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Analysis of Two Dissimilar – Unit Cold Standby Redundant System Subject to Inspection And two Types of Repair

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Abstract:- This paper deals with the analysis of a two dissimilar unit cold standby redundant system subject to inspection and two types of repair where each unit of the system has two modes normal and failed. Assuming that the failure, repair, replacement and inspection times are stochastically independent random variables each having an arbitrary distribution .The cold standby unit replaces the failed operative unit after a random amount of time. Inspection is required to decide whether it needs type 1 (minor repair) or type 2 (major repair). In this system the repairman is not always available with the system but is called whenever the operative unit fails. The system is analyzed by the semi – Markov process technique. Some reliability measures of interest to system designers as well as operations managers have been obtained. Point wise availability, steady state availability, busy period by a server and expected cost per unit time of the system are obtained. Certain important results have been derived as particular cases.

Keywords: Cold standby ,Reliability measures, Point wise availability, Expected cost.

I. INTRODUCTION AND DESCRIPTION OF THE SYSTEM

Many authors have studied the two – unit cold standby redundant system . The repairmen are always available with the system for inspection repair of the failed unit and replace the failed unit by the standby unit if it is available instantaneously. Models have been formulated to treat many situations and obtain various reliability parameters by using the theory of regenerative process, Markov renewal process and semi – Markov process [4, 7].

This paper deals with a model of a two – dissimilar – unit cold standby redundant system with two modes of operation–normal and failure. The failure, repair, replacement and inspection time are stochastically independent random variables each having an arbitrary distribution. In this system the repairman is not always available with the system but is called whenever the operative unit fails. The cold standby unit replaces the failed operative unit after a random amount of time. Inspection is required whether it need types 1 repair or type 2 repair of a failed unit acts like a new one. Using the semi – Markov process technique and the results of the regenerative process, several reliability characteristics such as point wise availability, steady state availability, buys period by a server and expected cost per unit time of the system are obtained.

The results obtained by [15] are derived from the present paper as particular cases. In this system the following assumptions and notations are used to analyses the system.

1) The system consists of two – dissimilar units, the first is operative and the second is kept as cold standby, which of course does not fail unless it goes into operation.
2) A unit has two possible modes – normal and failure.
3) Failure, repair, replacement and inspection times are stochastically independent random variables each having an arbitrary distribution.
4) The system has two types of repair – type 1 (minor repair) and type 2 (major repair).
5) The repair facility is not always available with the system but is called whenever the operative unit fails.
6) The cold standby unit replaces the failed operative unit after a random amount of time.
7) Inspection is required to decide whether it need type 1 repair or type 2 repairs.
8) On repair of a failed unit from type 1 or type 2, it acts like a new one.
9) All random variables are mutually independent.

II. NOTATIONS AND STATES OF THE SYSTEM

E_0 — state of the system at epoch $t = 0$,

E — set of regenerative states; $\{S_2, S_3, S_4, S_5, S_6, S_7, S_{14}, S_{15}, S_{16}, S_{17}\}$

$\overline{E}$ — set of non – regenerative states; $\{S_0, S_1, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}\}$

$f_i(t), F_i(t)$ — pdf and cdf of failure time of the i-th unit; i =1,2 ,

$l_i(t), L_i(t)$ — pdf and cdf of availability time of the repairman of the i-th unit ; i = 1,2 .

$k_i(t), K_i(t)$ — pdf and cdf of replacement time of the i-th failed unit i= 1,2 .

$h_i(t), H_i(t)$ — pdf and cdf of inspection time of the i-th unit ; i=1,2.

$g_{ji}(t), G_{ji}(t)$ — pdf and cdf of type j repair time of the i-th failed unit j = 1,2 ; i= 1,2 .

P_i — Probability that the i-th failed enters type 1 repair ; i = 1,2,

q_i — Probability that the i-th failed enters type 2 repair, where $p_i + q_i = 1$; i = 1,2.

$q_{ij}(t), Q_{ij}(t)$ — pdf and cdf of first passage time from regenerative State i to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0,t]; i, j \in E$.

$q_{ij}^{(k)}(t), Q_{ij}^{(k)}(t)$ — pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j visiting state k only once in $(0,t]; i, j \in E$; $k \in \overline{E}$

$q_{ij}^{(k.h)}(t), Q_{ij}^{(k.h)}(t)$ — pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j Visiting state k and h in $(0,t]$ respectively; $i, j \in E \;\; h, k \in \overline{E}$.

p_{ij} — one step transition probability from state i to state j ; $i, j \in E$,

$p_{ij}^{(K)}$ — probability that the system in state i goes to state j passing through state $k; i, j \in E; k \in \overline{E}$,

$p_{ij}^{(k.h)}$ — Probability that the system in state i goes to state j ppassing through tes $k, h; i, j \in E; k, h \in \overline{E}$.

$A_i(t)$ — Probability that the system is in up state at instant t given that the system started from regenerative state i at time $t = 0$,

$M_i(t)$ — Probability that the system, having started from state i is up at time t without making any transition into any other regenerative state,

$B_i^j(t)$	Probability that the repairman is busy at time t in type j repair given that the system entered regenerative stat i at time $t=0$; $j=1,2,$
$V_i(t)$	Expected number of visits by the repairman given that the system started from regenerative state at time $t=0$
μ_{ij}	Contribution mean sojourn time in state I when transition is to state j is $-\tilde{Q}_{ij}(0)=q_{ij}^{*}(0),$
$\sqcup$	Symbol for laplace-stieltjes, e.g. $\tilde{f}(s)=\int e^{-st}F(t)$
$*$	Symbol for laplace transform, e.g. $f^{*}(s)=\int e^{-st}f(t)dt$
&	Symbol for stieltjes convolution,e.g $A(t)\&B(t)=\int_0^t B(t-u)dA(u),$
©	Symbol for ordinary convolution ,e.g. $a(t)$©b(t)=$\int_0^t$ a(u)b(t-u)du

For simplicity, whenever integration limits are $(0,\infty)$ they are not written.

Symbols used for the states:

N_{oi}	The i-th unit is in N-mode and operative ; $i=1,2,$
N_{si}	The i-th unit is in N-mode and standby ; $i=1,2,$
F_{Ii}	The i-th unit is in F-mode and under inspection ; $i=1,2,$
F_{WIi}	The i-th unit is in F-mode and waiting for inspection ; $i=1,2,$
$F_{rj,i}$	The i-th unit is in F-mode and under repair of type j ; $j=1,2,\ i=1,2,$
N_{di}	The i-th unit is in normal mode and under replacement; $i=1,2,$
N_{Wdi}	The i-th unit is in normal mode and waiting for replacement ; $i=1,2,$

Considering these symbols for the two units, the system may be in one of the following states:

$$S_0\equiv(N_{01},N_{s2}),S_1\equiv(N_{s1},N_{s2}),S_2\equiv(F_{WI1},N_{Wd2}),\quad S_3\equiv(N_{Wd1},F_{WI2}),$$
$$S_4\equiv(N_{WI1},N_{d2}),S_5\equiv(N_{d1},N_{WI2}),S_6\equiv(N_{01},N_{I2}),\quad S_7\equiv(N_{I1},N_{02}),$$

$S_8 \equiv (N_{ri,1}, N_{02}), S_9 \equiv (N_{01}, N_{ri,2}), S_{10} \equiv (N_{r2,1}, N_{02}), S_{11} \equiv (N_{01}, N_{r2,2}),$

$S_{12} \equiv (N_{I1}, N_{WI2}), S_{13} \equiv (N_{WI1}, N_{I2}), S_{14} \equiv (N_{rI,1}, N_{WI2}),$

$S_{15} \equiv (N_{WI1}, N_{r1,2}), S_{16} \equiv (N_{r2,1}, N_{WI2}), \quad S_{17} \equiv (N_{WI1}, N_{r2,2}),$

Up states: $S_0, S_1, S_6, S_7, S_8, S_9, S_{10}, S_{11}$.

