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Reliability Analysis of a two Dissimilar Unit Warm Standby Redundant System with Imperfect Switch

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ABSTRACT: *This paper investigates the reliability analysis of a two-dissimilar unit warm standby redundant system with two modes (normal and total failure) and imperfect switching device has been studied. Failure, repair of units and repair of a switch time distributions are stochastically independent random variables each having an arbitrary distribution. The system is analyzed by the semi-Markov process technique. Some reliability measures of interest to system designers as well as operations managers have been obtained. Explicit expressions for the Laplace-Stieltjes transforms of the distribution function of the first passage time, mean time to system failure, steady state availability and the busy period analysis of the system are obtained. Certain important results have been derived as particular cases.*
Keywords: *Mean time to system failure, Availability, Busy period, Reliability.*

I. INTRODUCTION AND DESCRIPTION OF THE SYSTEM

Many authors [4 , 7 , 12] have studied the two – unit warm standby system operating under different model formulations by using the theory of semi-Markov process , regenerative process and Markov renewal process. They obtained the mean time to system failure, the pointwise availability and the steady state availability of the system using the theory of regenerative process . The purpose of the present paper is to investigate a two-dissimilar-unit warm standby system where each unit works in two different modes normal and total failure. The failure , the repair of units and the repair of a switch time distributions are assumed to different arbitrary distributed , the probability that the switch works at the time of need is $p\ (=1-q)$. Repair of a total failure unit, failed from operative or standby state continues when the other unit enters the total failure mode, when an operative unit fails , switch is used to disconnect the failed unit and connect the standby unit if it is operative . A single repair facility is available , priority for repair being given to transfer switch , after repair of a unit or the switch works like a new one , repair time distribution of a unit failed from the standby state is different from that of the unit failed from operative state. Switch failure occurs in a non-regenerative state , for the first time . Using the semi-Markov process technique and the results of the regenerative process , various measures of the system effectiveness as mean time to system failure , pointwise availability , steady state availability and busy period analysis are found out. The results by [15] are derived from the present results as a special case . In this system the following assumptions and notations are used to analyze the system .

(1) The system consists of two-dissimilar units in warm standby configuration, so That a unit can fail during its standby state.
(2) Each unit has two modes normal and total failure.
(3) Failure of units, repair of units and repair of a switch times are stochastically independent random variables each having an arbitrary distribution.
(4) The probability that the switch works at the time of need is $p\ (=1-q)$.
(5) A single repair facility is available. Priority for repair given to transfer switch. After repair of a unit or the switch works like a new one .
(6) Repair of a total failure unit ., failed from operative or standby state continues when the other unit enters the total failure mode from the normal mode .
(7) When an operative unit fails, switch is used to disconnect the failed unit and connect the standby unit(if it is operative).
(8) Switch failure occurs in a non-regenerative state , for the first time .
(9) Repair time distribution of a unit failed from the standby state is different from that of the unit failed from operative state .

II. NOTATIONS AND STATES OF THE SYSTEM

E_0 state of the system at time point $t = 0$,

E set of regenerative states $\{0,1,2,3,4,5,6,7\}$,

$\bar{E}$ set of non-regenerative states; $\{s_8, s_9, s_{10}, s_{11}\}$,

$f_i(t)$, $F_i(t)$ pdf and cdf of failure time of the i -th operative unit from normal mode to total failure mode ; $i = 1,2$,

$l_i(t)$, $L_i(t)$ pdf and cdf of failure time of the i -th standby unit from normal mode to total failure mode ; $i = 1,2$,

$g_i(t)$, $G_i(t)$ pdf and cdf of time to repair for the i -th unit failed while operative i= 1, 2 ,

$h_i(t)$, $H_i(t)$ pdf and cdf of time repair for a unit failed while in standby ; $i = 1,2$,

$j(t)$, $J(t)$ pdf and cdf of time to repair for the switch,

$q_{ij}(t)$, $Q_{ij}(t)$ pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0,t]; i, j \in E$,

$q_{ij}^{(k)}(t)$, $Q_{ij}^{(k)}(t)$ pdf and cdf of first passage time from regenerative state i to a regenerative state j or to failed state j visiting state k only once in $(0,t]; i, j \in E; k \in \bar{E}$,

$q_{ij}^{(k,h)}(t), Q_{ij}^{(k,h)}(t)$ pdf and cdf of first passage time from regenerative state i to a regenerative state j or to failed state j visiting state k and h in $(0,t]$ respectively; $i, j \in; h, k \in \bar{E}$

P_{ij} one step transition probability from state i to state $j; i, j \in E$,

$P_{ij}^{(k)}$ probability that the system in state i goes to state j passing through state $k; i.j \in E; k \in \bar{E}$,

$P_{ij}^{(k,h)}$ probability that the system in state i goes to state j passing through states $k, h; i, j \in E; k, h \in \bar{E}$,

$\pi_i(t)$ cdf of first passage time from regenerative state i to failed state,

$A_i(t)$ probability that the system is in up state instant t given that the system started from regenerative state i at time $t = 0$,

$M_i(t)$ probability that the system , having started from state i is up at time t without making any transition into any other regenerative state ,

$B_i(t)$ probability that the repairman is busy at time t given that the system entered regenerative state i at time $t = 0$,

