-objective optimization problem. To convert it as a single objective optimization problem we use Global Criteria method.
Then the above problem reduces to

$$\text{Minimize} \quad GC_6 \quad \text{…………… (44b)}$$
$$\text{Subject to} \quad 0 \le \alpha \le 1$$

Hence, the optimal solution is obtained by using Lingo software.

4.7 Model with Considering Random Demand:

Here, we consider the model in stochastic environment. Let the Demand X is a random variable with mean demand as

$$\mu(x) = \int_a^b x \ f(x) \ dx \qquad \text{…………. (45)}$$

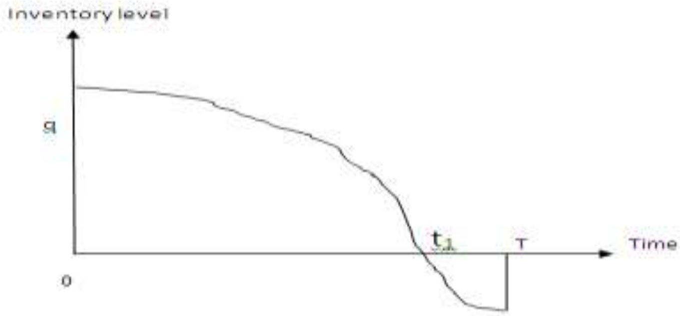

Fig. 6 graphically show the inventory level

Then from equation (3) & (4) by solving we have

$$Q(t) = q e^{-\theta t} - \frac{x}{\theta}\left(1 - e^{-\theta t}\right), \ 0 \le t \le t_1 \qquad \text{…………. (46)}$$

And $$Q(t) = x(t_1 - t), \ t_1 \le t \le T \qquad \text{…………. (47)}$$

Corresponding expected costs are:
Setup cost = C_0
Purchased cost = cq
Expected Holding cost:

$$\text{EHC} = C_h \int_a^b \left\{ \int_0^{t_1} Q(t) \ dt \right\} f(x) dx$$

$$= \frac{C_h}{\theta} \left\{ q\left(1 - e^{-\theta t_1}\right) - \frac{\mu(x)}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right) \right\} \qquad \text{………… (48)}$$

Expected Deterioration cost:

$$\text{EDC} = C_d \int_a^b \left\{ \int_0^{t_1} \theta(t) \ Q(t) \ dt \right\} f(x) dx$$

$$= C_d \left\{ q \left(1 - e^{-\theta t_1}\right) - \frac{\mu(x)}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right) \right\} \qquad \text{………… (49)}$$

Expected Shortage cost:

$$\text{ESC} = C_s \int_a^b \left\{ \int_{t_1}^{T} Q(t) \ dt \right\} f(x) dx$$

$$= C_s \ \mu(x) \left\{ T t_1 - \frac{1}{2}\left(T^2 + t_1^2\right) \right\} \qquad \text{………… (50)}$$

Therefore, the Expected Total Cost over the cycle is

$$\text{ETC}_6 = C_0 + cq + \frac{C_h}{\theta} \left\{ q\left(1 - e^{-\theta t_1}\right) - \frac{\mu(x)}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right) \right\} + C_d \left\{ q \left(1 - e^{-\theta t_1}\right) - \frac{\mu(x)}{\theta}\left(e^{-\theta t} + \theta t_1 - 1\right) \right\}$$

$$+ \ C_s \ \mu(x) \left\{ T t_1 - \frac{1}{2}\left(T^2 + t_1^2\right) \right\} \qquad \text{………… (51)}$$

Case I: If X~ Uniform Distribution $f(x) = \dfrac{1}{b-a}$ $\ a \le x \le b$ then, the mean demand

$$\mu(x)=\frac{1}{a+b}$$

Hence, the Expected Total Cost over the cycle is

$$ETC_{6I}=C_0+cq+\frac{C_h}{\theta}\left\{q\left(1-e^{-\theta t_1}\right)-\frac{1}{\theta(a+b)}\left(e^{-\theta t}+\theta t_1-1\right)\right\}+C_d\left\{q\left(1-e^{-\theta t_1}\right)-\frac{1}{\theta(a+b)}\left(e^{-\theta t}+\theta t_1-1\right)\right\}$$
$$+\frac{C_s}{(a+b)}\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\} \qquad \ldots\ldots\ldots\ldots (52)$$

