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Probabilistic Analysis of A Single Server Multi-Component Two-Dissimilar Unit Cold Standby Redundant System Subject to Inspection And Slow Switch

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ABSTRACT:- This paper investigates the analysis of a single server multi-component two-dissimilar unit cold standby redundant system subject to inspection and slow switch. Each unit consists of n independent separately maintained components, there is a single repair facility for inspection and repair of the failed unit. The inspection is carried out to detect which component of the unit has failed. The inspection, failure and repair times are stochastically independent random variables each having an arbitrary distribution. Using the regenerative point technique various reliability characteristics are obtained to carry out the cost-benefit analysis . Certain important results have been derived as particular cases.

I. INTRODUCTION AND DESCRIPTION OF THE SYSTEM

Many authors [1, 4, 7] deals with a multi-component system using the supplementary variable technique. They have assumed that on failure of the operative unit. The cold standby unit becomes operative with the help of the switch, which may be perfect or imperfect at the time of need with respective probabilities p and q . It is assumed that the repair of a failed unit starts after its inspection, which is carried out to determine which component of the unit has failed so as to repair that particular component.

The purpose of the present paper is to introduce the idea of slow switch and to investigate a two-dissimilar-unit cold standby redundant system. Each unit consists of n independent components, under the assumption that on failure of the operative unit the switch puts the standby unit into operation after a fixed amount of time t_s .The inspection. Failure and repair times are stochastically independent random variables each having an arbitrary distribution. A unit is good if all of its components are good and any one component fails the unit too fails. There is a single repair facility, the repair of a failed unit starts after its inspection. The inspection is carried out to find which component of the unit has failed so as to repair that component. A single server is available for inspection and repair of the failed unit upon repair the units work as good as new. Using the semi- Markov process technique and the results of the regenerative process. Several reliability characteristics such as pointwise availability. Steady state availability. the busy period by the server, the expected number of visits by the server and the cost per unit time in a steady state are obtained. The results obtained by [8] are derived from the present paper as special cases. In this system the following assumptions and notations are used to analyze the system.

(1) The system consists of two-dissimilar units. The first is operative and the second is kept as cold standby. Which of course does not fail unless it goes into operation.
(2) A unit has two possible modes (normal and failed).
(3) Each unit has n independent components.
(4) On failure of the operative unit the switch puts the standby unit into operation after a fixed amount of time t_s .
(5) A unit is good if all of its components are good and if any component of it fails the unit too fails.
(6) The inspection is carried out to find which component of the unit has failed so as to repair that particular component.
(7) A single server is available for inspection and repair of the failed unit.

(8) Failure, repair and inspection times are stochastically independent random variables each having an arbitrary distribution.
(9) Upon repair of a unit and the switch work as good as new.
(10) All random variables are mutually independent.

II. NOTATIONS AND STATES OF THE SYSTEM

E_0	state of the system at epoch $t = 0$,
E	Set of regenerative state; $\{s_2, s_3, s_6, s_7, s_{10j}, s_{11j}, s_{12}, s_{13}, s_{14j}, s_{15j}\}$
$\bar{E}$	set of non-regenerative states; $\{s_0, s_1, s_{4j}, s_{5j}, s_8, s_9\}$,
$f_i(t), F_i(t)$	pdf and cdf of failure time of the j-th component of the i -th operative unit; $j = 1, 2, ..., n; i = 1, 2,$
$h_i(t), H_i(t)$	pdf and cdf of inspection time of the i -th failed unit; $i = 1, 2,$
$g_{ji}(t), G_{ji}(t)$	pdf and cdf of repair of j -th component of the i-th failed unit; $j = 1, 2, ..., n; i = 1, 2,$
t_s	fixed amount of time required for the switch to replace the failed unit by the standby unit.
$q_{ij}(t), Q_{ij}(t)$	pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0.t]; i.j \in E,$
$q_{ij}^{k}(t), Q_{ij}^{k}(t)$	pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j visiting state k only once in $(0.t]; i.j \in E, k \in \bar{E}$
$q_{ij}^{(k,h)}(t), Q_{ij}^{(k,h)}(t)$	lf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j visiting state k and h in $(0,t]$ respectively $i,j \in E; h,k \in \bar{E},$
p_{ij}	one step transition probability from state i to state $j; \quad i,j \in E,$
p_{ij}^{k}	Probability that the system in state i goes to state j passing through state $k; \quad i,j \in E; \quad h,k \in \bar{E},$
$p_{ij}^{(k,h)}$	probability that the system in state i goes to state j passing through states $h,k; \quad i,j \in E; \quad h,k \in \bar{E},$
$\pi_i(t)$	cdf of first passage time from regenerative state i to a failed state,
$A_i(t)$	probability that the system is in up state at instant i given that the system started from regenerative state i at time $t = 0,$
$M_i(t)$	probability that the system. having started from state i is up at time t without making any transition into any other regenerative state,

$B_i(t)$	probability that the server is busy at time t given that the system entered regenerative state i at time $t=0$,
$V_i(t)$	expected number of visits by the server given that the system started from regenerative state i at time $t=0,\ i=1,2$
$p_{ji}(t)$	probability of success of the inspection in investigation of the failure of the j -th component of the i -th unit; $j=1,2,...,n; i=1,2,$
μ_{ij}	contribution mean sojourn time in state i when transition is to state j is $-\tilde{Q}_{ij}(0)=q_{ij}^{*}(0),$
μ_i	mean sojourn time in state i , $\mu_i=\sum[\mu_{ij}+\sum\mu_{ij}^{k}],$
~	Symbol for Laplace-Stieltjes transform, e.g. $\bar{F}(s)=\int e^{st}dF(t).$
*	symbol for Laplace transform, e.g. $F^{*}(s)=\int e^{st}f(t)dt.$
&	symbol for Stieltjes convolution, e.g $A(t)\&B(t)=\int_0^t B(t-u)dA(u),$
©	symbol for ordinary convolution, e.g. $a(t)©b(t)=\int_0^t a(u)b(t-u)du$

For simplicity. Whenever integration limits are $(0,\infty)$, they are not written

Symbols used for the states:

N_{0i}	the i -th unit is in N-mode and operative; i = 1, 2,
$N_{\bar{S}i}$	the i -th unit is in N-mode and standby; i = 1, 2,
F_{wpi}	the i -th unit is in F-mode and waiting for inspection; i = 1, 2,
F_{pi}	The i -th unit is in F-mode and under inspection; i =1,2,
$N_{\bar{S}wi}$	the i -th unit is in N-mode as standby and being switched; i = 1, 2,
F_{rji}	the i -th unit is in F-mode and under repair due to the failure of the j -th component; i = 1, 2; j = 1, 2, 3, …, n,

