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Multimodeling Interconnected Systems Application for the Stabilisation Control of the Inverted Pendulum

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***ABSTRACT**:- The 'Divide and rule' principle or the so called multimodeling in control engineering is often used to simplify the control task of non-linear systems. The aim of this paper is to take advantage of the multimodeling approach to stabilise a non-linear interconnected system, usually used as a benchmark: the Inverted pendulum (IP). The multimodeling approach applied to interconnected systems is to divide the global system into different subsystems; the whole system multimodel is made by the association of the subsystems multimodels. The Simulation results of the stabilisation control of the IP show a suitable stabilisation of the IP by the proposed method and a good robustness against perturbations.*

Keywords: *Interconnected systems- Inverted pendulum - Multimodeling - Stabilisation control.*

I. INTRODUCTION

Modern industrial mechanical systems are highly non-linear and interconnected. The control task of such systems is not always affordable, especially for highly non-linear and strongly interconnected ones.

Over the last two decades, many researches have been done to alleviate the control design problem of non-linear interconnected systems. The use of Neural Networks for the decentralised control of non-linear interconnected systems was introduced by [1] in 1999. Fuzzy control was used in [2]. Some authors used the model reduction techniques in order to reduce the interconnected model complexity [3].

A commonly used and practical approach to model non-linear systems is the multimodeling strategy [4]-[6]. In [4] a new multimodeling approach was proposed to model interconnected systems. This method deals with mechanical systems having weakly interconnected subsystems, like Inverted Pendulum, industrial manipulators, simple robots...

This paper is focused on applying the proposed multimodeling approach to the control of the cart inverted pendulum.

The Inverted Pendulum (IP) considered in the paper is mounted on a cart; the displacement of the cart induces the rotation of the pendulum. The pendulum control comprises two phases, the swing-up and the stabilisation control.

The swinging-up phase is non-linear, we used in [7] a Brunovsky tracking trajectory method, and we built our stabilisation control after a recall of the swing-up control algorithm already done.

The stabilisation control of the IP around its unstable equilibrium was largely studied in the literature, in the majority of the researches a classical model linearization around the operating point was used [8]-[11].

The multimodeling approach adopted in this paper is based on the decomposition of a complex system (fig1) onto several subsystems; the global system multimodel is made by the use of the subsystems association rules applied to the elementary multimodels. The association's rules are for example cascade, parallel…

As, for example [4], if the system is made by the parallel association of two systems S1 and S2, with respectively n1 and n2 models, the resulting modelling base of the global system have n1*n2 models. The validity vij of the novel base model Mij is done by the multiplication of S1 and S2 validities v1i*v2j.

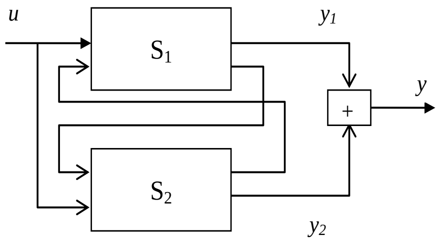

Fig1: A general subsystems association in a complex system

II. NON LINEAR MODEL OF THE CART INVERTED PENDULUM

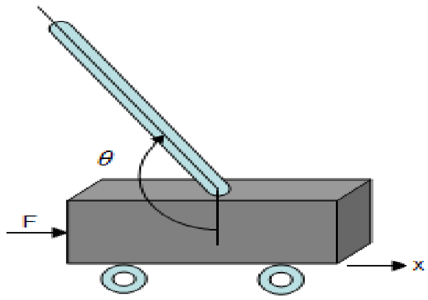

Fig. 2: Cart Inverted Pendulum system

The inverted pendulum shown in fig2 is mounted on a moving cart actuated via a control force F=G.u, the displacement of the cart induces the rotation of the pendulum.
The system motion is described by the following interconnected differential equations [12]:

$$(M+m).\ddot{x}+b.\dot{x}+m.l.\ddot{\theta}\cos(\theta)-m.l.\dot{\theta}^{2}.\sin(\theta)=G.u \qquad (1)$$

$$(I+m.l^{2}).\ddot{\theta}+m.g.l.\sin(\theta)=-m.l.\ddot{x}.\cos(\theta) \qquad (2)$$

Where:
- x, $\dot{x}$ and $\ddot{x}$ are respectively the cart displacement, velocity and acceleration.
- θ, $\dot{\theta}$ and $\ddot{\theta}$ are respectively the pendulum angle, velocity and acceleration.
- M and m are respectively the cart and pendulum masses.
- l is the pendulum length.