Case II: If X~Exponential Distribution $f(x)=a\,e^{bx}\quad 0\le x\le\infty$ then, the mean demand

$$\mu(x)=\frac{a}{b^2}\left\{\left(b^2-1\right)e^{b^2}-(ab-1)e^{ab}\right\}$$

Hence, the Expected Total Cost over the cycle is

$$ETC_{6II}=C_0+cq+\left|\frac{C_h}{\theta}\left\{q\left(1-e^{-\theta t_1}\right)-\frac{1}{\theta(a+b)}\left(e^{-\theta t}+\theta t_1-1\right)\right\}+C_d\left\{q\left(1-e^{-\theta t_1}\right)-\frac{1}{\theta(a+b)}\left(e^{-\theta t}+\theta t_1-1\right)\right\}\right.$$
$$\left.+\frac{C_s}{(a+b)}\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\}\right|_{\mu(x)=\frac{a}{b^2}\left\{\left(b^2-1\right)e^{b^2}-(ab-1)e^{ab}\right\}} \qquad \ldots\ldots\ldots\ldots (53)$$

Our object is to minimize the total cost (52) & (53). Differentiating equation (52) & (53) partially with respect to q two times, we get

$$\frac{d^2ETC_{7I}}{dq^2}>0\ \ \&\ \ \frac{d^2ETC_{7II}}{dq^2}>0$$

Consequently, ETC_{7I} & ETC_{7II} are convex function of ‘q’. Thus, there exists a unique value of ‘q’ which minimizes ETC_{7I} & ETC_{7II}.

4.8 Model With Considering Expectation Of A Fuzzy Random Variable:

Here, we consider the model in fuzzy-stochastic environment. Let, X be a trapezoidal fuzzy number. Then by Liu and Liu (2002) and using Equation (2), the expected value of fuzzy variable $\tilde{X}=[d_1,d_2,d_3,d_4],\ 0<\rho<1$ is defined as

$$E\left[\tilde{X}\right]=\frac{1}{2}\left[(1-\rho)(d_1+d_2)+\rho(d_3+d_4)\right] \qquad \ldots\ldots\ldots\ldots\ldots (54)$$

Then, the Expected Total Cost over the cycle is

$$ETC_8=C_0+cq+\left|\frac{C_h}{\theta}\left\{q\left(1-e^{-\theta t_1}\right)-\frac{D}{\theta}\left(e^{-\theta t}+\theta t_1-1\right)\right\}+C_d\left\{q\left(1-e^{-\theta t_1}\right)-\frac{D}{\theta}\left(e^{-\theta t}+\theta t_1-1\right)\right\}\right.$$
$$\left.+C_s\,D\left\{Tt_1-\frac{1}{2}\left(T^2+t_1^2\right)\right\}\right|_{E[\tilde{X}]} \qquad \ldots\ldots\ldots\ldots (55)$$

Differentiating equation (55) partially with respect to q two times, we get

$$\frac{d^2ETC_8}{dq^2}>0$$

Consequently, ETC_8 are convex function of ‘q’. Thus, there exists a unique value of ‘q’ which minimizes ETC_8.

V. NUMERICAL EXAMPLES

5.1Numerical Example:

Case I: *model without shortage:*

Let C_0 = Rs.100 per cycle, c=Rs. 10 per unit, D=500 units, C_h = Rs7. per unit, C_d = Rs.5 per unit, θ= 0.06, T=1years. Then order quantity, and total cost are respectively as:

q = 515.3046 units, and TC = Rs. 7115.100

Table 1. Sensitivity Analysis of the model without shortage of Deterioration			
Change in	changes	q	TC
Θ	0.05	512.710	7070.199
	0.06	515.310	7115.100
	0.07	517.928	7160.292
	0.08	520.540	7205.778
	0.09	523.190	7251.560
C_d	4.0	515.310	7099.795
	4.5	515.310	7107.447
	5.0	515.310	7115.100
	5.5	515.310	7122.752
	6.0	515.310	7130.404