$V_i(t)$	expected number of visits by the repairman given that the system started from regenerative state at time $t=0$,
μ_{ij}	contribution to mean sojourn time in state i when transition is to state j is $-\tilde{Q}_{ij}(0)=q_{ij}^{*}(0)$,
μ_i	mean sojourn time in state i , $\mu_i=\sum_j\left[\mu_{ij}+\sum_k \mu_{ij}^{(k)}\right]$
$\sim$	symbol for Laplace – Stieltjes transform , e.g. $\tilde{F}(s)=\int e^{-st}dF(t)$,
$*$	symbol for Laplace transform , e.g. $F^{*}(s)=\int e^{-st}F(t)dt$
&	symbol for stieltjes convolution e.g. $A(t)\&B(t)=\int_0^t B(t-u)dA(u)$
©	symbol for ordinary convolution , e.g. $a(t)©b(t)=\int_0^t a(u)b(t-u)du$

For simplicity, whenever integration limits are $(0,\infty)$ they are not written.

symbols used for the states :

N_{0i}	the i-th unit is operative in normal mode $i=1,2$,
$N_{\bar{s}i}$	the i -th unit is standby in normal mode $i=1,2$,
F_{ori}	the i -th unit operative in total failure mode and under repair ; $i=1,2$,
$F_{\bar{s}ri}$	the i -th unit is standby in total failure mode and under repair ; $i=1,2$,
F_{oRi}	the i -th unit is operative in total failure mode , with repair continued from earlier state; $i=1,2$,
$F_{\bar{s}Ri}$	the i -th unit is standby in total failure mode, with repair continued from earlier state; $i=1,2$,
F_{owi}	the i -th unit operative in total failure mode and waiting for repair ; $i=1,2$,
s_r	switching device is under repair.

Considering these symbols, the system may be in one of the following states:

$s_0\equiv(N_{01},N_{\bar{s}2})$, $s_1\equiv(N_{\bar{s}1},N_{02})$, $s_2\equiv(F_{0r1},N_{02})$, $s_3\equiv(N_{01},F_{0r2})$,
$s_4\equiv(N_{01},F_{\bar{s}r2})$, $s_5\equiv(F_{\bar{S}r1},N_{02})$, $s_6\equiv(F_{0W1},N_{\bar{s}2},S_r)$, $s_7\equiv(N_{\bar{s}1},F_{0w2},S_r)$
$s_8\equiv(F_{0R1},F_{0w2})$, $s_9\equiv(F_{0w1},F_{0Rr})$, $s_{10}\equiv(F_{0W1},F_{\bar{s}R2})$, $s_{11}\equiv(F_{\bar{s}R1},F_{0w2})$.

Up states : S_0-S_5. Down states: S_6-S_{11} .
States and possible transitions between them are shown in Fig. 4.1.

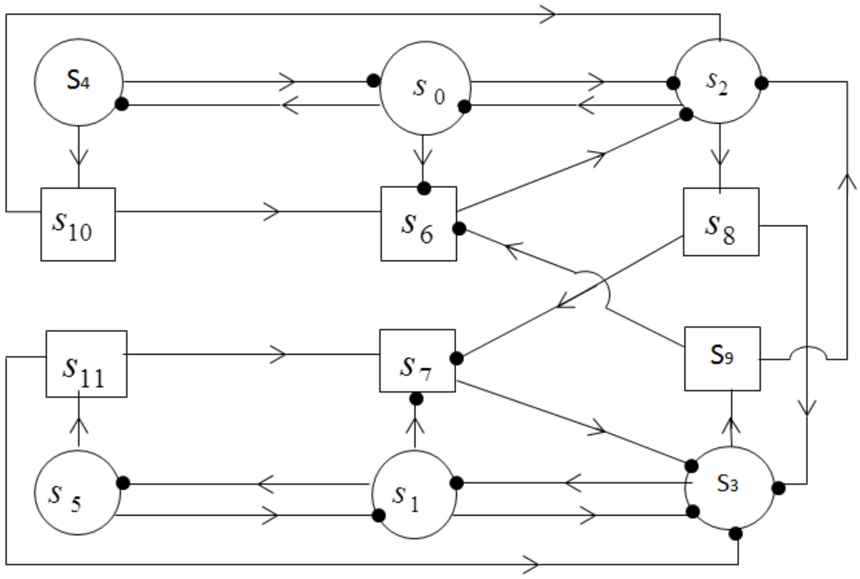


Fig .1.

Up state Down state Regeneration point

III. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

It can be observed that the time points of entry into $S_i \in E$ are regenerative points so these states are regenerative . Let $T_0(\equiv 0), T_1, T_2, \ldots$ denote the time points at which the system enters any state $S_i \in E$ and X_n denotes the state visited at the time point T_{n+1}, i.e. just after the transition at T_{n+1}, then $\{X_n, T_n\}$ is a Markov renewal process with state space E and $Q_{ij} = p[X_{n+1} = j, T_{n+1} - T_n < t \mid X_n = i]$ is a semi-Markov kernel over E . The stochastic matrix of the embedded Markov chain is $P = (P_{ij}) = (Qij(\infty)) = Q(\infty)$ and the nonzero elements P_{ij} are

$$P_{02} = P\int \bar{L}_2(t)d\,F_1(t) \quad , P_{04} = \int \bar{F}_1(t)dL_2(t) \quad , P_{06} = q\int \bar{L}_2(t)dF_1(t) \quad ,$$