III. MULTIMODELING CONTROL OF THE INVERTED PENDULUM

We consider that the pendulum is previously swang-up from its downward stable position to its unstable upright position see [7]. To stabilise the IP a Multimodel controller is proposed here.
The multimodeling approach is made by decomposing the operating system into interconnected subsystems. The system decomposition is fair when some subsystems are linear, so the whole system modelling is reduced to the modelling of the non-linear part. The global system multimodel is obtained by associating multimodels of the nonlinear part to the linear part. For more details of the multimodeling of interconnected systems see [4].

The Cart-Inverted Pendulum is a complex and highly non-linear system; we propose to decompose it on two interconnected subsystems: the Cart and the Pendulum.
From the system equations (1) and (2) one can guess the nature of interconnection between the Cart and the Pendulum. When the cart is actuated by a control force u, the pendulum is under-actuated via the cart's acceleration $\ddot{x}$, the pendulum rotation also acts on the cart displacement by the angle θ and its derivatives. These interconnections are illustrated by fig3.

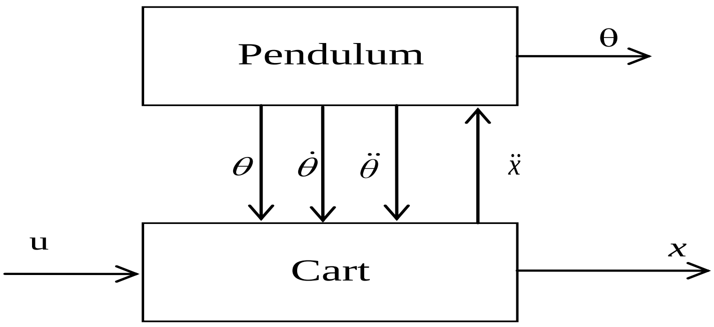

Fig.3: Interconnection scheme between the pendulum and the cart

By disconnecting the pendulum from the cart, the cart model becomes linear as it will be explained later.
For the pendulum modelling a multimodel is proposed to linearise the model around three functioning points.

A. The linear Cart Model:
To model the cart we consider that the pendulum is stabilised at □□□. So the cart displacement is then described by the following linear equation (3). The transfer function having the control signal u as input and the acceleration $\ddot{x}$ as output is done by (4).

$$(M+m).\ddot{x}+b.\dot{x}=G.u \quad (3)$$

$$H_C=\frac{\ell[\ddot{x}]}{\ell[\mathrm{u}]}=\frac{G.s}{((M+m)s+b)} \quad (4)$$

Where ℓ is the Laplace operator.

B. The linearized Pendulum Model:

To isolate the pendulum from the cart we consider that the cart is not moving so x=cte, the pendulum model (2) is then done by equation (5).

$$(I+m.l^2).\ddot{\theta}=-m.l.g.\sin(\theta) \quad (5)$$

After linearization of (5) around $\theta=\theta_0$ and replacing □ by □□ □□□ where □□□ is a small deviation of □□□ the pendulum model becomes:

$$\left(I+ml^2\right)\delta\ddot{\theta}=-m.l.g\cos(\theta_0)\delta\theta \quad (6)$$

If we consider that the pendulum is actuated by a control force $u_p=m.l.\cos(\theta).\ddot{x}$, equation (2) becomes:

$$(I+ml^2).\ddot{\theta}+m.g.l.\sin(\theta)=-u_p \quad (7)$$

A small deviation □□□ of the pendulum angle □ is actuated by a small variation □ u_p of the control signal u_p.
Equation (7) is then replaced by equation (8).