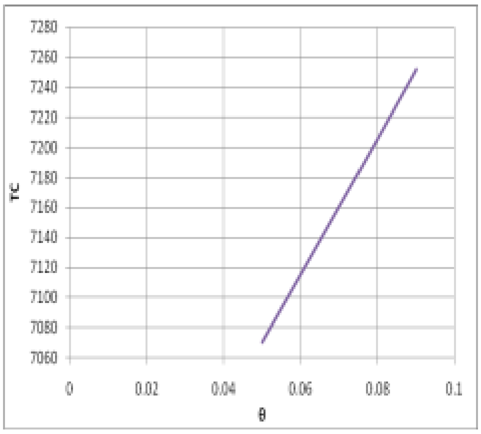

Fig. 7 graphically TC changes with θ

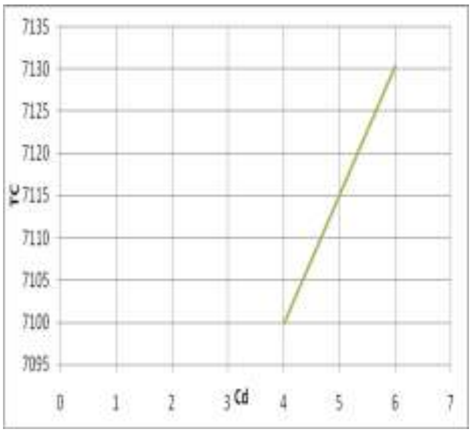

Fig. 8 graphically TC changes with C_d

Case II: *model with shortage:*

Let C_0 = Rs.100 per cycle, c=Rs. 10 per unit, D=500 units, C_h = Rs.7 per unit, C_d = Rs.5 per unit, C_s = Rs.1 per unit θ = 0.06, t_1=0.6 year, T=1year. Then order quantity, and total cost are respectively as:

q = 305.4654 units, and TC = Rs. 6552.303

Table 2. Sensitivity Analysis of the model without shortage of Deterioration			
Change in	changes	q	TC
t_1	0.60	305.46	6552.303
	0.70	357.45	8308.437
	0.80	409.76	10262.2
	0.85	436.02	11313.56
	0.90	462.37	12414.78
C_s	0.6	305.46	5459.226
	0.8	305.46	6005.764
	1.0	305.46	6552.303
	1.2	305.46	7098.842
	1.4	305.46	7645.381

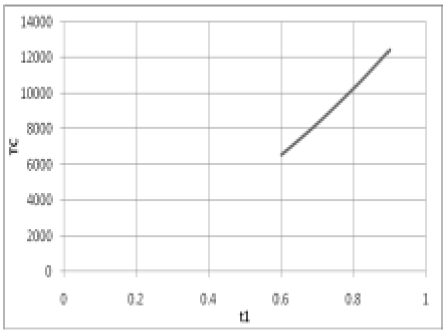

Fig. 9 graphically TC changes with t_1

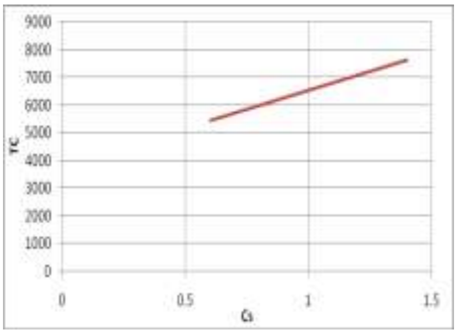

Fig. 10 graphically TC changes with C_s

5.2 Numerical Example of the model with time dependent HC:

Let C_0 = Rs.100 per cycle, c=Rs. 10 per unit, D=500 units, C_h = Rs.7 per unit, C_d = Rs.5 per unit, C_s = Rs.1 per unit θ = 0.06, t_1=0.6 year, T=1year. Then order quantity, and total cost are respectively as: *q = 305.4654 units and TC = Rs. 6038.324*