Down states: $S_2, S_3, S_4, S_5, S_{12}, S_{13}, S_{14}, S_{15} S_{16}, S_{17}$.

States and possible transitions between them are shown in Fig. 5.1.

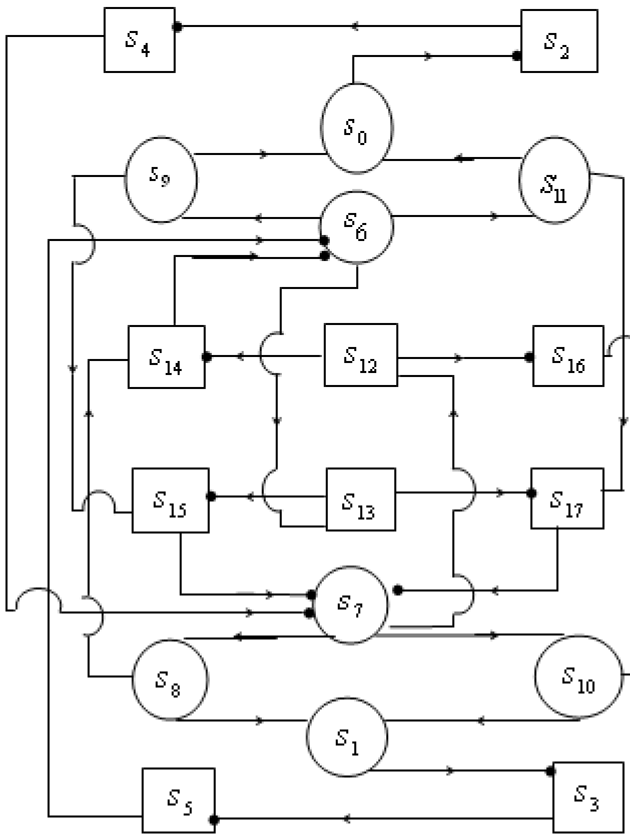
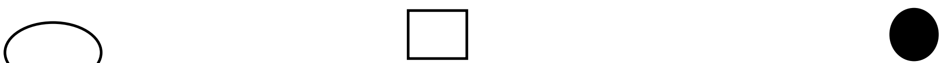


Fig.5.1



Up state, Down state, regenerative point

III. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

It can be observed that the time points of entry into $S_i \in E$ are regenerative points so these states are regenerative. Let $T_0 (\equiv 0), T_1, T_2, \ldots$ denote the time points at which the system enters any state $S_i \in E$ and X_n denotes the state visited at the time point T_{n+1} i.e. just after the transition at T_{n+1} then $\{X_n, T_n\}$ is a Markov – renewal process with state space E and

$Q_{ij} = P[X_{n+1} = j, T_{n+1} - T_n < t \mid X_n = i]$ is a semi – Markov kernel over E . the stochastic matrix of the embedded Markov chain is $P = (p_{ij}) = (Q_{ij}(\infty)) \equiv Q(\infty)$ and the nonzero elements P_{ij} are

$$P_{02} = P_{13} = P_{24} = P_{35} = P_{47} = P_{56} = 1 \quad ,$$

$$P_{62}^{(9,0)} = p_2 \int \int_0^W \int_0^V dF_1(W) dG_{12}(v-u) dH_2(u) \quad ,$$

$$P_{62}^{(11,0)} = q_2 \int \int_0^W \int_0^V dF_1(W) dG_{22}(v-u) dH_2(u) \quad ,$$

$$P_{73}^{(8,1)} = p_1 \int \int_0^W \int_0^V dF_2(W) dG_{11}(v-u) dH_1(u) \quad ,$$

$$P_{73}^{(10,1)} = q_1 \int \int_0^W \int_0^V dF_2(W) dG_{21}(v-u) dH_1(u) \quad ,$$

$$P_{67}^{(9,15)} = p_2 \int \int_0^W \int_0^V dG_{12}(w-v) dF_1(v) d\overline{G}_{12}(v-u) dH_2(u) \quad ,$$

$$P_{67}^{(11,17)} = p_2 \int \int_0^W \int_0^V dG_{22}(w-v) dF_1(v) d\overline{G}_{22}(v-u) dH_2(u) \quad ,$$

$$P_{76}^{(8,14)} = p_1 \int \int_0^W \int_0^V dG_{11}(w-v) dF_2(v) d\overline{G}_{11}(v-u) dH_1(u) \quad ,$$

$$P_{76}^{(10,16)} = q_1 \int \int_0^W \int_0^V dG_{21}(w-v) dF_2(v) d\overline{G}_{21}(v-u) dH_1(u) \quad ,$$

$$P_{6.15}^{(13)} = p_2 \int \int_0^V dF_1(u) dH_2(v) \quad ,$$

$$P_{6.17}^{(13)} = q_2 \int \int_0^V dF_1(u) dH_2(v) \quad ,$$

$$P_{7.14}^{(12)} = p_1 \int \int_0^V dF_2(u) dH_1(v) \quad ,$$

$$P_{7.16}^{(12)} = q_1 \int \int_0^V dF_2(u) dH_1(v) \quad ,$$

$$P_{14.6} = \int g_{11}(t) \overline{L}_1(t) dt \quad ,$$

$$P_{15.7} = \int g_{12}(t) \overline{L}_2(t) dt \quad ,$$

$$P_{16.6} = \int g_{21}(t) \overline{L}_1(t) dt \quad ,$$

$$P_{17.7} = \int g_{22}(t)\overline{L_2}(t)dt \qquad , \qquad (3.1\text{-}3.17)$$

The mean sojourn times μ_i in state S_i are

$$\mu_0 = \int \overline{F_1}(t)dt \quad , \quad \mu_1 = \int \overline{F_2}(t)dt \quad ,$$
$$\mu_2 = \int \overline{L}_2(t)dt \quad , \quad \mu_3 = \int \overline{L}_1(t)dt \quad ,$$
$$\mu_4 = \int \overline{K}_2(t)dt \quad , \quad \mu_5 = \int \overline{K}_1(t)dt \quad ,$$
$$\mu_6 = \int \overline{F}_1(t)\overline{H}_2(t)dt \quad , \quad \mu_7 = \int \overline{F}_2(t)\overline{H}_1(t)dt \quad ,$$
$$\mu_{14} = \int \overline{G}_{11}(t)dt \quad , \quad \mu_{15} = \int \overline{G}_{12}(t)dt \quad ,$$
$$\mu_{16} = \int \overline{G}_{21}(t)dt \quad , \quad \mu_{17} = \int \overline{G}_{22}(t)dt \quad ,$$

(3.18-3.29)