$$\left(I+m.l^2\right)\delta\ddot{\theta}=-ml.g\cos(\theta_0)\delta\theta-\delta u_P \quad (8)$$

The transfer function of the pendulum having $\delta\ddot{x}$ as input and □□□ as output is done by (9).

$$H_P=\frac{\ell[\delta\theta]}{\ell[\delta\,\ddot{\mathrm{x}}]}=\frac{-ml\cos(\theta_0)}{\left(I+ml^2\right)p^2+mgl\cos(\theta_0)} \quad (9)$$

C. The Cart-pendulum System Model:

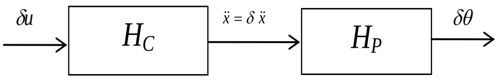

Fig.4: Interconnected Cart-pendulum system model at the stabilisation point

The linearized model of the cart-pendulum system (fig4) is made by the cascade association of the cart and the pendulum linearized models, its transfer function is done by the following expression (10).

$$H = H_P H_C = \frac{-ml\cos(\theta_0).G.s}{\left(\left(I + ml^2\right)s^2 + mgl\cos(\theta_0)\right)\left((M+m)s + b\right)}$$
(10)

A state space representation of the IP linearized model is presented as follows:

$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\\ \dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\alpha\cos(\theta_0) & 0 & 0 & \frac{\beta\cos(\theta_0)b}{M+m}\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & -\frac{b}{M+m}\end{bmatrix} * \begin{bmatrix}\theta\\ \dot{\theta}\\ x\\ \dot{x}\end{bmatrix} + \begin{bmatrix}0\\ -\frac{\beta\cos(\theta_0)G}{M+m}\\ 0\\ \frac{G}{M+m}\end{bmatrix} * u$$

With:

$$\begin{cases}\alpha = \dfrac{mgl}{\left(I + ml^2\right)}\\[2ex] \beta = \dfrac{ml}{\left(I + ml^2\right)}\end{cases}$$

D. Multimodel of the Cart-inverted Pendulum

To obtain the Cart-Inverted Pendulum multimodel first we replace the non-linear model of the pendulum by three linear models built around the stabilisation point; the operating points are □□ □ □ □ , □ and □ □ □ .

As mentioned previously the global system multimodel is made by the cascade association of the linear model of the cart and the multimodels of the pendulum. The state space representation of the CIP system multimodel is obtained by replacing the operating angle θ_0 by its value at each operating point.

Using the CIP system multimodel, an LQR control is proposed to stabilise the pendulum at its unstable equilibrium and a PID controller is used to control the cart displacement.

IV. SIMULATION RESULTS

The proposed stabilisation control explained above is simulated in Matlab Simulink.

The simulation parameters of the pendulum model are: M=0.5 kg, m=0.2kg, l=0.3m, g=9.8 m.s^{-2}, I=0.06 kg.m² and b=0.1 N/m/s^{-}

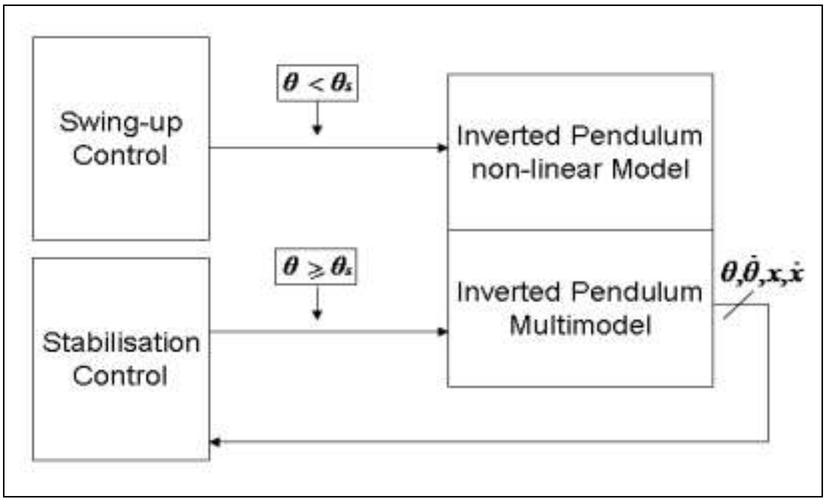

Fig5: Block diagram of the Swing-up and the Stabilisation controls of the IP system