Table 3. Total cost with constant and time varying HC

Model	HC	Time dependent HC
q	305.4654	305.4654
TC	6552.303	6038.324

5.3 Numerical Example of the model with time dependent DC:

Let C_0 = Rs.100 per cycle, c=Rs. 10 per unit, D=500 units, C_h = Rs.7 per unit, C_d = Rs.5 per unit, C_s = Rs.1 per unit θ = 0.06, t_1=0.6 year, T=1year. Then order quantity, and total cost are respectively as:

q = 305.4654 units, and TC = Rs. 6530.275

Table 4. Total cost with constant and time varying DC

Model	DC	Time dependent DC
q	305.4654	305.4654
TC	6552.303	6530.275

5.4 Numerical Example of the model Permissible delay in payment (PDP):

Let C_0 = Rs.100 per cycle, c=Rs. 10 per unit, D=500 units, C_h = Rs.7 per unit, C_d = Rs.5 per unit, C_s = Rs.1 per unit θ = 0.06, t_1=0.6 year, T=1year, m=0.5year, Ic=0.2, Ie=0.16. Then order quantity, and total cost are respectively as:

q = 305.4654 units, and TC = Rs. 5041.500

Table 5. Total cost with permissible delay in payment (PDP)

Model	Without PDP	With PDP
q	305.4654	305.4654
TC	6552.303	5041.500

5.5 Numerical Example of the model with Fuzzy Demand:

Let C_0 = Rs.100 per cycle, c=Rs. 10 per unit, C_h = Rs.7 per unit, C_d = Rs.5 per unit, C_s = Rs.1 per unit, θ = 0.06, t_1=0.6 year, T=1year, $\tilde{D}$ = (300, 500, 800), α=0.2. Then, $D^- = 340$ and $D^+ = 760$ .Corresponding $MinTC = 6552.303$, $MaxTC = 6887.778$, $MinTC^- = 12067.42$, $MaxTC^- = 12724.66$, $MinTC^+ = 3892.932$, $MaxTC = 4080.495$ and hence by Global Criteria Method for usual value of p=2 we get, total Cost is 17.15895.

5.6 Numerical Example of the model with Fuzzy Deterioration & Purchasing cost:

Let C_0 = Rs.100 per cycle, D=500 units, C_h = Rs.7 per unit, C_d = Rs.5 per unit, C_s = Rs.1 per unit, t_1=0.6 year, T=1year, $\tilde{\theta}$ = (0.02, 0.06, 0.10), α=0.2, then, $\theta^- = 0.0208$ & $\theta^+ = 0.0992$. And $\tilde{c}$ = (5, 10 15) Corresponding $MinTC = 6552.303$, $MaxTC = 6887.778$, $MinTC^- = 9947.832$, $MaxTC^- = 10465.17$, $MinTC^+ = 3221.521$, $MaxTC = 3380.627$ and hence by Global Criteria Method for usual value of p=2 we get, total Cost is 22.99926.

Table 6. Total cost with demand as fuzzy

Model	Fuzzy Demand	Fuzzy Deterioration & Purchasing cost
TC	17.15895	22.99926

5.7 Numerical Example of the model with stochastic demand:

Let C_0 = Rs.100 per cycle, c=Rs. 10 per unit, C_h = Rs.7 per unit, C_d = Rs.5 per unit, C_s = Rs.1 per unit θ = 0.06, t_1=0.6 year, T=1year, m=0.5year, Ic=0.2, Ie=0.16,

Case I: X~ Uniform Distribution:

For a=250, b=600, μ(x)=425, Then order quantity, and total cost are respectively as:

q = 259.6456 units, and ETC = Rs. 5236.039

Table 7. Sensitivity Analysis of the model without Uniform Demand				
Change in a	Change in b	q	μ(x)	TC
200	500	213.8253	350	4042.746
225	550	236.7357	387.5	4624.021
250	600	259.6456	425	5236.039
275	650	282.5555	462.5	5878.80
300	700	305.4654	500	6552.303