IV.AVAILABILITY ANALYSIS

Elementary probability arguments yield the following relations for $A_i(t)$,

$$A_0(t) = M_0(t) + q_{02}(t)©A_2(t) \quad ,$$
$$A_1(t) = M_1(t) + q_{13}(t)©A_3(t) \quad ,$$
$$A_2(t) = q_{24}(t)©A_4(t) \quad ,$$
$$A_3(t) = q_{35}(t)©A_5(t) \quad ,$$
$$A_4(t) = q_{47}(t)©A_7(t) \quad ,$$
$$A_5(t) = q_{56}(t)©A_6(t) \quad ,$$
$$A_6(t) = M_6(t) + \left[q_{62}^{(9.0)}(t) + q_{62}^{(11.0)}(t)\right]©A_2(t) + \left[q_{67}^{(9.15)}(t) + q_{67}^{(11.17)}(t)\right]©A_7(t) + q_{6.15}^{(13)}(t)©A_{15}(t) + q_{6.17}^{(13)}(t)©A_{17}(t) \quad ,$$

$$A_7(t) = M_7(t) + \left[q_{73}^{(8.1)}(t) + q_{73}^{(10.1)}(t)\right]©A_3(t) + \left[q_{76}^{(8.14)}(t) + q_{76}^{(10.16)}(t)\right]©A_6(t) + q_{7.14}^{(12)}(t)©A_{14}(t) + q_{7.16}^{(12)}(t)©A_{16}(t) \quad ,$$

$$A_{14}(t) = q_{14.6}(t)©A_6(t) \quad ,$$
$$A_{15}(t) = q_{15.7}(t)©A_7(t) \quad ,$$
$$A_{16}(t) = q_{16.6}(t)©A_6(t) \quad ,$$

$$A_{17}(t) = q_{17.7}(t) © A_7(t) \quad , \quad (4.1\text{-}4.12)$$

where;

$$M_0(t) = \bar{F}_1(t) \quad , \quad M_1(t) = \bar{F}_2(t) ,$$
$$M_6(t) = \bar{F}_1(t)\bar{H}_2(t) \quad , \quad M_7(t) = \bar{F}_2(t)\bar{H}_1(t) . \quad (4.13\text{-}4.16)$$

Taking Laplace transforms of equations (4.1-4.12) and solving for $A_0^*(s)$, it follows

$$A_0^*(s) = N_1(s)/D_1(s) \quad , \quad (4.17)$$

where ;

$N_1(s) = M_0^*(1-a_{67}a_{76}) + q_{02}^* q_{24}^* q_{47}^* (M_7^* + a_{76}M_6^*)$,and $D_1(s) = 1 - a_{67}a_{76}$,

$$a_{67} = q_{24}^* q_{47}^* (q_{62}^{(9.0)*} + q_{62}^{(11.0)*}) + q_{67}^{(9.15)*} + q_{67}^{(11.17)*} + q_{6.15}^{(13)*} q_{15.7}^* + q_{6.17}^{(13)*} q_{17.7}^* \quad ,$$

$$a_{76} = q_{35}^* q_{56}^* (q_{73}^{(8.1)*} + q_{73}^{(10.1)*}) + q_{76}^{(8.14)*} + q_{76}^{(10.16)*} + q_{7.14}^{(12)*} q_{14.6}^* + q_{7.16}^{(12)*} q_{16.6}^* \quad ,$$

The steady state availability of the system is $A_0(\infty) = N_1 / D_1$,

where; $N_1 = \mu_0(1-\hat{a}_{67}\hat{a}_{67}) + \mu_7 + \hat{a}_{76}\mu_6$,and $D_1 = -\hat{a}'_{67}\hat{a}_{76} - \hat{a}_{67}\hat{a}'_{76}$,

$$\hat{a}_{67} = P_{62}^{(9.0)} + P_{62}^{(11.0)} + P_{67}^{(9.15)} + P_{67}^{(11.17)} + P_{6.15}^{(13)} P_{15.7} + P_{6.17}^{(13)} P_{17.7} ,$$

$$\hat{a}_{76} = P_{73}^{(8.1)} + P_{73}^{(10.1)} + P_{76}^{(8.14)} + P_{76}^{(10.16)} + P_{7.14}^{(12)} P_{14.6} + P_{7.16}^{(12)} P_{16.6} ,$$

$$\hat{a}'_{76} = -[(\mu_{24} + \mu_{47})(P_{62}^{(9.0)} + P_{62}^{(11.0)}) + \mu_{62}^{(9.0)} + \mu_{62}^{(11.0)} + \mu_{67}^{(9.15)} + \mu_{67}^{(11.17)} (\mu_{6.15}^{(13)} P_{15.7} + P_{6.15}^{(13)} \mu_{15.7} + \mu_{6.17}^{(13)} P_{17.7} + P_{6.17}^{(13)} \mu_{17.7})] \quad ,$$

$$\hat{a}'_{76} = -[(\mu_{35} + \mu_{56})(P_{73}^{(8.1)} + P_{73}^{(10.1)}) + \mu_{73}^{(8.1)} + \mu_{73}^{(10.1)} + \mu_{76}^{(8.14)} + \mu_{76}^{(10.16)} \mu_{7.14}^{(12)} P_{14.6} + P_{7.14}^{(12)} \mu_{14.6} + \mu_{7.16}^{(12)} P_{16.6} + P_{7.16}^{(12)} \mu_{16.6}] \quad ,$$

$$(4.23\text{-}4.28)$$

The expected up time of the system in $(0,t]$ is $\mu_{up}(t) = \int_0^t A_0(u)\, du$,

so that $\mu_{up}^*(s) = A_0^*(s)/s$.

Thus one can evaluate $\mu_{up}(t)$ by taking inverse Laplace transform of $\mu_{up}^*(s)$.

V. EXPECTED PERIOD OF SERVER FOR REPAIR OF TYPE 1 DURING (0,T]

Elementary probability arguments yield the following relations for $B_i^1(t)$

$$B_0^1(t)=q_{02}(t)©B_2^1(t),\ B_1^1(t)=q_{13}(t)©B_3^1(t),B_2^1(t)=q_{24}(t)©B_4^1(t),$$
$$B_3^1(t)=q_{35}(t)©B_5^1(t),B_4^1(t)=q_{47}(t)©B_7^1(t),B_5^1(t)=q_{56}(t)©B_6^1(t),$$
$$B_6^1(t)=[q_{62}^{(9.0)}(t)+q_{62}^{(11.0)}]©B_2^1(t)+$$
$$[q_{67}^{(9.15)}(t)+q_{67}^{(11.17)}]©B_7^1(t)+ \quad ,$$
$$q_{6.15}^{(13)}(t)©B_{15}^1(t)+q_{6.17}^{(13)}(t)©B_{17}^1(t)$$
$$B_{14}^1(t)=V_{14}(t)+q_{14.6}(t)©B_6^1(t) \quad ,$$
$$B_{15}^1(t)=V_{15}(t)+q_{15.7}(t)©B_7^1(t) \quad ,$$
$$B_{16}^1(t)=q_{16.6}(t)©B_6^1(t) \quad ,$$
$$B_{17}^1(t)=q_{17.7}(t)©B_7^1(t) \quad ,$$

(5.1- 5.12)

where;

$$V_{14}(t)=\bar{G}_{11}(t)\ ,V_{15}(t)=\bar{G}_{12}(t) \quad , \qquad (5.13\text{- }5.14)$$

Taking Laplace transforms of equations (5.1-5.12) and solving for $B_0^{1*}(s)$ it gives

$$B_0^{1*}(s)=N_2^1(s)/D_1(s) \quad , \qquad (5.15)$$

where;

$$N_2^1(s)=q_{02}^*q_{24}^*q_{47}^*(q_{6.15}^{(13)*}V_{17}^*a_{76}+q_{7.14}^{(12)*}V_{16}^*). \qquad (5.16)$$

In long run, the fraction of time for which the server is busy is given by

$$B_0^1(\infty)=N_2^1/D_1 \qquad (5.17)$$

where; $N_2^1=P_{6.15}^{(13)}\mu_{15}\hat{a}_{76}+P_{7.14}^{(12)}\mu_{14}$. (5.18)