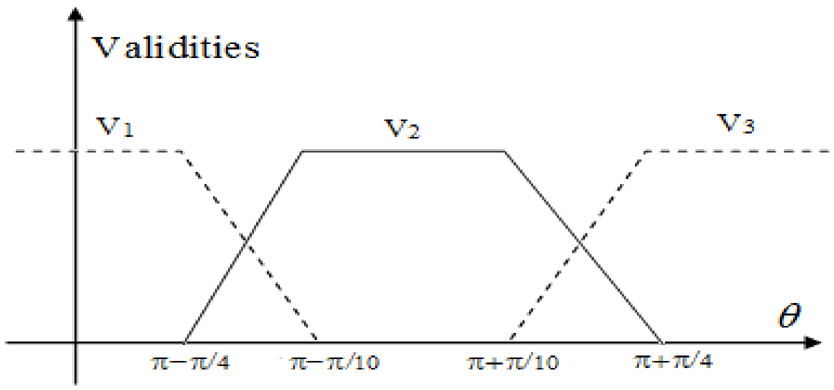

Fig6: Inverted Pendulum Multimodel validities

In fig5 is shown the block diagram of the IP control. The swing-up control is used to bring the pendulum from its stable equilibrium nearby its unstable equilibrium. When the pendulum position reaches an angle θ_s close to $\theta = \pi$. thecontroller switches to the stabilisation control. While the swing-up control is non-linear and open loop the stabilisation control is linear and closed loop control.

The model validities used to build the multimodel of the CIP system are mentioned in fig6.
The simulated pendulum angle, cart position and the control signal are presented in figures 7 to 9.
We notice from the simulation results that both pendulum and cart are stable after a brief swinging phase.
When the cart is propelled from the origin to a chosen stabilisation position (x=9), the pendulum swings-up from its stable equilibrium, it reaches the upper vertical and stabilises at $\theta = \pi$.
In figures 10 and 11 are shown the simulated pendulum angle and the cart position when a perturbation is made on the cart displacement. We notice that cart stabilises at its new position and the pendulum responds very quickly after the perturbation, and stabilises at $\theta = \pi$.

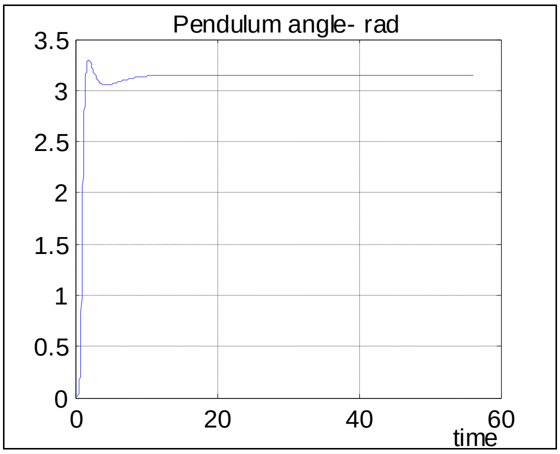

Fig.7: **Pendulum angle (θ)**

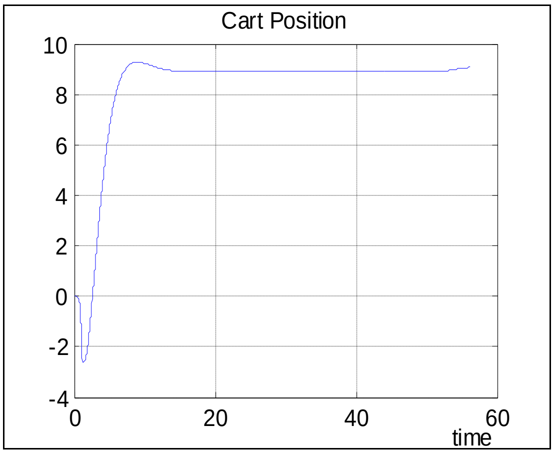

Fig.8: Cart position (x)