Considering these symbols, the system may be in one of the following states:

$$S_0=(N_{01},N_{\bar{S}2}),\quad S_1=(N_{\bar{S}1},N_{02}),$$
$$S_2=(N_{p1},N_{\bar{S}w2}),\quad S_3=(N_{\bar{S}w1},N_{p2}),$$
$$S_{4j}=(F_{rj1},N_{\bar{S}w2}),\quad S_{5j}=(N_{\bar{S}w1},F_{rj2}),$$

$S_6 = (F_{p1}, N_{02})$, $S_7 = (N_{01}, F_{p2})$,
$S_8 = (N_{\bar{S}1}, N_{\bar{S}w2})$, $S_9 = (N_{\bar{S}w2}, N_{\bar{S}2})$,
$S_{10j} = (F_{rj1}, N_{02})$, $S_{11j} = (N_{01}, F_{rj2})$,
$S_{12} = (F_{p1}, F_{wp2})$, $S_{13} = (F_{wp1}, F_{p2})$,
$S_{14j} = (F_{rj1}, F_{wp2})$, $S_{15j} = (F_{wp1}, F_{rj2})$,

Up states: $S_0, S_1, S_6, S_7, S_{10j}, S_{11j}$.

Down states: $S_2, S_3, S_{4j}, S_{5j}, S_8, S_9, S_{10j}, S_{11j}, S_{12}, S_{13}, S_{14j}, S_{15j}$.

States and possible transitions between them are shown in Fig. 1.

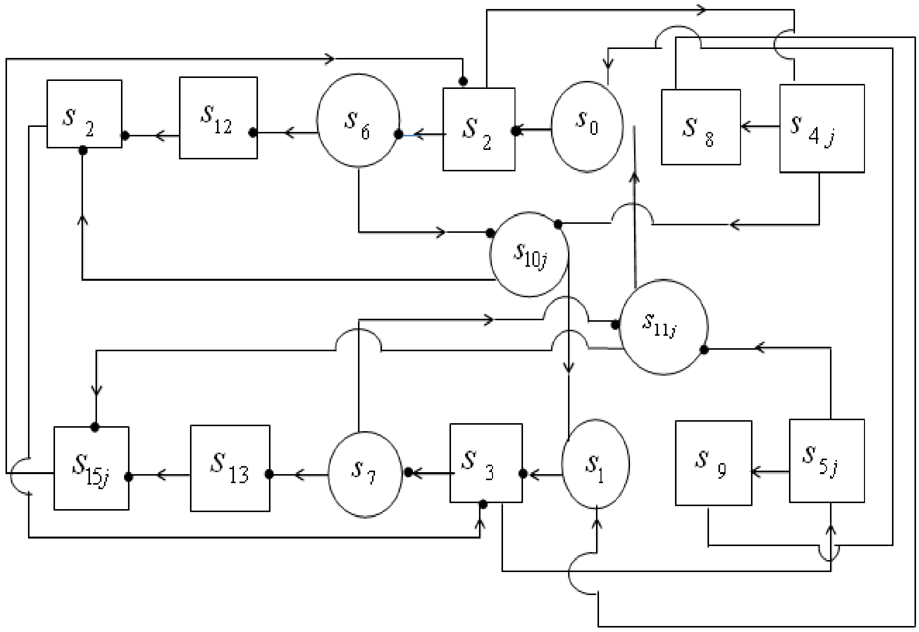
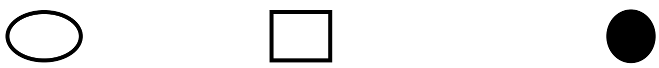


Fig.1.

Up state Down state Regeneration point



III. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

It can be observed that the time points of entry into $S_i \in E$ are regenerative points so these states are regenerative. Let $T_0 (\equiv 0). T_1, T_2, \ldots$ denote the time points at which the system enters any state $S_i \in E$ and X_n denotes the state visited at the time point X_{n+1}, i.e. just after the transition at T_{n+1}, then $\{X_n, T_n\}$ is a Markov renewal process with state space E and

$$Q_{ij} = p[X_{n+1} = j, T_{n+1} - T_n < t \mid X_n = i]$$

is a semi-Markov kernel over E. The stochastic matrix of the embedded Markov chain is

$$P = (p_{ij}) = (Q_{ij}(\infty)) = Q(\infty)$$

and the nonzero elements p_{ij} are

$$p_{02} = p_{13} = p_{14j,3} = p_{15j,2} = 1 \quad , \quad p_{13,15j} = p_{j2} \quad ,$$

$$p_{12,14j} = p_{j1} , \quad p_{26} = \bar{H}_1(t_S) \quad , \quad p_{37} = \bar{H}_2(t_S) \quad ,$$

$$p_{6,10j} = p_{j1} \int \bar{F}_2(t) dH_1(t) \quad , \quad p_{7,11j} = p_{j2} \int \bar{F}_1(t) dH_2(t) \quad ,$$

$$p_{6,12} = p_{6,12} \int \bar{H}_1(t) dF_2(t) \quad , \quad p_{7,13} = \int \bar{H}_2(t) dF_1(t) \quad ,$$

$$p_{10j,14j} = \int \bar{G}_{j1}(t) dF_2(t) \quad , \quad p_{11j,15j} = \int \bar{G}_{j2}(t) dF_1(t) \; ,$$

$$p_{10j,3}^{(1)} = \int \bar{F}_2(t) dG_{j1}(t) , p_{11j,2}^{(0)} = \int \bar{F}_1(t) dG_{j2}(t) \; ,$$

$$p_{2,10j}^{(4)} = p_{j1} \int_0^t h_1(t_s - u) \bar{G}_{j1}(u) du$$

$$p_{3,11j}^{(5j)} = p_{j2} \int_0^t h_2(t_s - u) \bar{G}_{j2}(u) du$$

$$p_{21}^{(4j,8)} = p_{j1} \int_0^t \int_0^u h_1(u - w) G_{j1}(w) du dw \quad ,$$

$$p_{30}^{(5j,9)} = p_{j2} \int_0^t \int_0^u h_2(u - w) G_{j2}(w) du dw \quad . \qquad (3.1)$$

The mean sojourn times μ_i in state S_i are

$$\mu_0 = \int \bar{F}_1(t) dt \quad , \quad \mu_1 = \int \bar{F}_2(t) dt \quad ,$$

$$\mu_6 = \int \bar{H}_1 \bar{F}_2(t) dt \quad , \quad \mu_7 = \int \bar{H}_2 \bar{F}_1(t) dt \quad ,$$

$$\mu_{10j} = \int \bar{G}_{j1} \bar{F}_2(t) dt \; , \quad \mu_{11j} = \int \bar{G}_{j2} \bar{F}_1(t) dt \quad ,$$

$$\mu_{12} = \int \bar{H}_1(t) dt \quad , \quad \mu_{13} = \int \bar{H}_2(t) dt \quad ,$$

$$\mu_{14j} = \int \bar{G}_{j1}(t) dt \quad , \quad \mu_{15j} = \int \bar{G}_{j2}(t) dt \; . \qquad (3.2)$$