The expected busy period of the server for repair of type l during $(0,t]$ is

$$\mu_b^1(t)=\int_0^t B_0^1(u)du, \text{ so that } \mu_b^{1*}(s)=B_0^{1*}(s)/s \quad .$$

VI. EXPECTED PERIOD OF SERVER FOR REPAIR OF TYPE 2 DURING (0, T]

Elementary probability arguments yield the following relations for $B_i^2(t)$

$$B_0^2(t)=q_{02}(t)©B_2^2(t),B_1^2(t)=q_{13}(t)©B_3^2(t),B_2^2(t)=q_{24}(t)©B_4^2(t),$$
$$B_3^2(t)=q_{35}(t)©B_5^2(t),B_4^2(t)=q_{47}(t)©B_7^2(t),\ B_5^2(t)=q_{56}(t)©B_6^2(t),$$
$$B_6^2(t)=[q_{62}^{(9.0)}(t)+q_{62}^{(11.0)}(t)]©B_2^2(t)+[q_{67}^{(9.15)}(t)+q_{67}^{(11.17)}(t)]©B_7^2(t) \quad ,$$
$$+q_{67}^{(13)}(t)©B_{15}^2(t)+q_{6.17}^{(13)}(t)©B_{17}^2(t)$$

$$B_7^2(t)=[q_{73}^{(8.1)}+q_{73}^{(10.1)}(t)]©B_3^2(t)+[q_{76}^{(8.14)}(t)+q_{76}^{(10.16)}(t)]©B_6^2(t)+q_{7.14}^{(12)}(t)©B_{14}^2(t)+q_{7.16}^{(12)}(t)©B_{16}^2(t),$$

$$B_{14}^2(t)=q_{14.6}(t)©B_6^2(t), B_{15}^2(t)=q_{15.7}(t)©B_7^2(t),$$

$$B_{16}^2(t)=V_{16}(t)+q_{16.6}(t)©B_6^2(t), B_{17}^2(t)=V_{17}(t)+q_{17.7}(t)©B_7^2(t),$$

(6.1-6.12)

where;

$$V_{16}(t)=\bar{G}_{21}(t), V_{17}(t)=\bar{G}_{22}(t).$$

Taking Laplace transforms of equations (6.1-6.12) and solving for $B_0^{2*}(s)$, it gives

$$B_0^{2*}(s)=N_2^2(s)/D_1(s), \quad (6.15)$$

where;

$$N_2^2(s)=q_{02}^*q_{24}^*q_{47}^*(q_{6.17}^{(13)*}V_{17}^*a_{76}+q_{7.16}^{(12)*}V_{16}^*). \quad (6.16)$$

In long run , the fraction of time which the server is busy is given by

$$B_0^2(\infty)=N_2^2/D_1, \quad (6.17)$$

where;

$$N_2^2=P_{6.17}^{(13)}\mu_{17}\hat{a}_{76}+P_{7.16}^{(12)}\mu_{16}. \quad (6.18)$$

The expected busy period of the server for repair of type 2 during $(0,1]$ is

$$\mu_b^2(t)=\int_0^t B_0^2(u)du \text{ ,so that } \mu_b^{2*}(s)=B_0^{2*}(s)/s.$$

VII. EXPECTED NUMBER OF VISITED BY THE REPAIRMAN

According to the definition of $V_i(t)$, by elementary probability arguments it follows

$$V_0(t)=Q_{02}(t)\&V_2(t),$$

$$V_1(t)=Q_{13}(t)\&V_3(t), V_2(t)=Q_{24}(t)\&[1+V_4(t)],$$

$$V_3(t)=Q_{35}(t)\&[1+V_5(t)], V_4(t)=Q_{47}(t)\&V_7(t),$$

$$V_5(t)=Q_{56}(t)\&V_6(t),$$

$$V_6(t)=[Q_{62}^{(9.0)}(t)+Q_{62}^{(11.0)}(t)]\&V_2(t)+[Q_{67}^{(9.15)}(t)+Q_{67}^{(11.17)}(t)]\&V_7(t)+Q_{6.15}^{(13)}(t)\&V_{15}(t)+Q_{6.17}^{(13)}(t)\&V_{17}(t),$$

$$V_7(t)=[Q_{73}^{(8.1)}(t)+Q_{73}^{(10.1)}(t)]\&V_3(t)+[Q_{76}^{(8.14)}(t)+Q_{76}^{(10.16)}(t)]\&V_6(t)+Q_{7.14}^{(12)}(t)\&V_{14}(t)+Q_{7.16}^{(12)}(t)\&V_{16}(t),$$

$$V_{14}(t)=Q_{14.6}(t)\&V_6(t), V_{15}(t)=Q_{15.7}(t)\&V_7(t),$$

$$V_{16}(t)=Q_{16.6}(t)\&V_6(t), V_{17}(t)=Q_{17.7}(t)\&V_7(t).$$

(7.1-7.12)

Taking Laplace-Stieltjes transforms of equations (7.1-7.12) and solving for $V_0^*(s)$, dropping the argument "s" for brevity, it follows

$$\tilde{V}(s) = N_3(s)/D_2(s) \quad , \qquad (7.13)$$

where

$$N_3(s) = \tilde{Q}_{02}\tilde{Q}_{24}\left\{(1-b_{67}b_{76}) + \tilde{Q}_{47}\left[b_{76}\left(\tilde{Q}_{62}^{(9.0)} + \tilde{Q}_{62}^{(11.0)}\right) + \left(\tilde{Q}_{73}^{(8.1)} + \tilde{Q}_{73}^{(10.1)}\right)\right]\right\} \quad ,$$

$$D_2(s) = 1 - b_{67}b_{76} \quad ,$$

$$b_{67} = \left(\tilde{Q}_{62}^{(9.0)} + \tilde{Q}_{62}^{(11.0)}\right)\tilde{Q}_{24}\tilde{Q}_{47} + \tilde{Q}_{67}^{(9.15)} + \tilde{Q}_{67}^{(11.17)} + \tilde{Q}_{6.15}^{(13)}\tilde{Q}_{15.7} + \tilde{Q}_{6.17}^{(13)}\tilde{Q}_{17.7},$$

$$b_{76} = \left(\tilde{Q}_{73}^{(9.0)} + \tilde{Q}_{73}^{(10.1)}\right)\tilde{Q}_{35}\tilde{Q}_{56} + \tilde{Q}_{76}^{(8.14)} + \tilde{Q}_{67}^{(10.16)} + \tilde{Q}_{7.14}^{(12)}\tilde{Q}_{14.6} + \tilde{Q}_{7.16}^{(12)}\tilde{Q}_{16.6}.$$

(7.14-7.17)

In steady state, number of visits per unit is given by

$$V_0(\infty) = N_3/D_2 \quad , \qquad (7.18)$$

where; $N_3 = 1 - \hat{b}_{67}\hat{b}_{67} + \hat{b}_{76}\left(p_{62}^{(9.0)} + p_{62}^{(11.0)}\right) + p_{73}^{(8.1)} + p_{73}^{(10.1)}$,

$$D_2 = 1 - b_{67}b_{76} \quad ,$$

$$\hat{b}_{67} = P_{62}^{(9.0)} + P_{62}^{(11.0)} + P_{67}^{(9.15)} + P_{67}^{(11.17)} + P_{6.15}^{(13)}P_{15.7} + P_{6.17}^{(13)}P_{17.7} \quad ,$$

$$\hat{b}_{76} = P_{73}^{(8.1)} + P_{73}^{(10.1)} + P_{76}^{(8.14)} + P_{67}^{(10.16)} + P_{7.14}^{(12)}P_{14.6} + P_{7.16}^{(12)}P_{16.6} \quad ,$$

(7.19-7.23)

VIII. COST ANALYSIS

The cost function of the system obtained by considering the mean- up time of the system , expected busy period of the server and the expected number of visits by the server , therefore the expected profit incurred in (0,t] is $C(t)$ = expected total revenue in (0,t]- expected total service cost in (0,t]

- expected cost of visits by server (0,t]

$$= k_1\mu_{up}(t) - k_1\mu_b^1(t) - k_3\mu_b^2(t) - k_4 v_0(t). \qquad (8.1)$$

The expected profit per unit time in steady – state is

$$C = k_1A_0 - k_2B_0^1 - k_3B_0^2 - k_4v_0 \quad . \qquad (8.2)$$

Where k_1 is the revenue per unit up time, k_2 and k_3 are the cost per unit time for which the system is under type 1 and type 2 repair . Respectively, and k_4 is the cost per visit by repair facility .