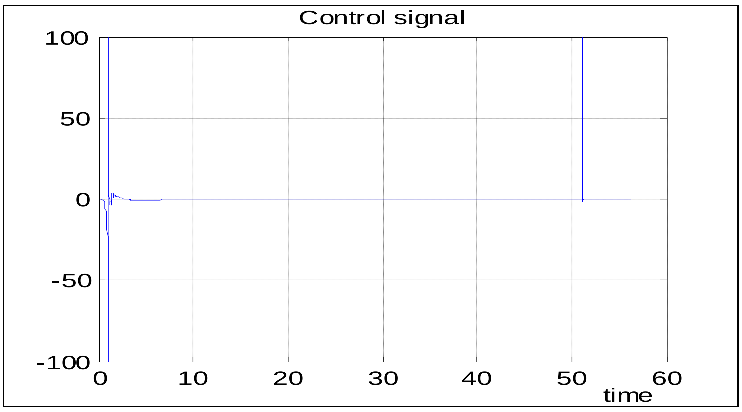

Fig. 9: Control signal

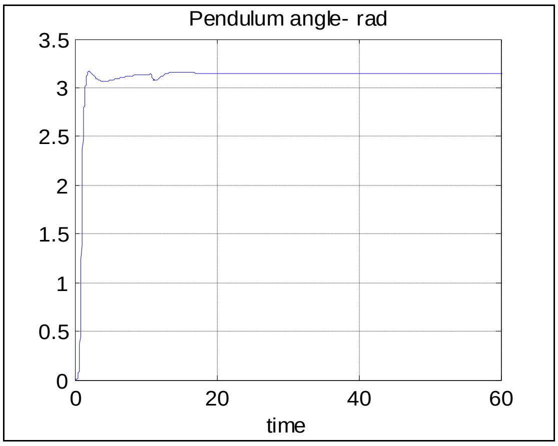

Fig.10: Pendulum angle(θ) under a perturbation made on the cart displacement

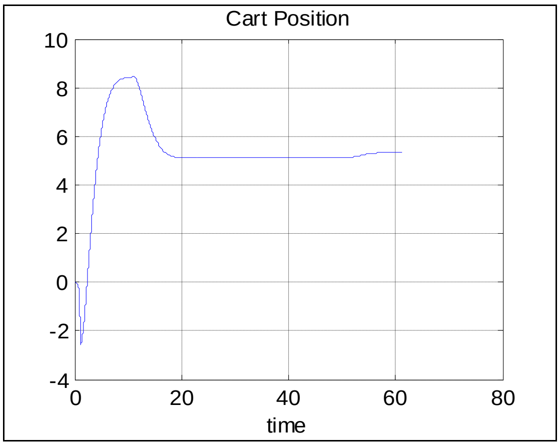

Fig.11: Cart displacement(x) under a perturbation made on the cart displacement

V. CONCLUSION

In this paper a novel multimodeling approach is proposed for interconnected complex non-linear systems. By decomposing a complex system on several subsystems, the multimodeling task becomes easier. For the inverted pendulum system, by dissociating the cart from the pendulum, the cart model becomes linear, and the multimodeling task is reduced to the determination of the pendulum multimodels. The whole system multimodel is obtained by associating different subsystems linear multimodels. After validation of the multimodeling task, we use simple and classical linear controls (like LQR and PID) to stabilise the pendulum around its unstable equilibrium, and the cart at a chosen position. Simulation results show a fair stabilisation and a good robustness of the proposed method against perturbations.

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