$$P_{13} = P\int \bar{L}_1(t)dF_2(t), P_{15} = \int \bar{F}_2(t)dL_1(t), \quad P_{17} = q\int \bar{L}_1(t)dF_2(t),$$

$$P_{20} = \int \bar{F}_2(t)dG_1(t) \quad , P_{28} = \int \bar{G}_1(t)dF_2(t), P_{23}^{(8)} = P\int F_2(t)dG_1(t),$$

$$P_{27}^{(8)} = q\int F_2(t)dG_1(t), \quad P_{31} = \int \bar{F}_1(t)dG_2(t), P_{39} = \int \bar{G}_2(t)dF_1(t),$$

$$P_{32}^{(9)} = P\int F_1(t)dG_2(t),\quad P_{36}^{(9)} = q\int F_1(t)dG_2(t), P_{40} = \int \bar{F}_1(t)dH_2(t),$$
$$P_{4,10} = \int \bar{H}_2(t)dF_1(t), P_{42}^{(10)} = P\int F_1(t)dH_2(t), P_{46}^{(10)} = q\int F_1(t)dH_2(t),$$
$$P_{51} = \int \bar{F}_2(t)dH_1(t), P_{5,11} = \int \bar{H}_1(t)dF_2(t), P_{53}^{(11)} = P\int F_2(t)dH_1(t),$$
$$P_{57}^{(11)} = q\int F_2(t)dH_1(t),\ P_{62} = P_{73} = 1 \qquad (3.1 - 3.23)$$

The mean sojourn times μ_i in state S_i are

$$\mu_0 = \int \bar{F}_1(t)\bar{L}_2(t)dt, \mu_1 = \int \bar{F}_2(t)\bar{L}_1(t)dt, \mu_2 = \int \bar{G}_1(t)\bar{F}_2(t)dt\ \ \mu_3 = \int \bar{G}_2(t)\bar{F}_2(t)dt,$$
$$\mu_4 = \int \bar{F}_1(t)\bar{H}_2(t)dt,\ \mu_5 = \int \bar{F}_2(t)\bar{H}_1(t)dt, \mu_6 = \mu_6 = \int \bar{J}(t)dt,\ \mu_8 = \int \bar{G}_1(t)dt,$$
$$\mu_9 = \int \bar{G}_2(t)dt,\ \mu_{10} = \int \bar{H}_2(t)dt,\ \mu_{11} = \int \bar{H}_1(t)dt. \qquad (3.24 – 3.34)$$

IV. MEAN TIME TO SYSTEM FAILURE

Time to system failure can be regarded as first passage to the failed states $S_6, S_7, S_8, S_9, S_{10}, S_{11}$ which are considered as absorbing. By probabilistic arguments, the following recursive relations for $\pi_i(t)$ are obtained.

$$\pi_0(t) = Q_{02}(t)\&\pi_2(t) + Q_{04}(t)\&\pi_4(t) + Q_{06}(t),$$
$$\pi_1(t) = Q_{13}(t)\&\pi_3(t) + Q_{15}(t)\&\pi_5(t) + Q_{17}(t),$$
$$\pi_2(t) = Q_{20}(t)\&\pi_0(t) + Q_{28}(t),\ \pi_3(t) = Q_{31}(t)\&\pi_1(t) + Q_{39}(t),$$
$$\pi_4(t) = Q_{40}(t)\&\pi_0(t) + Q_{4.10}(t),\ \pi_5(t) = Q_{51}(t)\&\pi_1(t) + Q_{5.11}(t). \qquad (4.1 – 4.6)$$

Taking Laplace – Stieltjes transforms of equations (4.1 – 4.6) and solving for $\tilde{\pi}_0(s)$, dropping the argument "s" for brevity, it follows.

$$\tilde{\pi}_0(s) = N_0(s)/D_0(s), \qquad (4.7)$$
$$N_0(s) = (1 - \tilde{Q}_{13}\tilde{Q}_{31} - \tilde{Q}_{15}\tilde{Q}_{51})(\tilde{Q}_{06} + \tilde{Q}_{04}\tilde{Q}_{4.10}),$$
$$D_0(s) = (1 - \tilde{Q}_{13}\tilde{Q}_{31} - \tilde{Q}_{15}\tilde{Q}_{51})(1 - \tilde{Q}_{04}\tilde{Q}_{4.10}). \qquad (4.8-4.9)$$

The mean time to system failure with starting state S_0 is given by

$$MTSF = N_0/D_0. \qquad (4.10)$$
$$N_0 = p_{02}(\mu_{13}p_{31} + p_{13}\mu_{31} + \mu_{15}p_{51} + p_{15}\mu_{51}) + (\mu_{04} + p_{04}\mu_4 + \mu_{06})(1 - p_{13}p_{31} - p_{15}p_{51}),$$
$$D_0 = (1 - p_{13}p_{31} - p_{15}p_{51})(1 - p_{04}p_{40}). \qquad (4.11 – 4.12)$$