IX. SPECIAL CASES

9.1 The two units are dissimilar with exponential distributions:

Let

α_i failure rate of the i-th unit ; i= 1,2,

β_i availability rate of the repairman of the i-th unit , i=1,2,

γ_i replacement rate of the i-th unit i=1,2,

θ_i inspection rate of the i-th unit i=1,2,

Ω_{ji} repair rate of type j of i-th failed unit j =1,2, i = 1,2,

P_i probability that the i-th failed unit enters type 1 repair
i = 1,2 ,

q_i probability that the i-th failed unit enters type 2 repair

$p_i + q_i = 1$; i=1,2

Transition probabilities are

$$p_{02} = p_{13} = p_{24} = p_{35} = p_{47} = p_{56} = p_{15.7} = p_{17.7} = p_{14.6} = p_{16.6} = 1,$$

$$p_{62}^{(9.0)} = p_2\theta_2\Omega_{12} / (\alpha_1 + \Omega_{12})(\alpha_1 + \theta_2), p_{62}^{(11.0)} = q_2\theta_2\Omega_{22} / (\alpha_1 + \Omega_{22})(\alpha_1 + \theta_2),$$

$$p_{73}^{(8.1)} = p_1\theta_1\Omega_{11} / (\alpha_2 + \Omega_{11})(\alpha_2 + \theta_1), p_{73}^{(10.1)} = q_1\theta_1\Omega_{21} / (\alpha_2 + \Omega_{21})(\alpha_2 + \theta_1),$$

$$p_{67}^{(9.15)} = p_2\alpha_1\theta_2 / (\alpha_1 + \Omega_{12})(\alpha_1 + \theta_2), p_{67}^{(11.17)} = q_2\alpha_1\theta_2 / (\alpha_1 + \Omega_{22})(\alpha_1 + \theta_2) \quad ,$$

$$p_{76}^{(8.14)} = p_1\alpha_2\theta_1 / (\alpha_2 + \Omega_{11})(\alpha_2 + \theta_1), \; p_{76}^{(10.16)} = q_1\alpha_2\theta_1 / (\alpha_2 + \Omega_{21})(\alpha_2 + \theta_1) \quad ,$$

$$p_{6.15}^{(13)} = p_2\alpha_1 / (\alpha_1 + \theta_2), p_{6.17}^{(13)} = q_2\alpha_1 / (\alpha_1 + \theta_2), p_{7.14}^{(12)} = p_1\alpha_2 / (\alpha_2 + \theta_1),$$

$$p_{7.16}^{(12)} = q_1\alpha_2 / (\alpha_2 + \theta_1), p_{15.7} = \Omega_{11} / (\Omega_{11} + \beta_1), p_{17.7} = \Omega_{12} / (\Omega_{12} + \beta_2),$$

$$p_{14.6} = \Omega_{21} / (\Omega_{21} + \beta_1), p_{16.6} = \Omega_{22} / (\Omega_{22} + \beta_2) \quad ,$$

The mean sojourn times μ_i in state S_i are

$$\mu_0 = 1/\alpha_1 \;, \; \mu_1 = 1/\alpha_2, \mu_2 = 1/\beta_2 \;, \; \mu_3 = 1/\beta_1 \;, \mu_4 = 1/\gamma_2,$$

$$\mu_5 = 1/\gamma_1, \mu_6 = 1/(\alpha_1 + \theta_2) \;, \mu_7 = 1/(\alpha_2 + \theta_1) \;, \mu_{14} = 1/\Omega_{11},$$

$$\mu_{15} = 1/\Omega_{12} \;, \mu_{16} = 1/\Omega_{21} \;, \quad \mu_{17} = 1/\Omega_{22} \quad ,$$

In this case, $\hat{M}_i(t)$ are

$$\hat{M}_0(t) = e^{-\alpha_1 t}, \hat{M}_1(t) = e^{-\alpha_2 t}, \hat{M}_6(t) = e^{-(\alpha_1+\theta_2)t}, \hat{M}_7(t) = e^{-(\alpha_2+\theta_1)t}.$$

The steady state availability of the system is $\hat{A}_0(\infty) = \hat{N}_1 / \hat{D}_1$,

where $\hat{N}_1 = \frac{1}{\alpha_1}(1 - \hat{a}_{67}\hat{a}_{76}) + \frac{1}{(\alpha_2 + \theta_1)} + \frac{\hat{a}_{76}}{(\alpha_1 + \theta_2)}$,

$$\hat{D}_1 = \hat{a}'_{67}\hat{a}_{76} - \hat{a}_{67}\hat{a}'_{76} \quad ,$$

$$\hat{a}_{67} = \frac{1}{(\alpha_1 + \theta_2)}\left\{\theta_2 + \alpha_1\left[\frac{p_2\Omega_{11}}{(\Omega_{11} + \beta_1)} + \frac{q_2\Omega_{12}}{(\Omega_{12} + \beta_2)}\right]\right\},$$

$$\hat{a}_{76} = \frac{1}{(\alpha_2 + \theta_1)}\left\{\theta_1 + \alpha_1\left[\frac{p_1\Omega_{22}}{(\Omega_{22} + \beta_2)} + \frac{q_1\Omega_{21}}{(\Omega_{21} + \beta_1)}\right]\right\},$$

$$\hat{a}'_{67} = -\left\{\left(\frac{1}{\beta_2}+\frac{1}{\gamma_2}\right)\frac{\theta_2+\Omega_{21}}{(\alpha_1+\Omega_{12})(\alpha_1+\theta_2)}+\frac{\Omega_{12}\theta_2}{\alpha_1^2(\theta_2-\Omega_{12})}\right.$$

$$\left[\frac{2\alpha_1+\Omega_{12}}{(\alpha_1+\Omega_{12})^2}-\frac{2\alpha_1+\theta_2}{(\alpha_1+\theta_2)^2}\right]+\frac{\Omega_{12}\alpha_1\theta_2}{(\theta_2-\Omega_{12})}$$

$$\left[\frac{\alpha_1+2\Omega_{12}}{\Omega_{12}^2(\alpha_1-\Omega_{12})^2}+\frac{\theta_2+\alpha_1+\Omega_{12}}{\Omega_{12}^2(\theta_2-\alpha_1)^2}\right]+\frac{\alpha_1(\alpha_1+2\theta_2)}{\theta_2(\alpha_1+\theta_2)},$$

$$\left.+\frac{\alpha_1}{(\alpha_1+\theta_2)}\left(\frac{p_2}{\Omega_{12}}+\frac{q_2}{\Omega_{22}}\right)\right\}$$

$$\hat{a}'_{76} = -\left\{\left(\frac{1}{\beta_1}+\frac{1}{\gamma_1}\right)\frac{\theta_1+\Omega_{21}}{(\alpha_2+\Omega_{21})(\alpha_2+\theta_1)}+\frac{\Omega_{21}\theta_1}{\alpha_2^2(\theta_1-\Omega_{21})}\right.$$

$$\left[\frac{2\alpha_2+\Omega_{21}}{(\alpha_2+\Omega_{21})^2}-\frac{2\alpha_2+\theta_1}{(\alpha_2+\theta_1)^2}\right]+\frac{\Omega_{21}\alpha_2\theta_1}{(\theta_2-\Omega_{21})},$$

$$\left[\frac{\alpha_2+2\Omega_{21}}{\Omega_{21}^2(\alpha_2-\Omega_{21})^2}+\frac{\theta_1+\alpha_2+\Omega_{21}}{\Omega_{21}^2(\theta_1-\alpha_2)^2}\right]+\frac{\alpha_2(\alpha_2+2\theta_1)}{\theta_1(\alpha_2+\theta_1)}$$

$$\left.+\frac{\alpha_2}{(\alpha_2+\theta_1)}\left(\frac{p_1}{\Omega_{21}}+\frac{q_1}{\Omega_{11}}\right)\right\}$$

