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Analytical Analysis of Vorticity Transport in Maxwellian Viscoelastic Fluid

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ABSTRACT: *The transport of vorticity in Maxwellian viscoelastic fluid in the presence of suspended magnetic particles through porous medium is considered here. Equations governing the transport of vorticity in Maxwellian viscoelastic fluid in the presence of suspended magnetic particles are obtained from the equations of magnetic fluid flow. From these equations it follows that the transport of solid vorticity is coupled with the transport of fluid vorticity in a porous medium. Further, we find that because of thermokinetic process, fluid vorticity may exist in the absence of solid vorticity, but when fluid vorticity is zero, then solid vorticity is necessarily zero. A two-dimensional case is also studied and found that the fluid vorticity is indirectly influenced by the temperature and the magnetic field gradient.*

Keywords: *Maxwellian viscoelastic fluid, Porous Medium, Suspended magnetic particles, Vorticity*

I. INTRODUCTION

Magnetic fluids are suspensions of small magnetic particles in a liquid carrier. Under normal conditions, the material behaves like a viscous fluid. When it is exposed to a magnetic field, the particles inside align and it responds to the field, exhibiting magnetized behaviour. There are a number of uses for magnetic fluids, ranging from medicine to industrial manufacturing. It is, therefore, a two-phase system consisting of solid and liquid phases. The net effect of the particles suspended in the fluid is an extra dragging force acting on the system. This is due to relative velocity between the solid and fluid particles. In recent years, there has been considerable interest in the study of magnetic fluids. Saffman [1] proposed the equations of the flow of suspension of non-magnetic particles. These equations were modified by Wagh [2] to describe the flow of magnetic fluid, by including the magnetic body force $\mu_0 M\nabla H$. Wagh and Jawandhia [3] have studied the transport of vorticity in a magnetic fluid. Transport and sedimentation of suspended particles in inertial pressure-driven flow has been considered by Yan and Koplik [4].

In all the above studies, the fluid was considered as Newtonian, but many industrially important fluids (molten plastics, polymers, pulps and foods) exhibit a non-Newtonian fluid behaviour. Many common materials (paints and plastics) and more exotic ones (silicic magma, saturated soils, and the Earth's lithosphere) behave as viscoelastic fluids. With the growing importance of non-Newtonian materials in various manufacturing and processing industries, considerable effort has been directed towards understanding their flow. Widely used theoretical models (models A and B, respectively) for certain classes of viscoelastic fluids have been proposed by Oldroyd [5]. Vest and Arpaci [6] have studied the stability of a horizontal layer of Maxwell's viscoelastic fluid heated from below. The thermal instability of Maxwellian viscoelastic fluid in the presence of a uniform rotation has been considered by Bhatia and Steiner [7], where rotation is found to have a destabilizing effect. This is in contrast to the thermal instability of a Newtonian fluid where rotation has a stabilizing effect. In another study, Bhatia and Steiner [8] have studied the problem of thermal instability of a viscoelastic fluid in hydromagnetics and have found that the magnetic field has the stabilizing influence on Maxwell fluid just as in the case of Newtonian fluid.

The medium has been considered to be non-porous in all the above studies. In recent years, the investigations of flow of fluids through porous media have become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in the books by Phillips [9], Ingham and Pop [10], and Nield and Bejan [11]. When the fluid slowly permeates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy's law according to which the usual viscous term in the equations of fluid motion is replaced by the resistance term $-\left(\frac{\mu}{k_1}\right)\vec{q}$, where μ is the fluid viscosity, k_1 is the medium permeability and $\vec{q}$ is the

Darcian (filter) velocity of the fluid. Lapwood [12] has studied the stability of convective flow in hydrodynamics in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through porous medium has been considered by Wooding [13]. The physical properties of comets, meteorities and interplanetary dust strongly suggest the importance of porosity in astrophysical context (McDonnel [14]). Sharma and Sharma [15] have considered the thermal instability of a rotating Maxwell fluid through porous medium and found that, for stationary convection, the rotation has stabilizing effect whereas the permeability of the medium has both stabilizing as well as destabilizing effect, depending on the magnitude of rotation. Khare and Sahai [16] have considered the effect of rotation on the convection in porous medium in a horizontal fluid layer which was viscous, incompressible and of variable density. Sharma and Kumar [17] have considered the Hall effect on thermosolutal instability in a Maxwellian viscoelastic fluid in porous medium. The thermal instability of a rotating Maxwellian viscoelastic fluid permeated with suspended particles in porous medium has been studied by Kumar [18]. Kumar and Singh [19] have studied the stability of superposed Maxwellian viscoelastic fluids through porous media in hydromagnetics. In another study, Kumar [20] has studied the thermal instability of Maxwellian heterogeneous viscoelastic fluid layer through porous medium. Kumar [21] has also studied the slow, immiscible, Maxwellian viscoelastic liquid-liquid displacement in a permeable medium.