IV. AVAILABILITY ANALYSIS

Elementary probability arguments yield the following relations for $A_i(t)$

$$A_0(t) = M_0(t) + q_{02}(t) © A_2(t) \quad ,$$

$$A_1(t) = M_1(t) + q_{13}(t) © A_3(t) \quad ,$$

$$A_2(t) = \sum_{j=1}^{n} q_{21}^{(4j,8)}(t) \copyright \mathrm{A}_1(t) + q_{26}(t) \copyright \mathrm{A}_6(t) + \sum_{j=1}^{n} q_{2,10j}^{(4j)}(t) \copyright \mathrm{A}_{10j}(t) ,$$

$$A_3(t) = \sum_{j=1}^{n} q_{30}^{(5j,9)}(t) \copyright \mathrm{A}_0(t) + q_{37}(t) \copyright \mathrm{A}_7(t) + \sum_{j=1}^{n} q_{3,11j}^{(5j)}(t) \copyright \mathrm{A}_{11j}(t) ,$$

$$A_6(t) = M_6(t) \sum_{j=1}^{n} q_{6,10j}(t) \copyright \mathrm{A}_{10j}(t) + q_{6,12}(t) \copyright \mathrm{A}_{12}(t) ,$$

$$A_7(t) = M_7(t) \sum_{j=1}^{n} q_{7,11j}(t) \copyright \mathrm{A}_{11j}(t) + q_{17,13}(t) \copyright \mathrm{A}_{13}(t) ,$$

$$A_{10j}(t) = M_{10j}(t) + q_{10j,3}^{(1)} \copyright \mathrm{A}_3(t) + q_{10j,14j}(t) \copyright \mathrm{A}_{14j}(t) ,$$

$$A_{11j}(t) = M_{11j}(t) + q_{11j,2}^{(0)} \copyright \mathrm{A}_2(t) + q_{11j,15j}(t) \copyright \mathrm{A}_{15j}(t) ,$$

$$A_{12}(t) = \sum_{j=1}^{n} q_{12,14j}(t) \copyright \mathrm{A}_{14j}(t) ,$$

$$A_{13}(t) = \sum_{j=1}^{n} q_{13,15j}(t) \copyright \mathrm{A}_{15j}(t) ,$$

$$A_{14j}(t) = q_{14j,3}(t) \copyright \mathrm{A}_3(t) ,$$

$$A_{15j}(t) = q_{15j,2}(t) \copyright \mathrm{A}_2(t) , \qquad (4.1)$$

where;

$$M_0(t) = \bar{F}_1(t) , \quad M_1(t) = \bar{F}_2(t) ,$$

$$M_6(t) = \bar{H}_1(t)\bar{F}_2(t) , \quad M_7(t) = \bar{H}_2(t)\bar{F}_1(t) ,$$

$$M_{10j}(t) = \bar{G}_{j1}\bar{F}_2(t) , \quad M_{11j}(t) = \bar{G}_{j2}\bar{F}_1(t) ,$$

Taking Laplace transforms of equations (6.4.1) and solving for $A_0^*(s)$ it follows

$$A_0^*(s) = N_1(s)/D_1(s) , \qquad (4.2)$$

where ;

$$N_1(s) = M_0^*[1-(c_{32}+q_{36}^*c_{72})(c_{23}+q_{26}^*c_{63})] + q_{02}^*[(a_2+q_{26}^*a_6+c_{21}M_1^*)$$
$$+(a_3+q_{37}^*a_7)(c_{23}+q_{26}^*c_{63}+c_{21}q_{13}^*)] ,$$

and

$$D_1(s) = 1-(c_{32}+q_{37}^*c_{72})(c_{23}+q_{26}^*c_{63}) - c_{30}q_{02}^*(c_{23}+q_{26}^*c_{63}+c_{21}q_{13}^*),$$

$$a_2 = \sum_{j=1}^{n} q_{2,10j}^{(4j)*} M_{10j}^{*} \ , \quad a_3 = \sum_{j=1}^{n} q_{3,11j}^{(5j)*} M_{11j}^{*} \ ,$$

$$a_6 = M_6^{*} + \sum_{j=1}^{n} q_{6,10j}^{*} M_{10j}^{*} \ , \ a_7 = M_7^{*} + \sum_{j=1}^{n} q_{7,11j}^{*} M_{11j}^{*} \ ,$$

$$c_{21} = \sum_{j=1}^{n} q_{21,}^{(4j,8)*} \ , c_{23} = \sum_{j=1}^{n} q_{2,10j}^{(4j)*} (q_{10j,3}^{(1)*} + q_{10j,14j}^{*} q_{14j,3}^{*}),$$

$$c_{30} = \sum_{j=1}^{n} q_{30}^{(5j,9)*} \ ,$$

$$c_{32} = \sum_{j=1}^{n} q_{3,11j}^{(5j)*} (q_{11j,2}^{(0)*} + q_{11j,15j}^{*} q_{15j,2}^{*}) \ ,$$

$$c_{63} = \sum_{j=1}^{n} [q_{6,10j}^{*} (q_{10j,3}^{(1)*} + q_{10j,14j}^{*} q_{14j,3}^{*}) + q_{6,12}^{*} q_{1j,14j}^{*} q_{14j,3}^{*}],$$

$$c_{72} = \sum_{j=1}^{n} [q_{7,11j}^{*} (q_{11j,2}^{(0)*} + q_{11j,15j}^{*} q_{15j,2}^{*}) + q_{7,13}^{*} q_{13,15j}^{*} q_{15j,2}^{*}]. \qquad (4.3)$$