V. AVAILABILITY ANALYSIS

Elementary Probability arguments yield the following relations for $A_i(t)$

$$A_0(t)=M_0(t)+q_{02}(t)\text{Ó}A_2(t)+q_{04}(t)\text{Ó}A_4(t)+q_{06}(t)\text{Ó}A_6(t)\ ,$$
$$A_1(t)=M_1(t)+q_{013}(t)\text{Ó}A_3(t)+q_{15}(t)\text{Ó}A_5(t)+q_{17}(t)\text{Ó}A_7(t)\ ,$$
$$A_2(t)=M_2(t)+q_{20}(t)\text{Ó}A_0(t)+q_{23}^{(8)}(t)\text{Ó}A_3(t)+q_{27}^{(8)}(t)\text{Ó}A_7(t)\ ,$$
$$A_3(t)=M_3(t)+q_{31}(t)\text{Ó}A_1(t)+q_{32}^{(9)}(t)\text{Ó}A_2(t)+q_{36}^{(9)}(t)\text{Ó}A_6(t),$$
$$A_4(t)=M_4(t)+q_{40}(t)\text{Ó}A_0(t)+q_{42}^{(10)}(t)\text{Ó}A_2(t)+q_{46}^{(10)}(t)\text{Ó}A_6(t)\ ,$$
$$A_5(t)=M_5(t)+q_{51}(t)\text{Ó}A_1(t)+q_{53}^{(11)}(t)\text{Ó}A_3(t)+q_{57}^{(11)}(t)\text{Ó}A_7(t)\ ,$$
$$A_6(t)=q_{62}(t)\text{Ó}A_2(t)\ ,A_7(t)=q_{73}(t)\text{Ó}A_3(t)\ , \qquad . \qquad (5.1\text{-}5.8)$$
$$M_0(t)=\bar{F}_1(t)\bar{L}_2(t)\ ,\ M_1(t)=\bar{F}_2(t)\bar{L}_1(t)\ ,\ M_2(t)=\bar{G}_1(t)\bar{F}_2(t)\ ,$$
$$M_3(t)=\bar{G}_2(t)\bar{F}_1(t),M_4(t)=\bar{F}_1(t)\bar{H}_2(t),M_5=\bar{F}_2(t)\bar{H}_1(t). \qquad (5.9\text{-}5.14)$$

Taking Laplace transforms of equations (5.1-5.8) and solving for $A_0^*(s)$, we have

$$A_0^*(s)=N_1(s)/D_1(s) \qquad , \qquad (5.15)$$
$$N_1(s)=M_0^*I_0-M_1^*I_1+M_2^*I_2-M_3^*I_3+M_4^*I_4-M_5^*I_5\ ,$$
$$D_1(s)=I_0-q_{20}^*I_2-q_{40}^*I_4 \qquad ,$$
$$I_0=1-bq_{31}^*+q_{15}^*(q_{31}^*f\ -q_{51}^*)-cd\,(1-q_{15}^*q_{51}^*),$$
$$I_1=-q_{31}^*c\,(a+q_{04}^*e) \qquad ,$$
$$I_2=-[1+bq_{31}^*+q_{15}^*(q_{31}^*f\ -q_{51}^*)](a+q_{04}^*e),I_3=-c\,(1-q_{15}^*q_{51}^*)(a+q_{04}^*e)\ ,$$
$$I_4=q_{31}^*[1+bq_{31}^*+q(q_{31}^*f\ -q_{51}^*)]-cq_{04}^*d\,(1-q_{15}^*q_{51}^*),I_5=-q_{51}^*q_{31}^*c\,(a+q_{04}^*e),$$
$$a=-(q_{02}^*+q_{06}^*q_{62}^*) \quad , \quad b=-(q_{13}^*+q_{17}^*q_{73}^*) \quad ,c=-(q_{23}^{(8)*}+q_{27}^{(8)*}q_{73}^*),$$
$$d=-(q_{32}^{(9)*}+q_{36}^{(9)*}q_{62}^*),e=-(q_{42}^{(10)*}+q_{46}^{(10)*}q_{62}^*),f\ =-(q_{53}^{(11)*}+q_{57}^{(11)*}q_{73}^*).$$
(5.16 – 5.29)

The steady state availability of the system is

$$A_0(\infty)=N_1/D_1 \qquad , \qquad (5.30)$$
$$N_1=\mu_0\bar{I}_0-\mu_1\bar{I}_1+\mu_2\bar{I}_2-\mu_3\bar{I}_3+\mu_4\bar{I}_4-\mu_5\bar{I}_5\ ,$$
$$D_1=\bar{I}_0'-\mu_{20}\bar{I}_2-p_{21}\bar{I}_2'+\mu_{40}\bar{I}_4-p_{40}\bar{I}_4' \qquad ,$$
$$\bar{I}_0=1-b_0q_{31}+q_{15}(q_{31}^*f_0-q_{51})-c_0d_0(1-q_{15}q_{51}),\bar{I}_1=-p_{31}c_0(a_0+q_{04}e_0),$$

$$\bar{I}_2 = -[1+b_0q_{31}+q_{15}(q_{31}f_0-q_{51})](a_0+q_{04}e_0), \bar{I}_3 = -_0c(1-q_{15}q_{51})(a_0+q_{04}e\ ),$$

$$\bar{I}_4 = p_{04}[1+b_0q_{31}+p_{15}(q_{31}f_0-q_{51})]-c_0q_{04}d_0(1-q_{15}q_{51}) \quad ,$$