Keeping in mind the importance of non-Newtonian fluids in modern technology and industries and owing to the importance of porous medium in chemical engineering and geophysics, the present paper attempts to study the transport of vorticity in magnetic Maxwellian viscoelastic fluid-particle mixtures in porous medium.

II. BASIC ASSUMPTIONS AND MAGNETIC BODY FORCE

Particles of magnetic material are much larger than the size of the molecules of carrier liquid. Accordingly, considering the limit of a microscopic volume element in which a fluid can be assumed to be a continuous medium and the magnetic particles must be treated as discrete entities. Now, if one considers a cell of magnetic fluid containing a larger number of magnetic particles, then one must consider the micro-rotation of the cell in addition to its translations as a point mass. Thus, one has to assign average velocity $\vec{v}_d$ and the average angular velocity $\vec{\omega}$ of the cell. But, here as an approximation, we neglect the effect of micro-rotation.

We shall also make the following assumptions:

(i) Most of the ferrofluids are relatively poor conductors and hence free current density $\vec{J}$ is negligible and, hence $\vec{J} \times \vec{B}$ is assumed to be insignificant.

(ii) The magnetic field is assumed to be curl free (i.e. $\nabla \times \vec{H} = 0$).

(iii) In many practical situations liquid compressibility is not important. Hence contribution due to magnetic friction can be neglected. The remaining force of the magnetic field is referred to as the magnetization force.

(iv) All time-dependent magnetization effects in the fluid such as hysteresis are assumed to be negligible and the magnetization $\vec{M}$ is assumed to be collinear with $\vec{H}$.

From electromagnetic theory, the force per unit volume in MKS units on a piece of magnetized material of magnetization $\vec{M}$ (i.e. dipole moment per unit volume) in the field of magnetic intensity $\vec{H}$ is $\mu_0\left(\vec{M}.\nabla\right)\vec{H}$, where μ_0 is the free space permeability.

Here we note that $\nabla \, . \, \vec{a} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}$ and $\vec{a} \, . \, \nabla = a_1 \frac{\partial}{\partial x} + a_2 \frac{\partial}{\partial y} + a_3 \frac{\partial}{\partial z}$, where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$.

Using assumption (iv), we get

$$\mu_0\left(\vec{M}.\nabla\right)\vec{H} = \frac{\mu_0 M}{H}\left(\vec{H}.\nabla\right)\vec{H}, \text{ where } M = \left|\vec{M}\right| \text{ and } H = \left|\vec{H}\right| . \tag{1}$$

But $$\left(\vec{H}.\nabla\right)\vec{H} = \frac{1}{2}\nabla\left(\vec{H}.\vec{H}\right) - \vec{H} \times \left(\nabla \times \vec{H}\right) = \frac{1}{2}\nabla\left(\vec{H}.\vec{H}\right) \quad \text{[by assumption (ii)].} \tag{2}$$

Hence $\mu_0\left(\vec{M}.\nabla\right)\vec{H} = \left(\frac{\mu_0 M}{H}\right)\frac{1}{2}\nabla\left(\vec{H}.\vec{H}\right) = \mu_0 M\nabla H.$

Thus, the magnetic body force assumes the form (Rosensweig [22])

$$\vec{f}_m = \mu_0 M\nabla H. \tag{3}$$

III. DERIVATION OF EQUATIONS GOVERNING TRANSPORT OF VORTICITY IN MAGNETIC VISCOELASTIC MAXWELLIAN FLUID

Wagh (1991) modified the Saffman's equations for flow of suspension to describe the flow of magnetic fluid by including the body force $\mu_0 M\nabla H$ acting on the suspended magnetic particles. Let $\Gamma_{ij}, \tau_{ij}, e_{ij}, \mu, \lambda, p, \delta_{ij}, v_i, x_i$ *and* d/dt denote respectively the total stress tensor, the shear stress tensor, the rate-of-strain tensor, the viscosity, the stress relaxation time, the isotropic pressure, the Kroneckor delta, the velocity vector, the position vector and the convective derivative. Then the Maxwellian fluid is described by the constitutive relations

$$\left.\begin{aligned} T_{ij} &= -p\delta_{ij} + \tau_{ij} , \\ \left(1+\lambda\frac{d}{dt}\right)\tau_{ij} &= 2\mu e_{ij} , \\ e_{ij} &= \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) , \end{aligned}\right\} \tag{4}$$