The steady state availability of the system is

$$A_0(\infty) = N_1 / D_1 , \qquad (4.4)$$

where;

$$N_1 = \mu_0[1 - (\bar{c}_{32} + p_{37}\bar{c}_{72})(\bar{c}_{23} + p_{26}\bar{c}_{63})] + (\bar{a}_2 + p_{26}\bar{a}_6 + \bar{c}_{21}\mu_1) + (\bar{a}_3 + p_{37}\bar{a}_7)(\bar{c}_{23} + p_{26}\bar{c}_{63} + \bar{c}_{21}) \ ,$$

and

$$\begin{aligned} D_1 = & -(\bar{c}'_{32} + \mu_{37}\bar{c}_{72} + p_{37}\bar{c}'_{72})(\bar{c}_{23} + p_{26}\bar{c}_{63}) \\ & - (\bar{c}_{32} + p_{37}\bar{c}_{72})(\bar{c}'_{23} + \mu_{26}\bar{c}_{63} + p_{26}\bar{c}'_{63}) \\ & + (\bar{c}'_{30} + \bar{c}_{30}\mu_{02})(\bar{c}_{23} + p_{26}\bar{c}_{63} + \bar{c}_{21}) \\ & - \bar{c}_{30}(\bar{c}'_{32} - \mu_{26}\bar{c}_{63} + p_{26}\mu_{37}\bar{c}'_{63} + \bar{c}'_{63} + \bar{c}'_{21} - \bar{c}_{21}\mu_{13}) \end{aligned} \ , \bar{a}_2 = \sum_{j=1}^{n} p_{2,10j}^{(4j)} \mu_{10j} \ ,$$

$$\bar{a}_3 = \sum_{j=1}^{n} p_{3,11j}^{(5j)} \mu_{11j} \quad , \bar{a}_6 = \mu_6 + \sum_{j=1}^{n} p_{6,10j} \mu_{10j} \ , \ \bar{a}_7 = \mu_7 + \sum_{j=1}^{n} p_{7,11j} \mu_{11j} \ ,$$

$$\bar{c}_{21} = \sum_{j=1}^{n} p_{21}^{(4j,8)}, \quad \bar{c}'_{21} = \sum_{j=1}^{n} \mu_{21}^{(4j,8)},$$

$$\bar{c}_{23} = \sum_{j=1}^{n} p_{2,10j}^{(4j)} (p_{10j,3}^{(1)} + p_{10j,14j}),$$

$$\bar{c}'_{23} = -\sum_{j=1}^{n} [\mu_{2,10j}^{(4j)} (p_{10j,3}^{(1)} + p_{10j,14j}) + p_{2,10j}^{(4j)} (\mu_{10j,3}^{(1)} + \mu_{10j,3}^{(1)} + p_{10j,14j}\mu_{14j,3})],$$

$$\bar{c}_{30} = \sum_{j=1}^{n} p_{30}^{(5j,9)}, \qquad \bar{c}'_{30} = \sum_{j=1}^{n} \mu_{30}^{(5j,9)},$$

$$\bar{c}_{32} = \sum_{j=1}^{n} p_{3,11j}^{(5j)} (p_{11j,2}^{(0)} + p_{11j,15j}),$$

$$\bar{c}'_{32} = -\sum_{j=1}^{n} [\mu_{3,11j}^{(5j)} (p_{11j,2}^{(0)} + p_{11j,15j}) + p_{3,11j}^{(5j)} (\mu_{11j,2}^{(0)} + \mu_{11j,15j} + p_{11j,15j}\mu_{15j,2})],$$

$$\bar{c}_{63} = \sum_{j=1}^{n} [p_{6,10j} (p_{10j,3}^{(1)} + p_{10j,14j}) + p_{6,12}p_{12,14j}p_{14j,3}],$$

$$\bar{c}'_{63} = -\sum_{j=1}^{n} [\mu_{6,10j} (p_{10j,3}^{(1)} + p_{10j,14j}) + p_{6,10j} (\mu_{10j,3}^{(1)} + \mu_{10j,14j} + p_{10j,14j}\mu_{14j,3}) + \mu_{6,12}p_{12,14j} + p_{6,12}(\mu_{12,14j} + p_{12,14j}\mu_{14j,3})],$$

$$\bar{c}_{72} = \sum_{j=1}^{n} [p_{7,11j} (p_{11j,2}^{(0)} + p_{11j,15j}) + p_{7,13}p_{13,15j}p_{15j,2}],$$

$$\bar{c}'_{72} = -\sum_{j=1}^{n} [\mu_{7,11j} (p_{11j,2}^{(0)} + p_{11j,15j}) + p_{7,11j} (\mu_{11j,2}^{(0)} + \mu_{11j,15j} + p_{11j,15j}\mu_{15j,2}) + \mu_{7,13}p_{13,15j} + p_{7,13}(\mu_{13,15j} + p_{13,15j}\mu_{15j,2})]. \tag{4.5}$$

The expected up time of the system in $(0,t]$ is

$$\mu_{up}(t) = \int_0^t A_u(u)du \quad ,$$

so that

$$\mu_{up}^*(s) = A_0^*(s)/s \quad .$$

Thus one can evaluate $\mu_{up}(t)$ by taking inverse Laplace transform of $\mu_{up}^*(s)$.

The expected down time of the system in $(0,t]$ is

$$\mu_{dn}(t) = t - \mu_{up}(t) \quad ,$$

so that $\mu_{dn}^* = \frac{1}{s^2} - \mu_{up}^*(s)$.

V. BUSY PERIOD ANALYSIS

Elementary probability arguments yield the following relations for $B_i(t)$

$$B_0(t) = q_{02}(t)©\mathrm{B}_2(t) \quad ,$$

$$B_1(t) = q_{13}(t)©\mathrm{B}_3(t) \quad ,$$

$$B_2(t) = V_2(t) + \sum_{j=1}^{n} q_{21}^{(4j,8)}(t)©\mathrm{B}_1(t) + q_{26}(t)©\mathrm{B}_6(t) \quad ,$$

$$+ \sum_{j=1}^{n} q_{2,10j}^{(4j)}(t)©\mathrm{B}_{10j}(t)$$

$$B_3(t) = V_3(t) + \sum_{j=1}^{n} q_{30}^{(5j,9)}(t)©\mathrm{B}_0(t) + q_{37}(t)©\mathrm{B}_7(t)$$

$$+ \sum_{j=1}^{n} q_{3,11j}^{(5j)}(t)©\mathrm{B}_{11j}(t) \quad ,$$

$$B_6(t) = V_6(t) + \sum_{j=1}^{n} q_{6,10j}(t)©\mathrm{B}_{10j}(t) + q_{6,12}(t)©\mathrm{B}_{12}(t) \quad ,$$

$$B_7(t) = V_7(t) + \sum_{j=1}^{n} q_{7,11j}(t)©\mathrm{B}_{11j}(t) + q_{7,13}(t)©\mathrm{B}_{13}(t)$$

$$B_{10j}(t) = V_{10j}(t) + q_{10j,3}^{(1)}©\mathrm{B}_3(t) + q_{10j,14j}(t)©\mathrm{B}_{14j}(t)$$

$$B_{11j}(t) = V_{11j}(t) + q^{(0)}_{11j,2}(t) © B_2(t) + q_{11j,15j}(t) © B_{15j}(t)$$