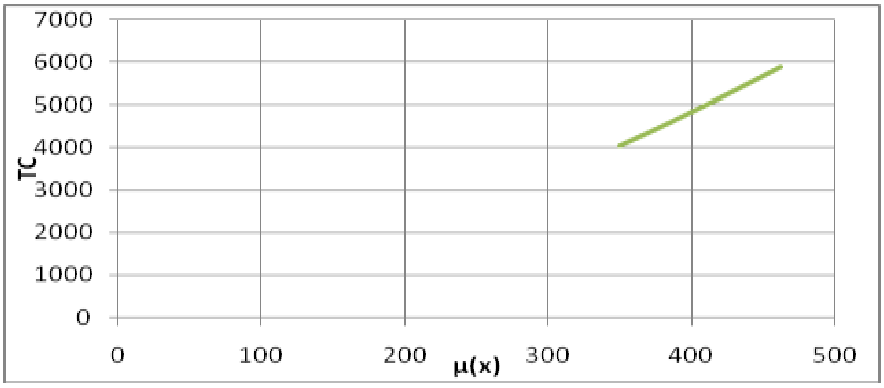

Fig.11 graphically TC changes with μ(x)

Case II: X~ Exponential Distribution:

For a=0.5, b=2.5, μ(x)=217.4956, Then order quantity, and total cost are respectively as:

q =132.875 units, and ETC = Rs.2235.070

Table 8. Sensitivity Analysis of the model without Uniform Demand				
Change in a	Change in b	q	μ(x)	TC
0.3	2.3	29.729	48.289	484.728
0.4	2.4	64.092	104.908	1000.739
0.5	2.5	132.875	217.496	2235.070
0.6	2.6	269.288	440.783	5502.816
0.7	2.7	540.434	884.607	15234.44

Fig.11 graphically TC changes with μ(x)

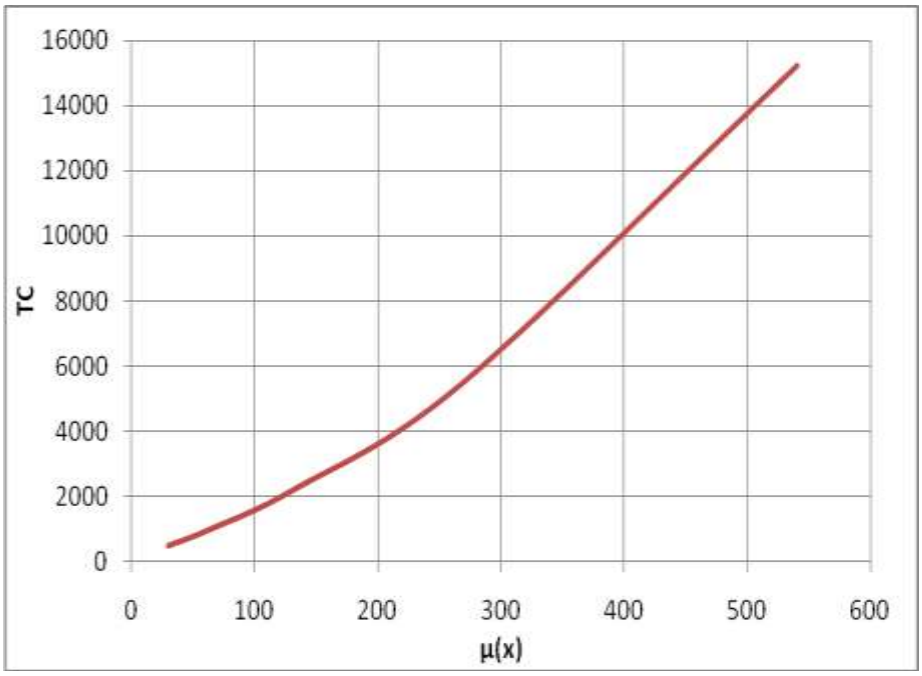

Table 9. Total cost with demand as stochastically

Model	In Crisp	With Uniform Demand	With Exponential Demand
q	305.465	259.646	132.875
μ(x)	500	425	217.496
ETC	6552.303	5236.039	2235.070