Now the equations expressing the flow of suspended magnetic particles and the flow of Maxwellian viscoelastic fluid in which magnetic particles are suspended in porous medium are therefore written as

$$\frac{mN}{\varepsilon}\left[\frac{\partial\vec{v}_d}{\partial t} + \frac{1}{\varepsilon}\left(\vec{v}_d.\nabla\right)\vec{v}_d\right] = mN\vec{g} + \mu_0 M\nabla H + \frac{KN}{\varepsilon}\left(\vec{v}-\vec{v}_d\right), \tag{5}$$

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial\vec{v}}{\partial t} + \frac{1}{\varepsilon}\left(\vec{v}.\nabla\right)\vec{v}\right] = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla P + \rho\vec{g} + \frac{KN}{\varepsilon}\left(\vec{v}_d-\vec{v}\right)\right] - \frac{\mu}{k_1}\vec{v}, \tag{6}$$

where $P, \rho, \mu, \vec{v}(u, v, w), \vec{g}(0, 0, -g); \vec{v}_d(l, r, s), m, N(\bar{x}, t)$ denote, respectively, the pressure less

the hydrostatic pressure, density, viscosity, velocity of the pure fluid, gravity force; velocity, mass and number density of the particles; $\bar{x} = (x, y, z), K = 6\pi\mu\eta, \eta$ being the particle radius, is the Stokes' drag coefficient and mN is the mass of particles per unit volume. Here ε is the medium porosity and is defined as $\varepsilon = \frac{volume\ of\ the\ voids}{total\ volume}$, $(0<\varepsilon<1)$. For very fluffy foam materials, ε is nearly one and in bed of packed spheres in the range 0.25-0.50.

In the equations of motion for the fluid, the presence of suspended particles adds an extra force term, proportional to the velocity difference between suspended particles and fluid. Since the force exerted by the fluid on the suspended particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the suspended particles.

Making use of the Lagrange's vector identities

$$\left(\vec{v}_d.\nabla\right)\vec{v}_d = \frac{1}{2}\nabla v_d^2 - \vec{v}_d\times\vec{\Omega}\,, \left(\vec{v}.\nabla\right)\vec{v} = \frac{1}{2}\nabla v^2 - \vec{v}\times\vec{\Omega}_1\,, \tag{7}$$

equations (5) and (6) become

$$\frac{mN}{\varepsilon}\left[\frac{\partial\vec{v}_d}{\partial t} - \frac{1}{\varepsilon}\left(\vec{v}_d\times\vec{\Omega}\right)\right] = -mNgz - \frac{1}{2\varepsilon^2}mN\nabla v_d^2 + \mu_0 M\nabla H + \frac{KN}{\varepsilon}\left(\vec{v}-\vec{v}_d\right), \tag{8}$$

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial\vec{v}}{\partial t}-\frac{1}{\varepsilon}\left(\vec{v}\times\vec{\Omega}_1\right)\right]=\left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla P-\nabla\rho gz-\frac{1}{2\varepsilon^2}\rho\nabla v^2+\frac{KN}{\varepsilon}\left(\vec{v}_d-\vec{v}\right)\right]-\frac{\mu}{k_1}\vec{v}, \quad (9)$$

where $\vec{\Omega}=\nabla\times\vec{v}_d$ and $\vec{\Omega}_1=\nabla\times\vec{v}$ are solid vorticity and fluid vorticity.

Taking the curl of these equations and recalling that the curl of a gradient is identically equal to zero, we obtain

$$\frac{mN}{\varepsilon}\left[\frac{\partial\vec{\Omega}}{\partial t}-\frac{1}{\varepsilon}\left(\nabla\times\vec{v}_d\times\vec{\Omega}\right)\right]=\mu_0\nabla\times M\nabla H+\frac{KN}{\varepsilon}\left(\vec{\Omega}_1-\vec{\Omega}\right), \quad (10)$$

$$\frac{\rho}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\partial\vec{\Omega}_1}{\partial t}-\frac{1}{\varepsilon}\left(\nabla\times\vec{v}\times\vec{\Omega}_1\right)\right]=-\frac{\mu}{k_1}\vec{\Omega}_1+\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{KN}{\varepsilon}\left(\vec{\Omega}-\vec{\Omega}_1\right). \quad (11)$$