$$B_{12}(t) = V_{12}(t) + \sum_{j=1}^{n} q_{12,14j}(t) © B_{14j}(t),$$

$$B_{13}(t) = V_{13}(t) + \sum_{j=1}^{n} q_{13,15j}(t) © B_{15j}(t) \quad ,$$

$$B_{14j}(t) = V_{14j}(t) + q_{14j,3}(t) © B_3(t) \quad ,$$

$$B_{15j}(t) = V_{15j}(t) + q_{15j,2}(t) © B_2(t) \quad , \qquad (5.1)$$

where;

$$V_2(t) = V_{12}(t) = \bar{H}_1(t), V_3(t) = V_{13}(t) = \bar{H}_2(t),$$

$$V_6(t) = \bar{H}_1(t)\bar{F}_2(t), V_7(t) = \bar{H}_2(t)\bar{F}_1(t),$$

$$V_{10j}(t) = \bar{G}_{j1}(t)\bar{F}_2(t), V_{11j}(t) = \bar{G}_{j2}(t)\bar{F}_1(t),$$

$$V_{14j}(t) = \bar{G}_{j1}(t), V_{16j}(t) = \bar{G}_{j2}(t) \quad ,$$

Taking Laplace transforms of equations (6.5.1) and solving for $B_0^*(s)$. it gives

$$B_0^*(s) = N_2(s)/D_1(s), \qquad (5.2)$$

where;

$N_2(s) = q^*_{02}[(b_2 + q^*_{26}b_6) + (b_3 + q^*_{37}b_7)(c_{23} + q^*_{26}c_{63} + c_{21}q^*_{13})]$ and

$D_1(s), c_{21}, c_{23}, c_{30}, c_{32}, c_{63}, c_{72}$ are given in (4.3),

$$b_2 = V_2^* + \sum_{j=1}^{n} q^{(4j)*}_{2,10j}(V^*_{10j} + q^*_{10j,14j}V^*_{14j}),$$

$$b_3 = V_3^* + \sum_{j=1}^{n} q^{(5j)*}_{3,11j}(V^*_{11j} + q^*_{11j,15j}V^*_{15j}) \quad ,$$

$$b_6 = V_6^* + \sum_{j=1}^{n}[q^*_{6,10j}(V^*_{10j} + q^*_{10j,14j}V^*_{14j}) + q^*_{6,12j}(V^*_{12j} + q^*_{12j,14j}V^*_{14j})]$$

$$b_7 = V_7^* + \sum_{j=1}^{n}[q^*_{7,11j}(V^*_{11j} + q^*_{11j,15j}V^*_{15j}) + q^*_{7,13}(V^*_{13} + q^*_{13,15j}V^*_{15j})]$$

(5.3)

in long run. The fraction of time for which the server is busy is given by

$$B_0(\infty) = N_2/D_1 \quad , \qquad (5.4)$$

where;

$$N_2 = (\bar{b}_2 + p_{26}\bar{b}_6) + (\bar{b}_3 + p_{37}\bar{b}_7)(\bar{c}_3 + p_{29}\bar{c}_{63} + \bar{c}_{21})$$

and

$D_1 = \bar{c}_{21}, \bar{c}'_{21}, \bar{c}_{23}, \bar{c}'_{23}, \bar{c}_{30}, \bar{c}'_{30}, \bar{c}_{32}, \bar{c}'_{32}, \bar{c}_{63}, \bar{c}'_{63}, \bar{c}_{72}, \bar{c}'_{72}$ are given in (4.3) ,

$$\bar{b}_2 = \mu_2 + \sum_{j=1}^{n} p_{2,10j}^{(4j)} (\mu_{10j} + p_{10j,15j}\mu_{14j}) \quad ,$$

$$\bar{b}_3 = \mu_3 + \sum_{j=1}^{n} p_{3,11j}^{(5j)} (\mu_{11j} + p_{11j,15j}\mu_{15j}) \quad ,$$

$$\bar{b}_6 = \mu_6 + \sum_{j=1}^{n} [p_{6,10j}(\mu_{10j} + p_{10j,14j}\mu_{14j}) + p_{6,12}(\mu_{12} + p_{12j,14j}\mu_{14j})] ,$$

$$\bar{b}_7 = \mu_7 + \sum_{j=1}^{n} [p_{7,11j}(\mu_{11j} + p_{11j,15j}\mu_{15j}) + p_{7,13}(\mu_{13} + p_{13j,15j}\mu_{15j})] ,$$

(5.5)

The expected busy period of the server in $(0,t]$ is

$$\mu_b(t) = \int_0^t B_0(u)du \quad ,$$

so that $\mu_b^*(s) = B_0^*(s)/s$.

Thus one can evaluate $\mu_b(t)$ by taking inverse Laplace transform of $\mu_b^*(s)$.

Expected idle time of the repairman in $(0,t]$ is

$$\mu_I(t) = t - \mu_b(t) \quad .$$

VI. EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

According to the definition of $V_i(t)$, by elementary probability arguments the following relations are obtained

$$V_0(t) = Q_{02}(t) \& [1 + V_2(t)] \quad ,$$

$$V_1(t) = Q_{13}(t) \& [1 + V_3(t)] \quad ,$$

$$V_2(t) = \sum_{j=1}^{n} Q_{21}^{(4j,8)}(t) \& V_1(t) + Q_{26}(t) \& V_6(t) + \sum_{j=1}^{n} Q_{2.10j}^{(4j)}(t) \& V_{10j}(t) \quad ,$$

$$V_3(t) = \sum_{j=1}^{n} Q_{30}^{(5j,9)}(t) \& V_0(t) + Q_{37}(t) \& V_7(t) + \sum_{j=1}^{n} Q_{3.11j}^{(5j)}(t) \& V_{11j}(t) \quad ,$$

$$V_6(t) = \sum_{j=1}^{n} Q_{6.10j}(t) \& V_{10j}(t) + Q_{6,12}(t) \& V_{12}(t) \quad ,$$

$$V_7(t) = \sum_{j=1}^{n} Q_{7.11j}(t) \& V_{11j}(t) + Q_{7,13}(t) \& V_{13}(t) \quad ,$$

$$V_{10j}(t) = Q_{10j,3}^{(1)}(t) \& [1+V_3(t)] + Q_{10j,14j}(t) \& V_{14j}(t),$$

$$V_{11j}(t) = Q_{11j,2}^{(0)}(t) \& [1+V_2(t)] + Q_{11j,15j}(t) \& V_{15j}(t),$$

$$V_{12}(t) = \sum_{j=1}^{n} Q_{12.14j}(t) \& V_{14j}(t) \quad ,$$

$$V_{13}(t) = \sum_{j=1}^{n} Q_{13.15j}(t) \& V_{15j}(t) \quad ,$$

$$V_{14j}(t) = Q_{14j,3}(t) \& V_3(t) \quad ,$$

$$V_{15j}(t) = Q_{15j,2}(t) \& V_2(t) \quad , \qquad (6.1)$$

Taking Laplace-Stieltjes transforms of equations (6.1) and solving for $\tilde{V}_0(s)$. Dropping the argument "s" for brevity. It follows