$$\bar{I}_5 = -p_{51}q_{31}c_0(a_0+q_{04}e_0) \quad ,$$

$$\bar{I}'_0 = b'_0p_{31}+b_0\mu_{31}-\mu_{31}(p_{31}f_0-p_{51})$$
$$-p_{15}(\mu_{31}f_0-p_{31}f'_0-\mu_{51})-(c_0d'_0+c'_0d_0)(1-p_{15}p_{51}) \quad ,$$
$$-c_0d_0(\mu_{15}p_{51}+p_{15}\mu_{51})$$

$$\bar{I}'_2 = [b_0\mu_{13}+b'_0p_{31}-\mu_{51}(p_{31}f_0-p_{51})$$
$$-p_{15}(\mu_{31}f_0-p_{31}f'_0-\mu_{51})](a_0+p_{04}e_0) \quad ,$$
$$-[1+b_0p_{31}+p_{51}(p_{31}f_0-p_{51})](a'_0-\mu_{04}e_0+p_{04}e'_0)$$

$$\bar{I}'_4 = -\mu_{04}[1+b_0p_{13}+p_{15}(p_{31}f_0-p_{51})]$$
$$-p_{04}(b'_0p_{31}-b_0\mu_{31}-\mu_{15}(p_{31}f_0-p_{51})$$
$$+p_{15}(p_{31}f'_0-\mu_{31}f_0+\mu_{51})] \quad ,$$
$$-[c'_0p_{04}d_0-c'_0p_{04}d_0-c_0(p_{04}d'_0-\mu_{04}d_0)$$
$$(1-p_{15}p_{51})c_0p_{04}d_0(\mu_{15}p_{51}+p_{15}\mu_{51})$$

$$a_0 = -(p_{02}+p_{06}), b_0 = -(q_{13}+q_{17}), c_0 = -(p_{23}^{(8)}+p_{27}^{(8)}), d_0 = -(p_{32}^{(9)}+p_{36}^{(9)}),$$

$$e_0 = -(p_{42}^{(10)}+p_{46}^{(10)}), f_0 = -(p_{53}^{(11)}+p_{57}^{(11)}), a'_0 = \mu_{02}+p_{06}\mu_{62}+\mu_{06},$$

$$b'_0 = \mu_{31}+p_{17}\mu_{63}+\mu_{17}, c'_0 = \mu_{43}^{(8)}+p_{27}^{(8)}\mu_{73}+\mu_{27}^{(8)}$$

$$d'_0 = \mu_{32}^{(9)}+p_{36}^{(9)}\mu_{62}+\mu_{36}^{(9)}, e'_0 = \mu_{42}^{(10)}+p_{46}^{(10)}\mu_{62}+\mu_{46}^{(10)}$$

$$f'_0 = \mu_{53}^{(11)}+p_{57}^{(11)}\mu_{73}+\mu_{57}^{(11)} \quad , \qquad (5.31\text{-}5.53)$$

VI. BUSY PERIOD ANALYSIS

Elementary probability arguments yield the following relations for $B_i(t)$

$$B_0(t) = q_{02}(t)\text{Ó}\,B_2(t)+q_{04}(t)\text{Ó}\,B_4(t)+q_{06}(t)\text{Ó}\,B_6(t) \quad ,$$

$$B_1(t) = q_{13}(t)\text{Ó}\,B_3(t)+q_{15}(t)\text{Ó}\,B_5(t)+q_{17}(t)\text{Ó}\,B_7(t) \quad ,$$

$$B_2(t) = V_2(t)+q_{20}(t)\text{Ó}\,B_0(t)+q_{23}^{(8)}(t)\text{Ó}\,B_3(t)+q_{27}^{(8)}(t)\text{Ó}\,B_7(t) \quad ,$$

$$B_3(t) = V_3(t)+q_{31}(t)\text{Ó}\,B_1(t)+q_{32}^{(9)}(t)\text{Ó}\,B_2(t)+q_{36}^{(9)}(t)\text{Ó}\,B_6(t) \quad ,$$

$$B_4(t) = V_4(t) + q_{40}(t)\text{Ó}B_0(t) + q_{42}^{(10)}(t)\text{Ó}B_2(t) + q_{46}^{(10)}(t)\text{Ó}B_6(t) \quad ,$$

$$B_5(t) = V_5(t) + q_{51}(t)\text{Ó}B_1(t) + q_{53}^{(11)}(t)\text{Ó}B_3(t) + q_{57}^{(11)}(t)\text{Ó}B_7(t),$$

$$B_6(t) = V_6(t) + q_{62}(t)\text{Ó}B_2(t) \;, B_7(t) = V_7(t) + q_{73}(t)\text{Ó}B_3(t) \quad ,$$

where

$$V_2(t) = \bar{G}_1(t)\bar{F}_2(t) \;, V_3(t) = \bar{G}_2(t)\bar{F}_1(t) \;, V_4(t) = \bar{F}_1(t)\bar{H}_2(t) \quad ,$$

$$V_5(t) = \bar{F}_2(t)\bar{H}_1(t) \quad , V_6(t) = V_7(t) = \bar{J}(t) \quad ,$$

Taking Laplace transforms of equations (4.6.1 –4.6.8) and solving for $B_0^*(s)$, we have

$$B_0^*(s) = N_2(s) / D_1(s) \quad , \tag{6.14}$$

$$\begin{aligned} N_2(s) = {} & q_{06}^* V_6^* I_0 - q_{17}^* V_7^* I_1 - (V_2^* + q_{27}^{(8)*} V_7^*) I_2 \\ & + (V_3^* + q_{36}^{(9)*} V_6^*) I_3 - (V_4^* + q_{46}^{(10)*} V_6^*) I_4 \quad , \\ & + (V_5^* + q_{57}^{(11)*} V_7^*) I_5 \end{aligned} \tag{6.15}$$

$D_1(s), I_j ; j = 0,1,2,3,4,5$ are given by (5.16 – 5.29)

Expected busy period of service facility in $(0,t]$

$\mu_b(t) =$ Expected busy time of the repairman in $(0,t]$.