By making use of the vector identities

$$\nabla\times\left(\vec{v}_d\times\vec{\Omega}\right)=\left(\vec{\Omega}.\nabla\right)\vec{v}_d-\left(\vec{v}_d.\nabla\right)\vec{\Omega}+\vec{v}_d\nabla.\vec{\Omega}-\vec{\Omega}\nabla.\vec{v}_d=\left(\vec{\Omega}.\nabla\right)\vec{v}_d-\left(\vec{v}_d.\nabla\right)\vec{\Omega}, \quad (12)$$

$$\nabla\times\left(\vec{v}\times\vec{\Omega}_1\right)=\left(\vec{\Omega}_1.\nabla\right)\vec{v}-\left(\vec{v}.\nabla\right)\vec{\Omega}_1+\vec{v}\nabla.\vec{\Omega}_1-\vec{\Omega}_1\nabla.\vec{v}=\left(\vec{\Omega}_1.\nabla\right)\vec{v}-\left(\vec{v}.\nabla\right)\vec{\Omega}_1, \quad (13)$$

equations (10) and (11) become

$$\frac{mN}{\varepsilon}\frac{D\vec{\Omega}}{Dt}=\mu_0\nabla\times M\nabla H+\frac{mN}{\varepsilon^2}\left(\vec{\Omega}.\nabla\right)\vec{v}_d+\frac{KN}{\varepsilon}\left(\vec{\Omega}_1-\vec{\Omega}\right), \quad (14)$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{D\vec{\Omega}_1}{Dt}=-\frac{\varepsilon\nu}{k_1}\vec{\Omega}_1+\frac{1}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\vec{\Omega}_1.\nabla\right)\vec{v}+\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{KN}{\rho}\left(\vec{\Omega}-\vec{\Omega}_1\right), \quad (15)$$

where ν is the kinematic viscosity and $\frac{D}{Dt}=\frac{\partial}{\partial t}+\frac{1}{\varepsilon}\left(\vec{q}_d.\nabla\right)$ is the convective derivative.

In equation (14),

$$\nabla\times\left(M\nabla H\right)=\left(\nabla M\times\nabla H\right)+\left(M\nabla\times\nabla H\right). \quad (16)$$

Since the curl of the gradient is zero, the last term in equation (16) is zero. Also since $M=M\left(H,T\right)$.

Therefore, $\nabla M=\left(\frac{\partial M}{\partial H}\right)\nabla H+\left(\frac{\partial M}{\partial T}\right)\nabla T.$ (17)

By making use of (17), equation (16) becomes

$$\nabla\times\left(M\nabla H\right)=\left(\frac{\partial M}{\partial H}\right)\nabla H\times\nabla H+\left(\frac{\partial M}{\partial T}\right)\nabla T\times\nabla H. \quad (18)$$

The first term on the right hand side of this equation is clearly zero, hence we get

$$\nabla\times\left(M\nabla H\right)=\left(\frac{\partial M}{\partial T}\right)\nabla T\times\nabla H. \quad (19)$$

Putting this in equation (14), we get

$$\frac{mN}{\varepsilon}\frac{D\vec{\Omega}}{Dt}=\mu_0\left(\frac{\partial M}{\partial T}\right)\nabla T\times\nabla H+\frac{mN}{\varepsilon^2}\left(\vec{\Omega}.\nabla\right)\vec{v}_d+\frac{KN}{\varepsilon}\left(\vec{\Omega}_1-\vec{\Omega}\right). \quad (20)$$

Here (15) and (20) are the equations governing the transport of vorticity in magnetic Maxwellian viscoelastic fluid-particle mixtures in porous medium.

In equation (20), the first term in the right hand side $\mu_0\left(\frac{\partial M}{\partial T}\right)\nabla T\times\nabla H$ describes the production of vorticity due to thermo-kinetic processes. The last term $\frac{KN}{\varepsilon}\left(\vec{\Omega}_1-\vec{\Omega}\right)$ gives the change in solid vorticity on account of exchange of vorticity between the liquid and solid in porous medium.

From equations (15) and (20), it follows that the transport of solid vorticity $\vec{\Omega}$ is coupled with the transport of fluid vorticity $\vec{\Omega}_1$ in porous medium.

From equation (20), we see that if solid vorticity $\vec{\Omega}$ is zero, then the fluid vorticity $\vec{\Omega}_1$ is not zero, but it is given by

$$\vec{\Omega}_1=-\frac{\varepsilon\mu_0}{KN}\left(\frac{\partial M}{\partial T}\right)\nabla T\times\nabla H. \tag{21}$$

This implies that due to thermo-kinetic process, fluid vorticity may exist in the absence of solid vorticity in porous medium. Equation (21) also shows that fluid vorticity decreases in the presence of porosity. In the absence of porous medium $(\varepsilon=1)$

$$\vec{\Omega}_1=-\frac{\mu_0}{KN}\left(\frac{\partial M}{\partial T}\right)\nabla T\times\nabla H. \tag{22}$$

This is in conformity with Wagh and Jawandhia (1996) result.