$$\tilde{V}(s) = N_3(s) / D_2(s) \quad , \qquad (6.2)$$

where;

$$N_3(s) = \tilde{Q}_{02}[1-(X_{32}+\tilde{Q}_{37}X_{72})(X_{23}+\tilde{Q}_{26}X_{63})]$$
$$+\tilde{Q}_{02}[(d_2+\tilde{Q}_{26}d_6+X_{21}\tilde{Q}_{13}) \quad ,$$
$$+(d_3+\tilde{Q}_{37}d_7)(X_{23}+\tilde{Q}_{26}X_{63}+X_{21}\tilde{Q}_{13})]$$

and

$$D_2(s) = 1-(X_{32}+\tilde{Q}_{37}X_{72})(X_{23}+\tilde{Q}_{26}X_{63})$$
$$-X_{32}\tilde{Q}_{02}(X_{23}+\tilde{Q}_{26}X_{63}+X_{21}\tilde{Q}_{13}) \quad ,$$

$$d_2 = \sum_{j=1}^{n} \tilde{Q}_{2,10j}^{(4j)} \tilde{Q}_{10j,3}^{(1)} \quad , \quad d_3 = \sum_{j=1}^{n} \tilde{Q}_{3,11j}^{(5j)} \tilde{Q}_{11j,2}^{(0)} \quad d_6 = \sum_{j=1}^{n} \tilde{Q}_{6,10j} \tilde{Q}_{10j,3}^{(1)}$$

$$d_7 = \sum_{j=1}^{n} \tilde{Q}_{7,11j} \tilde{Q}_{11j,2}^{(0)} \quad ,$$

$$X_{21} = \sum_{j=1}^{n} \tilde{Q}_{21}^{(4j,8)} \quad , X_{23} = \sum_{j=1}^{n} \tilde{Q}_{2,10j}^{(4j)} (\tilde{Q}_{10j,3}^{(1)} + \tilde{Q}_{10j,14j} \tilde{Q}_{14j,3}),$$

$$X_{30} = \sum_{j=1}^{n} \tilde{Q}_{30}^{(5j,9)}, \; X_{32} = \sum_{j=1}^{n} \tilde{Q}_{3,11j}^{(5j)} (\tilde{Q}_{11j,2}^{(0)} + \tilde{Q}_{11j,15j} \tilde{Q}_{15j,2}),$$

$$X_{63} = \sum_{j=1}^{n} [\tilde{Q}_{6,10j} (\tilde{Q}_{10j,3} + \tilde{Q}_{10j,14j} \tilde{Q}_{14j,3}) + \tilde{Q}_{6,12} \tilde{Q}_{12,14j} \tilde{Q}_{14j,3}],$$

$$X_{72} = \sum_{j=1}^{n} [\tilde{Q}_{7,11j} (\tilde{Q}_{11j,3}^{(0)} + \tilde{Q}_{11j,15j} \tilde{Q}_{15j,2}) + \tilde{Q}_{7,12} \tilde{Q}_{13,15j} \tilde{Q}_{15j,2}],$$

(6.3)

In steady state, number of visits per unit is given by

$$V_0(\infty) = N_3 / D_2, \quad (6.4)$$

where;

$$N_3 = 1 - (\bar{X}_{32} + p_{37} \bar{X}_{32})(\bar{X}_{32} + p_{37} \bar{X}_{32}) + (d_2 + p_{26} \bar{d}_2 + \bar{X}_{21}) + (d_3 + p_{37} \bar{d}_7)(\bar{X}_{23} + p_{26} \bar{X}_{63} + \bar{X}_{21})$$

and

$$\begin{aligned} D_2 = &-(\bar{X}'_{32} + \mu_{37} \bar{X}_{72} + p_{37} \bar{X}'_{72})(\bar{X}_{23} + p_{26} \bar{X}_{63}) \\ &-(\bar{X}_{32} + p_{37} \bar{X}_{72})(\bar{X}'_{23} + \mu_{26} \bar{X}_{63} + p_{26} \bar{X}'_{63}) \\ &-(\bar{X}'_{30} + \bar{X}_{30} \mu_{02})(\bar{X}_{23} + p_{26} \bar{X}_{63} + \bar{X}_{21}) \\ &- X_{30}(\bar{X}'_{23} - \mu_{26} \bar{X}_{63} + p_{26} \bar{X}'_{63} + \bar{X}_{21} \mu_{13} - \bar{X}_{21}) \end{aligned},$$

$$\bar{d}_2 = \sum_{j=1}^{n} p_{2,10j}^{(4j)} p_{10j,3}^{(1)}, \; \bar{d}_3 = \sum_{j=1}^{n} p_{3,11j}^{(5j)} p_{11j,2}^{(0)},$$

$$\bar{d}_6 = \sum_{j=1}^{n} p_{6,10j} p_{10j,3}^{(1)}, \; \bar{d}_7 = \sum_{j=1}^{n} p_{7,11j} p_{11j,3}^{(0)},$$

$$\bar{X}_{21} = \sum_{j=1}^{n} p_{21}^{(4j,8)}, \; \bar{X}'_{21} = -\sum_{j=1}^{n} \mu_{21}^{(4j,8)},$$

$$\bar{X}_{23} = \sum_{j=1}^{n} p_{2,10j}^{(4j)} (p_{10j,3}^{(1)} + p_{10j,14j}),$$

$$\bar{X}'_{23} = -\sum_{j=1}^{n} [\mu_{2,10j}^{(4j)} (p_{10j,3}^{(1)} + p_{10j,14j}) + p_{2,10j}^{(4j)} (\mu_{10j,3}^{(1)} + \mu_{10j,14j} + p_{10j,14j} \mu_{14j,3})],$$

$$\bar{X}_{30} = \sum_{j=1}^{n} p_{30}^{(5j,9)} \;,\; \bar{X}'_{30} = -\sum_{j=1}^{n} \mu_{30}^{(5j,9)} \;,$$

$$\bar{X}_{32} = \sum_{j=1}^{n} p_{3,11j}^{(5j)} (p_{11j,2}^{(0)} + p_{11j,15j}) \;,$$

$$\bar{X}'_{32} = -\sum_{j=1}^{n} [\mu_{3,11j}^{(5j)} (p_{11j,2}^{(0)} + p_{11j,15j}) \;,$$
$$+ p_{3,11j}^{(5j)} (\mu_{11j,2}^{(0)} + \mu_{11j,15j} + p_{11j,15j} \mu_{15j,2})]$$

$$\bar{X}_{63} = \sum_{j=1}^{n} p_{6,10j} (p_{10j,3}^{(1)} + p_{10j,14j}) + p_{6,12} p_{12,14j} \,,$$