In long run, the fraction of time which the server is busy in type 1 repair is given by $\hat{B}_0^1(\infty)=\hat{N}_2^1/\hat{D}_1$ where; $\hat{N}_2^1=\frac{p_2\alpha_1\hat{\hat{a}}_{76}}{\Omega_{12}(\alpha_1+\theta_2)}+\frac{p_1\alpha_2}{\Omega_{11}(\alpha_2+\theta_1)}$.

In long run, the fraction of time for which the server is busy in type 2 repair is given by $\hat{B}_0^2(\infty)=\hat{N}_2^2/\hat{D}_1$,where; $\hat{N}_2^2=\frac{q_2\alpha_1\hat{\hat{a}}_{76}}{\Omega_{22}(\alpha_1+\theta_2)}+\frac{q_1\alpha_2}{\Omega_{21}(\alpha_2+\theta_1)}$.

In steady – state, the number of visits per unit is given by $\hat{V}_0(\infty)=\hat{N}_3/\hat{D}_2$

where;

$$\hat{N}_3=1-\hat{b}_{67}\hat{b}_{76}+\frac{\hat{b}_{67}\theta_2}{(\alpha_1+\theta_2)}\left[\frac{p_2\Omega_{12}}{(\alpha_1+\Omega_{12})}+\frac{q_2\Omega_{22}}{(\alpha_1+\Omega_{22})}\right]+\frac{\theta_1}{(\alpha_1+\theta_1)}\left[\frac{p_1\Omega_{11}}{(\alpha_1+\Omega_{11})}+\frac{q_1\Omega_{21}}{(\alpha_1+\Omega_{21})}\right]$$

$$\hat{D}_2=1-\hat{b}_{67}\hat{b}_{76}$$

$$\hat{b}_{67} = \frac{\theta_2}{(\alpha_1+\theta_2)}\left\{\left[\frac{p_2\Omega_{12}}{(\alpha_1+\Omega_{12})}+\frac{q_2\Omega_{22}}{(\alpha_1+\Omega_{22})}\right]+\alpha_1\left[\frac{p_2}{(\alpha_1+\Omega_{12})}+\frac{q_2}{(\alpha_1+\Omega_{22})}\right]\right\}$$

$$+\frac{p_2\alpha_1\Omega_{11}}{(\alpha_1+\theta_2)(\Omega_{11}+\beta_1)}+\frac{q_1\alpha_2\Omega_{12}}{(\alpha_2+\theta_1)(\Omega_{12}+\beta_2)}$$

$$\hat{b}_{76} = \frac{\theta_1}{(\alpha_2+\theta_1)}\left\{\left[\frac{p_1\Omega_{11}}{(\alpha_2+\Omega_{11})}+\frac{q_1\Omega_{21}}{(\alpha_2+\Omega_{21})}\right]+\alpha_2\left[\frac{p_1}{(\alpha_2+\Omega_{11})}+\frac{q_1}{(\alpha_2+\Omega_{21})}\right]\right\}$$

$$+\frac{p_1\alpha_2\Omega_{21}}{(\alpha_2+\theta_1)(\Omega_{21}+\beta_1)}+\frac{q_1\alpha_2\Omega_{22}}{(\alpha_2+\theta_1)(\Omega_{22}+\beta_2)}$$

The expected cost per unit per unit time at steady state is $C = k_1\hat{A}_0 - k_2\hat{B}_0^2 - k_4\hat{V}_0$.

9.2 The two units are similar with general distributions:- Let

$F_i(t) = F(t)$, $L_i(t) = L(t)$, $K_i(t) = K(t)$, $H_i(t) = H(t)$,

$G_{ij}(t) = G_j(t)$, $P_i = P$, $q_i = q$, $p + q = 1$.

The transition probabilities are $p_{02} = p_{13} = p_{24} = p_{35} = p_{47} = p_{56} = 1$,

$$p_{62}^{(9.0)} = p\int_0^w\int_0^v dF(w)dG_1(v-u)dH(u),\; p_{62}^{(11.0)} = q\int_0^w\int_0^v dF(w)dG_1(v-u)dH(u) ,$$

$$p_{73}^{(8.1)} = p\int_0^w\int_0^v dF(w)dG_2(v-u)dH(u),\; p_{73}^{(10.1)} = q\int_0^w\int_0^v dF(w)dG_2(v-u)dH(u) ,$$

$$p_{67}^{(11.17)} = q\int_0^w\int_0^v dG_1(w-v)dF(v)\bar{G}_1(v-u)dH(u),$$

$$p_{76}^{(8.14)} = p\int_0^w\int_0^v dG_2(w-v)dF(v)\bar{G}_2(v-u)dH(u) ,$$

$$p_{76}^{(10.16)} = q\int_0^w\int_0^v dG_2(w-v)dF(v)\bar{G}_2(v-u)dH(u)$$

$$p_{6.15}^{(13)} = p_{7.14}^{(12)} = p\int\int_0^v dF(u)dH(v),\; p_{6.17}^{(13)} = p_{7.16}^{(12)} = q\int\int_0^v dF(u)dH(v) ,$$

$$p_{14.6} = p_{15.7} = \int g_1(t)\bar{L}(t)dt,\; p_{16.6} = p_{17.7} = \int g_2(t)\bar{L}(t)dt ,$$

The mean sojourn times are

$\mu_0 = \mu_1 = \int \bar{F}(t)dt$, $\mu_2 = \mu_3 = \int \bar{L}(t)dt$, $\mu_4 = \mu_5 = \int \bar{K}(t)dt$,
$\mu_6 = \mu_7 = \int \bar{F}(t)\bar{H}(t)dt$, $\mu_{14} = \mu_{15} = \int \bar{G}_1(t)dt$, $\mu_{16} = \mu_{17} = \int \bar{G}_2(t)dt$.

In this case $\hat{M}_i(t)$ are $\hat{\hat{M}}_0(t) = \hat{\hat{M}}_1(t) = \bar{F}(t)$, $\hat{\hat{M}}_0(t) = \hat{\hat{M}}_1(t) = \bar{F}(t)$.