From equation (14), we find that if $\vec{\Omega}_1$ is zero, then $\vec{\Omega}$ is also zero. This implies that **when fluid vorticity is zero, then solid vorticity is necessarily zero.**

In the absence of suspended magnetic particles, N is zero and magnetization M is also zero, so equation (20) is identically satisfied and equation (15) reduces to

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{D\vec{\Omega}_1}{Dt}=-\frac{\varepsilon\nu}{k_1}\vec{\Omega}_1+\frac{1}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\vec{\Omega}_1.\nabla\right)\vec{v}. \tag{23}$$

This equation is vorticity transport equation in porous medium. The last term on the right hand side of equation (23) represents the rate at which $\vec{\Omega}_1$ varies for a given particle, when the vortex lines move with the fluid, the strengths of the vortices remaining constant and the rate of change of vorticity which varies for a given particle due to stress relaxation time. The first term represents the rate of dissipation of vorticity through friction (resistance).

IV. TWO-DIMENSIONAL CASE

Here we consider the two-dimensional case:

Let $\vec{v}_d=v_{d_x}(x,y)\,\hat{i}+v_{d_y}(x,y)\,\hat{j}$, $\vec{v}=v_x(x,y)\,\hat{i}+v_y(x,y)\,\hat{j}$, (24)

where components v_{d_x},v_{d_y} *and* v_x,v_y are functions of x,y *and* t, then

$$\vec{\Omega}=\Omega_z\hat{k}\,,\;\; \vec{\Omega}_1=\Omega_{1z}\hat{k}\;. \tag{25}$$

In two-dimensional case, equation (21) becomes

$$\frac{D\Omega_z}{Dt}=\frac{\mu_0\varepsilon}{mN}\left(\frac{\partial M}{\partial T}\right)\left(\frac{\partial T}{\partial x}\frac{\partial H}{\partial y}-\frac{\partial H}{\partial x}\frac{\partial T}{\partial y}\right)+\frac{K}{m}\left(\Omega_{1z}-\Omega_z\right). \tag{26}$$

Similarly, equation (15) becomes

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{D\Omega_{1z}}{Dt}=-\frac{\varepsilon\nu}{k_1}\Omega_{1z}+\frac{KN}{\rho}\left(\Omega_z-\Omega_{1z}\right)+\frac{KN\lambda}{\rho}\frac{\partial}{\partial t}\left(\Omega_z-\Omega_{1z}\right), \tag{27}$$

Because it can be easily verified that

$$\left(\vec{\Omega}.\nabla\right)\vec{v}_d = 0 \; and \; \left(\vec{\Omega}_1.\nabla\right)\vec{v} = 0. \qquad (28)$$

The first term on the right hand side of equation (27) is the change of fluid vorticity due to internal friction (resistance), the second term is the change in fluid vorticity on account of exchange of vorticity between solid and liquid and the third term is the rate of change in fluid vorticity on account of exchange of vorticity between solid and liquid due to stress relaxation time. Equation (27) does not involve explicitly the term representing change of vorticity due to magnetic field gradient and/or temperature gradient. But equation (26) shows that solid vorticity Ω_z depends on these factors. Hence, it follows that **fluid vorticity is indirectly influenced by the temperature and the magnetic field gradient.**

In the absence of magnetic particles, N is zero and magnetization M is also zero, so equation (26) is identically satisfied and equation (27) reduces to classical equation of transport of vorticity for fluid in porous medium. If instead of magnetic field we consider a suspension of non-magnetic particles, then the corresponding equation for the transport of vorticity may be obtained by putting M equal to zero in the equations governing the transport of vorticity in magnetic fluids. If magnetization M of the magnetic particles is independent of temperature, then the first term of equations (20) and (26) vanish and so the equations governing the transport of vorticity in magnetic fluid in porous medium are same as those which govern the transport of vorticity in non-magnetic suspensions in porous medium.

If the temperature gradient ∇T vanishes or if the magnetic field gradient ∇H vanishes or if ∇T is parallel to ∇H, then also the first term of equations (20) and (26) vanish. Thus, we see that in this case also the transport of vorticity in magnetic fluid in porous medium is same as transport of vorticity in non-magnetic suspension in porous medium.

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