The repairman may be buys during $(0,t]$ starting from initial state S_0.

Hence $\mu_b(t) = \int_0^t B_0(u)du$,so that $\mu_b^*(s) = B_0^*(s)/s$ .Thus one can evaluate $\mu_b(t)$ by taking inverse Laplace transform of $\mu_b^*(s)$.

Expected idle time of the repairman in $(0,t]$ is $\mu_1(t) = t - \mu_b(t)$.

VII. SPECIAL CASES

7.1 The two units are dissimilar with exponential distributions: Let

α_i failure rate of the i –th operative unit from the normal mode to the total failure mode ; $i = 1,2$; $\alpha_i > 0$,

β_i failure rate of the i – th standby unit from the normal mode to the total failure mode ; $i = 1,2$; $\beta_i > 0$,

γ_i rat of repair of the i – th unit failed while operative ; $i = 1,2$; $\gamma_i > 0$,

θ_i rat of repair of the i – th unit failed while standby; $i = 1,2$; $\gamma_i > 0$,

δ rate of repair of the switch ; $\delta > 0$.

Transition probabilities are

$p_{02} = p\alpha_1/(\alpha_1+\beta_2), p_{04} = \beta_2/(\alpha_1+\beta_2), P_{06} = q\alpha_1/(\alpha_1+\beta_2)$,

$p_{13} = p\alpha_2/(\alpha_2+\beta_1), p_{15} = \beta_1/(\alpha_2+\beta_1), p_{17} = q\alpha_2/(\alpha_2+\beta_1)$,

$p_{20} = \gamma_1/(\gamma_1+\alpha_1), p_{28} = \alpha_2/(\gamma_1+\alpha_1), p_{23}^{(8)} = p\alpha_2/(\alpha_2+\gamma_1)$,

$p_{27}^{(8)} = q\alpha_2/(\alpha_2+\gamma_1), p_{31} = \gamma_2/(\gamma_2+\alpha_1), p_{39} = \alpha_1/(\gamma_2+\alpha_1)$

$p_{32}^{(9)} = p\alpha_1/(\alpha_1+\gamma_2)$, $p_{36}^{(9)} = q\alpha_1/(\alpha_1+\gamma_2)$, $p_{40} = \theta_2/(\alpha_1+\theta_2)$,

$p_{4,10} = \alpha_1/(\alpha_1+\theta_2), p_{42}^{(10)} = p\alpha_1/(\alpha_1+\theta_2), p_{46}^{(11)} = q\alpha_1/(\alpha_1+\theta_2)$,

$p_{51} = \theta_1/(\alpha_2+\theta_1), p_{5,11} = \alpha_2/(\alpha_2+\theta_1), p_{53}^{(11)} = p\alpha_2/(\alpha_2+\theta_1)$,

$p_{57}^{(11)} = q\alpha_2/(\alpha_2+\theta_1), p_{62} = p_{73} = 1$,

The mean sojourn times are

$\mu_0 = 1/(\alpha_1+\beta_2), \mu_1 = 1/(\alpha_2+\beta_1)$, $\mu_2 = 1/(\gamma_1+\alpha_2)$, $\mu_3 = 1/(\gamma_2+\alpha_1)$,

$\mu_4 = 1/(\alpha_1+\theta_2)$, $\mu_5 = 1/(\alpha_2+\theta_1)$, $\mu_6 = \mu_7 = 1/\delta$, $\mu_8 = 1/\gamma_1$,

$\mu_9 = 1/\gamma_2$, $\mu_{10} = 1/\theta_2$, $\mu_{11} = 1/\theta_1$, $\mu_{02} = p\alpha_1/(\alpha_1+\beta_2)^2$,

$\mu_{04} = \beta_2/(\alpha_1+\beta_2)^2$, $\mu_{06} = q\alpha_1/(\alpha_1+\beta_2)^2$, $\mu_{13} = p\alpha_2/(\alpha_2+\beta_1)^2$,

$\mu_{15} = \beta_1/(\alpha_2+\beta_1)^2$, $\mu_{17} = q\alpha_2/(\alpha_2+\beta_1)^2$, $\mu_{20} = \gamma_1/(\gamma_1+\alpha_2)^2$,

$\mu_{28} = \alpha_2/(\gamma_1+\alpha_2)^2$, $\mu_{23}^{(8)} = p\alpha_2/(\alpha_2+2\gamma_1)/\gamma_1(\gamma_1+\alpha_2)^2$ $\mu_{31} = \gamma_2/(\gamma_2+\alpha_1)^2$,