$$\bar{X}'_{63} = -\sum_{j=1}^{n} [\mu_{6,10j} (p_{10j,3}^{(1)} + p_{10j,14j})$$
$$+ p_{6,10j} (\mu_{10j,3}^{(1)} + \mu_{10j,14j} + p_{10j,14j} \mu_{14j,3}) \;,$$
$$+ \mu_{6,12} p_{12,14j} + p_{6,12} (\mu_{12,14j} + p_{12j,14j} \mu_{14j,3})]$$

$$\bar{X}_{72} = \sum_{j=1}^{n} [p_{7,11j} (p_{11j,2}^{(0)} + p_{11j,15j}) + p_{7,13} p_{13,15j}],$$

$$\bar{X}'_{72} = -\sum_{j=1}^{n} [\mu_{7,11j} (p_{11j,2}^{(0)} + p_{11j,15j})$$
$$+ p_{7,11j} (\mu_{11j,2}^{(0)} + \mu_{11j,15j} + p_{11j,15j} \mu_{15j,2}) \;,$$
$$+ \mu_{7,13} p_{13,15j} + p_{7,13} (\mu_{13,15j} + p_{13,15j} \mu_{15j,2})]$$

(6.5)

VII. COST ANALYSIS

The cost function of the system obtained by considering the mean-up time of the system, expected busy period of the server and the expected number of visits by the server, therefore the expected profit incurred in $(0,t]$ is

$c(t)$ = expected total revenue in $(0,t]$ – expected total service cost in $(0,t]$ – expected cost of visits by server in $(0,t]$

$$= k_1 \mu_{up}(t) - k_2 \mu_b(t) - k_3 V_0(t) \;,$$

The expected profit per unit time in steady-state is

$$C = k_1 A_0 - k_2 B_0 - k_3 V_0 \,,$$

where K_1 is the revenue per unit up-time. K_2 is the cost per unit time for which the system is under repair and K_3 is the cost per visit by repair facility.

VIII. SPECIAL CASES

8.1 The two units are similar with exponential distributions:

Let

$$F_i(t)=1-e^{-\alpha t}\ ,H_i(t)=1-e^{-\beta t},G_{ji}(t)=1-e^{-\gamma_j t},$$

The transition probabilities are

$$p_{02}=p_{13}=p_{14j,3}=p_{15j,2}=1, p_{12,14j}=p_{13,15j}=p_j,$$

$$p_{26}=p_{37}=e^{-\beta ts},\ p_{6,10j}=p_{7,11j}=p_j\,\beta/(\alpha+\beta),$$

$$p_{6,12}=p_{7,13}=\alpha/(\alpha+\beta),$$

$$p_{10,14j}=p_{11,15j}=\alpha/(\gamma_j+\alpha),$$

$$p^{(1)}_{10j,3}=p^{(0)}_{11j,2}=\gamma_j/(\gamma_j+\alpha),$$

$$p^{(4j)}_{2,10j}=p^{(5j)}_{3,11j}=\frac{p_j}{(\beta+\gamma_j)}(e^{-\gamma_j ts}-e^{-\beta ts}),$$

$$p^{(4j,8)}_{21}=p^{(5j,9)}_{30}=\frac{p_j}{(\beta+\gamma_j)}[\beta(1-e^{-\gamma_j ts})-\gamma_j(t-e^{-\beta ts})],$$

The mean sojourn times are

$$\mu_0=\mu_1=\mu_{02}=\mu_{13}=1/\alpha\ ,\quad \mu_6=\mu_7=1/(\alpha+\beta),$$

$$\mu_{10j}=\mu_{11j}=1/(\alpha+\gamma_j)\ ,\quad \mu_{12}=\mu_{13}=1/\beta,$$

$$\mu_{14j}=\mu_{15j}=\mu_{14j,3}=\mu_{15j,2}=1/\gamma_j,$$

$$\mu_{12,14j}=\mu_{13,15j}=p_j/\beta^2,$$

$$\mu_{26}=\mu_{37}=1/\beta^2,$$

$$\mu_{6,10j}=\mu_{7,11j}=p_j\beta/(\alpha+\beta),$$

$$\mu_{6,12}=\mu_{7,13}=\alpha/(\alpha+\beta)^2,$$

$$\mu_{10j,14j}=\mu_{11j,15j}=\alpha/(\gamma_j+\alpha)^2,$$

$$\mu^{(1)}_{10j,3}=\mu^{(0)}_{11j,2}=\gamma_j/(\gamma_j+\alpha)^2,$$

$$\mu^{(4j)}_{2,10j}=\mu^{(5j)}_{3,11j}=p_j(\beta+\gamma_j)/\beta\gamma_j^2,$$

$$\mu_{21}^{(4j,8)} = \mu_{30}^{(5j,9)} = p_j(\beta+\gamma_j)\big/\beta\gamma_j$$.

The steady state availability of the system is

$$\hat{A}_0(\infty) = \hat{N}_1\big/\hat{D}_1$$,

where;

$$\hat{N}_1 = \frac{1}{\alpha} - \sum_{j=1}^{n}\frac{p_j\beta}{(\beta+\gamma_j)}(e^{-\gamma_j t} - e^{-\beta t s})\left[\frac{1}{\alpha} - \frac{1}{(\alpha+\gamma_j)}\right]$$ and

$$\hat{D}_1 = \sum_{j=1}^{n} p_j\left[\frac{(\beta+\gamma_j)}{\beta\gamma_j^2} + \frac{\beta}{(\beta+\gamma_j)(\gamma_j+\alpha)}(e^{-\gamma_j t} - e^{-\beta t s})\left(1+\frac{\alpha}{\gamma_j}\right)\right]$$

$$+\frac{1}{\beta^2}e^{-\beta t s}\sum_{j=1}^{n}\frac{p_j}{(\alpha+\beta)}\left[\beta + \frac{\beta}{(\gamma_j+\alpha)}\left(1+\frac{\alpha}{\gamma_j}\right) + \frac{\alpha}{(\alpha+\beta)} + \alpha\left(\frac{1}{\beta^2}+\frac{1}{\gamma_j}\right)\right]$$ In the long run, the

$$+\frac{1}{\alpha}\sum_{j=1}^{n}\frac{p_j}{(\beta+\gamma_j)}\left[\beta(1-e^{-\gamma_j t}) - \gamma_j(1-e^{-\beta t s})\right] + \sum_{j=1}^{n}\frac{p_j(\beta+\gamma_j)}{\beta\gamma_j}$$

fraction of time for which the server is busy is given by

$$\hat{B}_0(\infty) = \hat{N}_2\big/\hat{D}_1$$,

where,

$$\hat{N}_2 = 1 - \sum_{j=1}^{n}\frac{p_j\beta}{(\beta+\gamma_j)}(e^{-\gamma_j t} - e^{-\beta t s}) - \frac{e^{-\beta t s}p_j(\alpha-\beta)}{(\alpha-\beta)}$$

$$+\frac{1}{\beta} + \frac{e^{-\beta t s}}{(\alpha-\beta)}\sum_{j=1}^{n}\frac{p_j}{(\alpha+\gamma_j)}\left[\frac{\beta}{(\beta+\gamma_j)}(e^{-\gamma j t} - e^{-\beta t s}) + \frac{\beta e^{-\beta t s}}{(\alpha-\beta)}\right]$$

$$\left(1+\frac{\alpha}{\gamma_j}\right) + \frac{\alpha}{(\alpha-\beta)}\left(\frac{1}{\beta} + \sum_{j=1}^{n}\frac{p_j}{\gamma_j}\right)$$