The steady state availability of the system is $\hat{\hat{A}}_0(\infty) = \hat{\hat{N}}_1 / \hat{\hat{D}}_1$,

$$\hat{\hat{N}}_1 = \mu_0 \left\{1 - \left[p_{62}^{(9.0)} + p_{62}^{(11.0)} + p_{67}^{(9.15)} + p_{67}^{(11.17)} + p_{6.15}^{(13)} p_{15.7} + p_{6.17}^{(13)} p_{17.7}\right]\right\} + \mu_6,$$

$$\hat{\hat{D}}_1 = \left(\mu_{24} + \mu_{47}\right)\left(p_{62}^{(9.0)} + p_{62}^{(11.0)}\right) + \mu_{62}^{(9.0)} + \mu_{62}^{(11.0)} + \mu_{6.15}^{(13)} p_{15.7} + p_{6.15}^{(13)} \mu_{15.7} + \mu_{6.17}^{(13)} p_{17.7} + p_{6.17}^{(13)} \mu_{17.7}$$

In the long run, the fraction of time for which the server is busy in type 1 repair is given by $\hat{\hat{B}}_0^1(\infty) = \hat{\hat{N}}_2^1 / \hat{\hat{D}}_1$,where; $\hat{\hat{N}}_2 = p_{6.15}^{(13)} \mu_{15}$

Similarly, the fraction of time for which the server is busy in type 2 repair is given by $\hat{\hat{B}}_0^2(\infty) = \hat{\hat{N}}_2^2 / \hat{\hat{D}}_1$, where; $\hat{\hat{N}}_3 = p_{7.16}^{(12)} \mu_{16}$

In steady state, the number of visits per unit time is given by

$\hat{\hat{V}}_0(\infty) = \hat{\hat{N}}_3 / \hat{\hat{D}}_2$, where;

$$\hat{\hat{N}}_3 = 1 - \left(p_{67}^{(9.15)} + p_{67}^{(11.17)} + p_{6.15}^{(13)} p_{15.7} + p_{6.17}^{(13)} p_{17.7}\right),$$

$$\hat{\hat{D}}_2 = \left[1 - \left(p_{62}^{(9.0)} + p_{62}^{(11.0)} + p_{67}^{(9.15)} + p_{67}^{(11.17)} + p_{6.15}^{(13)} p_{15.7} + p_{6.17}^{(13)} p_{17.7}\right)\right]$$

The expected cost per unit per unit time at steady state is

$$C = k_1 \hat{\hat{A}}_0 - k_2 \hat{\hat{B}}_0^1 - k_3 \hat{\hat{B}}_0^2 - k_4 \hat{\hat{V}}_0.$$

9.3 The two units are similar with exponential distributions: Let

$F_i(t) = 1 - e^{-\alpha t}$, $L_i(t) = 1 - e^{-\beta t}$, $K_i(t) = 1 - e^{-\gamma t}$, $G_{ij}(t) = 1 - e^{-\Omega t}$, $p_i = p$

$q_i = q$, $p + q = 1$,

The transition probabilities are $p_{02} = p_{13} = p_{24} = p_{35} = p_{47} = p_{56} = 1$,

$p_{62}^{(9.0)} = p\Omega_1\theta / \alpha\left(\alpha + \Omega_1\right)\left(\alpha + \theta\right)$, $p_{62}^{(11.0)} = q\Omega_1\theta / \alpha\left(\alpha + \Omega_1\right)\left(\alpha + \theta\right)$,

$p_{73}^{(8.1)} = p\Omega_2\theta / \alpha\left(\alpha + \Omega_1\right)\left(\alpha + \theta\right)$, $p_{73}^{(10.1)} = q\Omega_2\theta / \alpha\left(\alpha + \Omega_1\right)\left(\alpha + \theta\right)$,

$p_{67}^{(9.15)} = p\alpha\theta / \left(\alpha + \Omega_1\right)\left(\alpha + \theta\right)$, $p_{67}^{(11.17)} = q\alpha\theta / \left(\alpha + \Omega_1\right)\left(\alpha + \theta\right)$,

$p_{76}^{(8.14)} = p\alpha\theta / \left(\alpha + \Omega_2\right)\left(\alpha + \theta\right)$, $p_{76}^{(10.16)} = q\alpha\theta / \left(\alpha + \Omega_2\right)\left(\alpha + \theta\right)$,

$p_{6.15}^{(13)} = p_{7.14}^{(12)} = p\alpha / \left(\alpha + \theta\right)$, $p_{6.17}^{(13)} = p_{7.16}^{(12)} = q\alpha / \left(\alpha + \theta\right)$,

$p_{14.6} = p_{15.7} = \Omega_1 / \left(\Omega_1 + \beta\right)$, $p_{16.6} = p_{17.7} = \Omega_2 / \left(\Omega_2 + \beta\right)$.

The mean sojourn times are

$\mu_0 = \mu_1 = 1/\alpha$, $\mu_6 = \mu_7 = 1/(\alpha+\theta)$, $\mu_{14} = \mu_{15} = 1/\Omega_1$, $\mu_{16} = \mu_{17} = 1/\Omega_2$,

The state availability of the system is $\tilde{A}_0(\infty) = \tilde{N}_1 / \tilde{D}_1$, where;

$$\tilde{N}_1 = \frac{1}{\alpha}\left\{1-\left[\frac{\theta(\alpha^2+\Omega_1)}{\alpha(\alpha+\Omega_1)(\alpha+\theta)} + \frac{\alpha}{(\alpha+\theta)}\left(\frac{p\Omega_1}{(\Omega_1+\beta)} + \frac{q\Omega_2}{(\Omega_1+\beta)}\right)\right]\right\} + \frac{1}{(\alpha+\theta)},$$

$$\tilde{D}_1 = \frac{\Omega_1\theta(\gamma+\beta)}{\gamma\beta\alpha(\alpha+\Omega_1)(\alpha+\theta)} + \frac{\Omega_1\theta}{\alpha^2(\alpha+\Omega_1)}\left[\frac{(2\alpha+\Omega_1)}{(\alpha+\Omega_1)^2} - \frac{(2\alpha+\theta)}{(\alpha+\theta)^2}\right]$$

$$+\frac{\alpha(\alpha+2\theta)}{\theta(\alpha+\theta^2)}\left[\frac{p\Omega_1}{(\Omega_1+\beta)} + \frac{q\Omega_2}{(\Omega_2+\beta)}\right] + \frac{\alpha}{(\alpha+\theta)}\left(\frac{p}{\Omega_1} + \frac{q}{\Omega_2}\right)$$

In the long run, the fraction of time for which the server is busy type 1 repair is given by $\tilde{B}_0^1(\infty) = \tilde{N}_2^1 / \tilde{D}_1$ where; $\tilde{N}_2^1 = p\alpha/\Omega_1(\alpha+\theta)$.

Similarity , the fraction of time for which the server is busy in type 2 repair is given by $\tilde{B}_0^2(\infty) = \tilde{N}_2^2 / \tilde{D}_1$ where ; $\tilde{N}_2^2 = q\alpha/\Omega_2(\alpha+\theta)$.