$\mu_{39} = \alpha_1/(\gamma_2+\alpha_1)^2$, $\mu_{32}^{(9)} = p\alpha_1/(\alpha_1+2\gamma_2)/\gamma_2(\gamma_2+\alpha_1)^2$,

$\mu_{36}^{(9)} = q\alpha_1/(\alpha_1+2\gamma_2)/\gamma_2(\gamma_2+\alpha_1)^2, \mu_{40} = \theta_2/(\alpha_1+\theta_2)^2$ $\mu_{4,10} = \alpha_1/(\alpha_1+\theta_2)^2$,

$\mu_{42}^{(10)} = p\alpha_1/(\alpha_1+2\theta_2)/\theta_2(\alpha_1+\theta_2)^2$, $\mu_{51} = \theta_1/(\alpha_2+\theta_1)^2$,

$\mu_{5,11} = \alpha_2/(\alpha_2+\theta_1)^2, \mu_{53}^{(11)} = p\alpha_2/(\alpha_2+2\theta_1)/\theta_1(\alpha_2+\theta_1)^2$,

$\mu_{57}^{(11)} = q\alpha_2/(\alpha_2+2\theta_1)/\theta_1(\alpha_2+\theta_1)^2$, $\mu_{62} = \mu_{73} = 1/\delta$.

The mean time to system failure with starting state S_0 is $MTSF = \hat{N}_0/\hat{D}_0$,

$$\hat{N}_0 = \frac{p\alpha_1}{(\alpha_1+\beta_2)(\alpha_2+\beta_1)}\left\{\frac{p\alpha\gamma}{(\gamma_2+\alpha_1)}\left[\frac{1}{(\alpha_2+\beta_1)}+\frac{1}{(\gamma_2+\alpha_1)}\right]+\frac{\beta_1\theta_1}{(\alpha_2+\theta_1)}\left[\frac{1}{(\alpha_2+\beta_1)}+\frac{1}{(\alpha_2+\theta_1)}\right]\right\}\frac{1}{(\alpha_1+\beta_2)}\left[\frac{(\beta_2+q\alpha_1)}{(\alpha_1+\beta_2)}+\frac{\beta_2}{(\alpha_1+\theta_2)}\right]\left\{1-\frac{1}{(\alpha_2+\beta_1)}\left[\frac{p\alpha_2\gamma_2}{(\gamma_2+\alpha_1)}+\frac{\beta_1\theta_1}{(\alpha_2+\theta_1)}\right]\right\},$$

$$\hat{D}_1 = \left\{1-\frac{1}{(\alpha_2+\beta_1)}\left[\frac{p\alpha\gamma}{(\gamma_2+\alpha_1)}+\frac{\beta_1\theta_1}{\alpha_2+\theta_1}\right]\right\}.\left[1-\frac{\beta_2\theta_2}{(\alpha_1+\beta_2)(\alpha_1+\theta_2)}\right]$$

In this case, $\hat{M}_i(t)$ are

$$\hat{M}_0(t)=e^{-(\alpha_1+\beta_2)t},\hat{M}_1(t)=e^{-(\alpha_2+\beta_1)t},\hat{M}_2(t)=e^{-(\gamma_1+\alpha_2)t},$$

$$\hat{M}_3(t)=e^{-(\gamma_2+\alpha_1)t},\hat{M}_4(t)=e^{-(\alpha_1+\theta_2)t},\ \hat{M}_5(t)=e^{-(\alpha_2+\theta_1)t},$$

the steady state availability of the system is $\hat{A}_0(\infty)=\hat{N}_1/\hat{D}_1$,

$$\hat{N}_1 = \frac{1}{(\alpha_1+\beta_2)}\hat{\bar{I}}_0-\frac{1}{(\alpha_2+\beta_1)}\hat{\bar{I}}_1+\frac{1}{(\gamma_1+\alpha_2)}\hat{\bar{I}}_2-\frac{1}{(\gamma_2+\alpha_1)}\hat{\bar{I}}_3+\frac{1}{(\alpha_1+\theta_2)}\hat{\bar{I}}_4-\frac{1}{(\alpha_2+\theta_1)}\hat{\bar{I}}_5,$$

$$\hat{D}_1=\hat{\bar{I}}_0'+\frac{\gamma_1}{(\gamma_1+\alpha_2)^2}\hat{\bar{I}}_2'-\frac{\gamma_1}{(\gamma_1+\alpha_2)}\hat{\bar{I}}_2'+\frac{\theta_2}{(\alpha_1+\theta_2)^2}\hat{\bar{I}}_4'-\frac{\theta_2}{(\alpha_1+\theta_2)}\hat{\bar{I}}_4',$$

$$\hat{I}_0' = 1 - \frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{b_0} - \frac{\beta_1}{(\alpha_2+\beta_1)}\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f_0} - \frac{\theta_1}{(\alpha_2+\theta_1)}\right]$$,

$$-\hat{c}_0\hat{d}_0\left[1-\frac{\beta_1\theta_1}{(\alpha_2+\beta_1)(\alpha_2+\theta_1)}\right]$$

$$\hat{I}_1' = 1 - \frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{c}_0 - \left[\hat{a}_0 + \frac{\beta_2}{(\alpha_1+\beta_2)}\hat{e}_0\right]$$,