In steady state, the number of visits per unit time is given by

$$\hat{V}_0(\infty) = \hat{N}_3\big/\hat{D}_1$$, where;

$$\hat{N}_3 = 1 - \sum_{j=1}^{n}\frac{\alpha}{(\alpha+\gamma_j)}\left[\frac{p_j\beta}{(\beta+\gamma_j)}(e^{-\gamma j t} - e^{-\beta t s}) + \frac{p_j\beta}{(\alpha+\beta)}e^{-\beta t s}\right] - \frac{\alpha}{(\alpha+\beta)}$$

The expected profit per unit time at steady state is

$$\hat{C} = K_1\hat{A}_0 - K_2\hat{B}_0 - K_3\hat{V}_0$$.

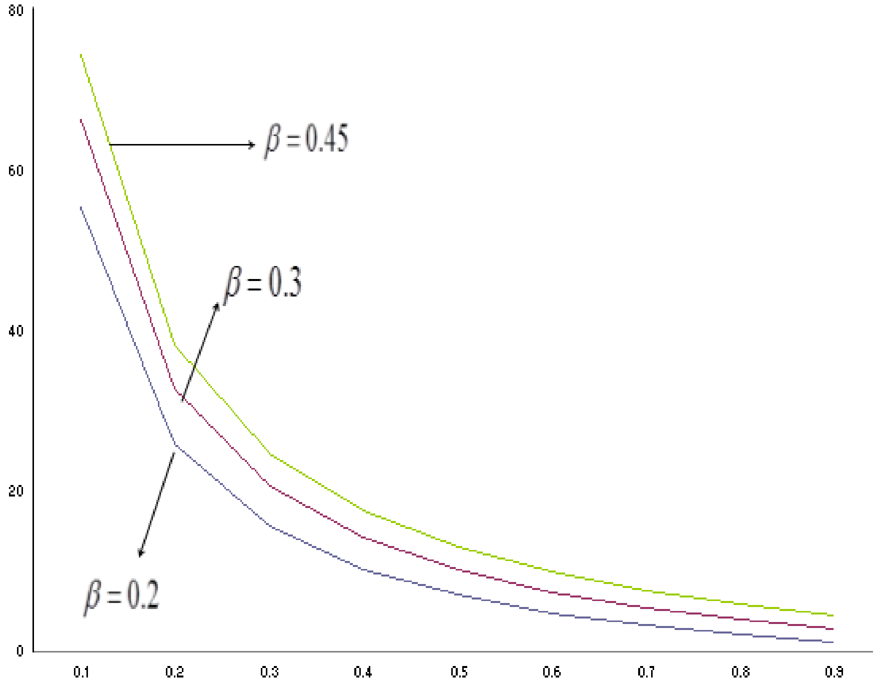
8.2. Numerical example:

Let the two units are similar with exponential distributions, and

$n = 2$, $p_j = 0.7$, $\gamma_j = 0.15$, $t_s = 10$

$K_1 = 1000$, $K_2 = 50$, $K_3 = 30$.

α	c		
	$\beta = 0.2$	$\beta = 0.3$	$\beta = 45$
0.1	55.48254	66.52853	74.60492
0.2	25.97565	32.92974	38.43232
0.3	15.60979	20.66413	24.86275
0.4	10.28482	14.21632	17.61510
0.5	7.03337	10.21486	13.06507
0.6	4.83847	7.48046	9.92743
0.7	3.25583	5.48982	7.62611
0.8	2.06005	3.97407	5.86273
0.9	1.12443	2.78050	4.46665



Failure rate of the j-th component of the unit

Fig.2

Relation between the failure rate of the j-th component of the unit and the expected profit per unit time of the system.

REFERENCES

[1]. GOEL, L.R., GUPTA. R. and GUPTA, P., “A single unit multicomponent system subject to various type of failures”, Microelectron. Reliab. Vol. 23, 813-816, 1983.

[2]. GOEL, L.R., and GUPTA. P. “A two-unit deteriorating standby system with inspection”. Microelectron. Reliab. Vol. 24, No 3, 435-438, 1984.

[3]. GOEL, L.R., and GUPTA. R. “Cost analysis of a two unit priority standby system with imperfect switch and arbitrary distributions”. Microelectron. Reliab. Vol. 25, No 1, 65-69, 1985.

[4]. GOEL, L.R., JAISWAL, N.K. and GUPTA, R., “A multistate system with two repair distributions”, Microelectron. Reliab. Vol. 23, 337-340, 1983.

[5]. GOPALAN. M.N. and NAIDU, R.S., “Stochastic behavior of a two-unit repairable system subject to inspection”, Microelectron. Reliab. Vol. 22, No. 4, 717-722, 1982.

[6]. GOPALAN. M.N. and NAIDU, R.S., “Cost benefit analysis of one-server two-unit cold standby system subject to inspection”, Microelectron. Reliab. Vol. 22, 699-705, 1982.

[7]. GUPTA, R., BAJAJ, C.P. and SINHA, S.M,. “A single server multi-component two-unit cold standby system with inspection and imperfect switching device”, Microelectron. Reliab. Vol.26, 873-877, 1986.

[8]. GUPTA, R., BAJAJ, C.P. and SINHA, S.M,. “Cost benefit analysis of a multi-component stochastic system with inspection and slow switch”, Microelectron. Reliab. Vol. 26, No. 5, 879-882, 1986.

[9]. MOKADDIS, G.S., "Reliability of a renewable system with redundant elements", The thirteenth annual conference in statistics, Computer Science and Operation Research I.S.S.R Cairo Univ. March 1978.
[10]. MOKADDIS, G.S., ELIAS, S.S. and LABIB, S.W,. "On a two dissimilar unit standby system with switychover time and proper initialization of connect switching". Microelectron. Reliab. Vol.27, No. 5, 819-822, 1987.