5.8 Numerical Example of the model with Expected value of a fuzzy variable:

Let C_0 = Rs.100 per cycle, c=Rs. 10 per unit, C_h = Rs.7 per unit, C_d = Rs.5 per unit, C_s = Rs.1 per unit θ = 0.06, t_1=0.6 year, T=1year and $\tilde{X} = [400, 450, 500, 550]$ Then order quantity, and total cost are respectively as:

q = 256.3643 units, and ETC = Rs. 5065.953

Table 10. Total cost with demand as fuzzy- stochastic

Model	Crisp	Stochastic with Uniform Demand	Fuzzy-stochastic demand
q	305.465	259.646	256.364
TC/ETC	6552.303	5236.039	5065.953

Table 8. Sensitivity Analysis of the model

Model under	Conditions	Initial inventory (q)	Mean Demand	TC or ETC
Crisp Environment	Without shortage	515.3046	500	7115.100
	Without shortage	305.4654	500	6552.303
	Time varying HC	305.4654	500	6038.324
	Time varying DC	305.4654	500	6530.275
	PDP	305.4654	500	5041.500
Fuzzy Environment	Demand as fuzzy	305.4654	[340, 760]	17.15895
	Deterioration & purchasing cost as fuzzy	305.4654	500	22.99926
Stochastic Environment	Uniform demand	259.6456	425	5236.039
	Exponential demand	132.875	217.4956	2235.070
Fuzzy-stochastic Environment	Expected value of a fuzzy variable	256.3643	500	5065.983

VI. Observations

- Increase of rate of deterioration, unit charge of deteriorating item & cost of shortage increase of total cost of the system. Consideration of deterioration & shortages in inventory make model more realistic and it helps the organization to decide how much profit wants during the cycle for the firm.
- Time dependent holding cost & time dependent deterioration cost decreases the total cost than static value of that.
- Permissible delay in payments reduced the annual total cost of the customer; this is a one of the great opportunity to him to buy more. This maximizes profit of both customer & retailer at the same time.
- Total cost increases in fuzzy deterioration & purchasing cost than fuzzy demand. The above result analyzes that fuzzy environment more practical in the realistic world.
- Fuzzy, Stochastic & fuzzy stochastic results are better than crisp result. That is very realistic in nature.
- Total costs are highly sensitive in variation of time to start of shortage, cost in different environments and sensible with other parameters shortage, deterioration, purchasing, mean demand etc.

VII. Concluding Remark

In this paper, we develop an EOQ inventory model for deteriorating item by considering eight different circumstances. The inventory problems for deteriorating items have attracted more and more attention and many researchers have conducted extensive studies in this area. Demand, deterioration & purchasing cost considered as constant first in Crisp then as uncertain, randomness and combinations of them. We can make a good comparative study between the results of the with-shortage case and without-shortage case. Also, we have developed an inventory model with time dependent holding cost & deterioration, which is a realistic situation for many items during storage period; finally we have taken permissible delay which is very real situation now a day. In real life situations, the supplier frequently offers a permissible delay to attract the customer for increasing their ordering quantity. We derive optimality conditions for finding the optimal solution. Finally we have given a numerical example to justify our model. Numerical example shows the applicability of the proposed model. In the numerical examples, it is found that the optimum total cost in without shortage case is more than that of the with shortage case. Fuzzy, Stochastic & fuzzy stochastic results are better than crisp result. Those are very realistic in nature.

The analytical model is developed based on the above stated concepts. The method used of solving the problem is partly analytical and partly computational and is user-friendly. The sensitivity of the solution to changes in the values of different parameters has been discussed.

The proposed model can be extended in several ways. For instance, we may extend the time dependent demand, deterioration rate. We could consider the demand as a function of selling price, stock- dependent, time product quantity, etc. Finally, we could generalize the model for quantity discount and others under different environments. In practical situation, the information about inventory is not always precise, most of the time it is vague or imprecise. So it is more reasonable to develop some fuzzy, stochastic and dynamic research methods and this also the future trend of the deteriorating inventory study.

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