In steady state , the number of visits per unit is given by $\tilde{V}_0(\infty) = \tilde{N}_3 / \tilde{D}_2$
where;

$$\tilde{N}_3 = 1 - \frac{\alpha}{(\alpha+\theta)}\left[\frac{\theta}{(\alpha+\Omega_1)} + \frac{p\Omega_1}{(\Omega_1+\beta)} + \frac{q\Omega_2}{(\Omega_2+\beta)}\right],$$

$$\tilde{D}_2 = -\left\{1 - \frac{\theta}{(\alpha+\theta)(\alpha+\Omega_1)}\left(\frac{\Omega_1}{\alpha}+\alpha\right) - \frac{\alpha}{(\alpha+\theta)}\left[\frac{p\Omega_1}{(\Omega_1+\beta)} + \frac{q\Omega_2}{(\Omega_2+\beta)}\right]\right\}$$

The expected cost per unit time at steady state is $C = k_1\tilde{A}_0 - k_2\tilde{B}_0^1 - k_3\tilde{B}_0^2 - k_4\tilde{V}$

9.4 Numerical Example

Let the two units are similar with exponential distributions and

$P = 0.8$, $q = 0.2$, $\theta = 0.3$, $\Omega_1 = 0.2$, $\Omega_2 = 0.1$, $\gamma = 0.5$, $K_1 = 1000$, $K_2 = 100$, $K_3 = 120$, $K_4 = 50$

Table 5.1

α	c		
	$\beta = 0.25$	$\beta = 0.25$	$\beta = 0.25$
0.40	324.356	345.043	359.924
0.45	296.093	330.182	356.100
0.50	278.341	322.850	352.147
0.55	265.602	312.952	343.075
0.60	252.914	300.222	330.010
0.65	239.794	285.777	314.663
0.70	226.569	270.651	298.367
0.75	213.612	255.569	282.007
0.80	201.192	240.973	266.114
0.85	189.461	227.110	250.980

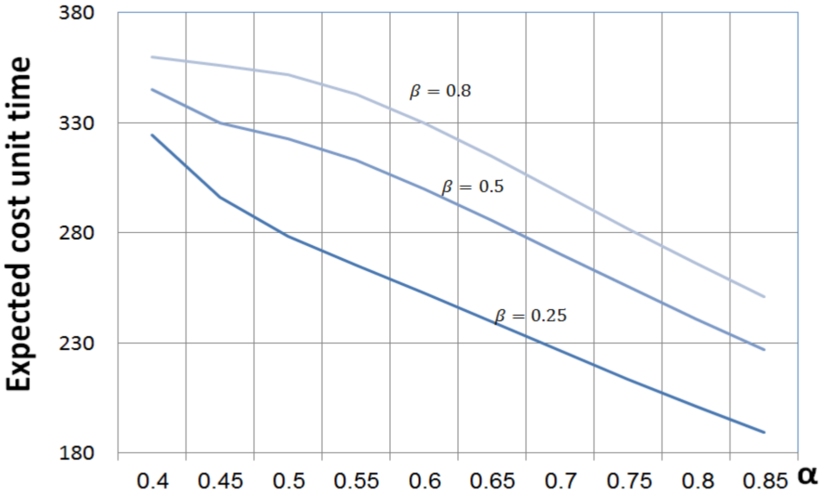


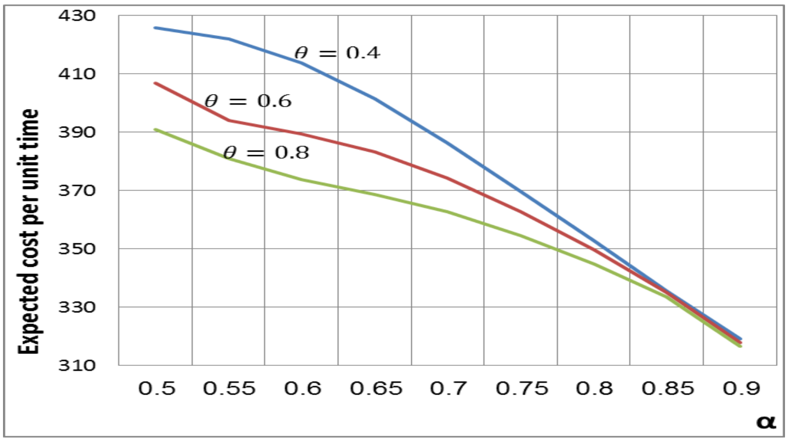
Fig.5.2 Relation between the failure rate of the unit and the expected cost per unit time

Let

$p = 0.8$, $q = 0.2$, $\beta = 0.4$, $\Omega_1 = 0.2$, $\Omega_2 = 0.1$, $\gamma = 0.5$, $k_1 = 1500$, $k_2 = 100$, $k_3 = 120$, $k_4 = 50$

Table 5.2

α	c		
	$\theta = 0.4$	$\theta = 0.6$	$\theta = 0.8$
0.50	425.881	406.827	390.905
0.55	422.118	394.149	380.952
0.60	413.904	389.596	373.823
0.65	401.537	383.382	368.622
0.70	386.421	374.269	362.685
0.75	369.836	362.796	354.671
0.80	352.719	349.782	344.781
0.85	335.699	335.000	333-556
0.90	319.165	317.832	316.537



The failure rate of the unitFig. 5.3.
Relation between the failure rates of the unit
and the expected cost per unit time.

REFERENCES

[1]. BARLOW, R.E. and PROSHAN. F., "Mathematical theory of reliability". John Wiely, 1965.

[2]. GOEL, L.R., and GUPTA. P. "A two-unit deteriorating standby system with inspection". Microelectron. Reliab. Vol. 24, No 3, 435-438, 1984.

[3]. GOEL, L.R., and GUPTA. R. "Analysis of a two-unit standby system with three modes and imperfect switching device". Microelectron. Reliab. Vol. 24, No 3, 425-429, 1984.

[4]. GOEL, L.R., KUMAR, A and RASTOGI, AK, "Analysis of two unit standby system with partial failure and two types of repair ". Microelectron Reliab, Vol.24, No 3, 873-876,1984.

[5]. GOEL, L.R., SHARMA, G.C. and GUPTA. P., "Stochastic behavior and profit analysis of a redundant system with slow switching device". Microelectron. Reliab. Vol. 26, No. 2, 215-219, 1986.

[6]. GOPALAN. M.N. and NAIDU, R.S., "Cost benefit analysis of one-server two-unit cold standby system subject to inspection", Microelectron. Reliab. Vol. 22, 699-705, 1982.

[7]. GUPTA, S.M., JAISWAL, N.K. and GOEL, L.R., "stochastic behaviour of standby redundant system with three modes ". Microelectron. Reliab. Vol. 23, No29, 329-331, 1983.

[8]. GUPTA, R., BAJAJ, C.P. and SINHA, S.M,. "A single server multi-component two-unit cold standby system with inspection and imperfect switching device", Microelectron. Reliab. Vol.26, 873-877, 1986.

[9]. GUPTA, R., BAJAJ, C.P. and SINHA, S.M,. "Cost benefit analysis of a multi-component stochastic system with inspection and slow switch", Microelectron. Reliab. Vol. 26, No. 5, 879-882, 1986.

[10]. MOKADDIS, G.S., "Reliability of a renewable system with redundant elements", The thirtheenth annual conference in statistics, Computer Science and Operation Research I.S.S.R Cairo Univ. March 1978.

[11]. MOKADDIS, G.S., ELIAS, S.S. and LABIB, S.W,. "On a two dissimilar unit standby system with switychover time and proper initialization of connect switching". Microelectron. Reliab. Vol.27, No. 5, 819-822, 1987.

[12]. MOKADDIS, G.S., ELIAS, S.S. and LABIB, S.W,. "On a two-unit standby redundant system subject to repair and preventive maintenance", Microelectron. Reliab. Vol.27, No. 4, 661-675, 1987.

[13]. MOKADDIS, G.S., LABIB, S.W. and El-SAID, K.H.M. "Two models for two-dissimilar-unit standby redundant system with three types of repair facilities and perfect or imperfect switch", Microelectron. Reliab. Vol.34, 1239-1247, 1994.

[14]. NAIDU, R.S. and GOPALAN, M.N., "On the stochastic behavior of a 1-server 2-unit system subject to arbitrary failure, random inspection and two failure modes". Microelectron. Reliab. Vol. 24, 375-378, 1984.

[15]. SINGH, S.P., KAPUR, P.K. and KAPOOR, K.R., "Two-unit redundant system with random switchover time and two types of repair". Microelectron. Reliab, Vol. 19, 325-328, 1979.