$$\hat{I}_2' = -\left\{1+\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{b_0} + \frac{\beta_1}{(\alpha_2+\beta_1)}\right.$$,

$$\left.\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f_0} - \frac{\theta_1}{(\alpha_2+\theta_1)}\right]\right\}\left[\hat{a}_0 + \frac{\beta_2}{(\alpha_1+\beta_2)}\hat{e}_0\right]$$

$$\hat{I}_3 = -\hat{c}_0\left[1-\frac{\beta_1\theta_1}{(\alpha_2+\beta_1)(\alpha_2+\theta_1)}\right]\left[\hat{a}_0 + \frac{\beta_2}{(\alpha_1+\beta_2)}\hat{e}_0\right]$$,

$$\hat{I}_4 = \frac{\beta_2}{(\alpha_1+\beta_2)}\left\{1+\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{b_0} + \frac{\beta_1}{(\alpha_2+\beta_1)}\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f_0} - \frac{\theta_1}{(\alpha_1+\theta_2)}\right]\right\}$$,

$$-\frac{\beta_2}{(\alpha_1+\beta_2)}\hat{c}_0\hat{d}_0\left[1-\frac{\beta_1\theta_1}{(\alpha_2+\beta_1)(\alpha_2+\theta_1)}\right]$$,

$$\hat{I}_5 = -\frac{\beta_1\gamma_2}{(\alpha_2+\beta_1)(\gamma_2+\alpha_1)}\hat{c}_0\left[\hat{a}_0 + \frac{\beta_2}{(\alpha_1+\beta_2)}\hat{e}_0\right]$$,

$$\hat{I}_0' = -\frac{\gamma_2}{(\gamma_2+\alpha_1)}\left[\hat{b_0}' - \frac{\hat{b_0}}{(\gamma_2+\alpha_1)}\right] - \frac{\beta_1}{(\alpha_2+\beta_1)} - \frac{\beta_1}{(\alpha_2+\beta_1)}\left\{\frac{1}{(\alpha_2+\beta_1)}\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f_0} - \frac{\theta_1}{(\alpha_2+\theta_1)}\right]\right.$$

$$\left.+\frac{\gamma_2}{(\gamma_2+\alpha_1)}\left[\frac{1}{(\gamma_2+\alpha_1)}\hat{f_0} - \hat{f_0}'\right] - \frac{\theta_1}{(\alpha_2+\theta_1)^2}\right\}$$,

$$-(\hat{c}_0\hat{d}_0' + \hat{c}_0'\hat{d}_0)\left[1-\frac{\beta_1\theta_1}{(\alpha_2+\beta_1)(\alpha_2+\theta_1)}\right]$$

$$-\frac{\beta_1\theta_1}{(\alpha_2+\beta_1)(\alpha_2+\theta_1)}\hat{c}_0\hat{d}_0\left[\frac{1}{(\alpha_2+\beta_1)(\alpha_2+\theta_1)}\right]$$

$$\hat{\bar{I}}_2' = -\left\{\frac{p\alpha_2}{(\alpha_2+\beta_1)}\left[\frac{1}{(\alpha_2+\beta_1)}\hat{b}_0 - \hat{b}_0'\right] + \frac{\beta_1}{(\alpha_2+\beta_1)^2}\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0 - \frac{\theta_1}{(\alpha_2+\theta_1)}\right] + \frac{\beta_1}{(\alpha_2+\beta_1)}\right.$$

$$\left.\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0 - \frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0' - \frac{\theta_1}{(\alpha_2+\theta_1)}\right]\right\}\left[\hat{a}_0 + \frac{\beta_2}{(\alpha_1+\beta_2)}\hat{e}_0\right] - \left\{1 + \frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{b}_0\right.,$$

$$\left. + \frac{\beta_1}{(\alpha_1+\beta_2)}\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0' - \frac{\theta_1}{(\alpha_2+\theta_1)}\right]\right\}\left\{\hat{a}_0 - \frac{\beta_2}{(\alpha_1+\beta_2)}\left[\frac{1}{(\alpha_1+\beta_2)}\hat{e}_0 + \hat{e}_0'\right]\right\}$$

$$\hat{\bar{I}}_4' = \frac{\beta_2}{(\alpha_1+\beta_2)}\left\{1 + \frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{b}_0 + \frac{\beta_1}{(\alpha_2+\beta_1)}\right.,$$

$$\left.\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0 - \frac{\theta_1}{(\alpha_2+\theta_1)}\right]\right\}$$

$$-\frac{\beta_2}{(\alpha_1+\beta_2)}\left\{\frac{\gamma_2}{(\gamma_2+\alpha_1)}\left[\hat{b}_0' - \frac{1}{(\gamma_2+\alpha_1)}\hat{b}_0\right] - \frac{\beta_1}{(\alpha_1+\beta_2)^2}\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0 - \frac{\theta_1}{(\alpha_2+\theta_1)}\right]\right.$$

$$\left. + \frac{\beta_1}{(\alpha_1+\beta_1)}\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0' - \frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0 + \frac{\theta_1}{(\alpha_2+\theta_1)}\right]\right\}$$

$$-\left\{\frac{\beta_2}{(\alpha_1+\beta_1)}\hat{c}_0'\hat{d}_0' - \frac{\beta_2}{(\alpha_1+\beta_2)}\hat{c}_0\left[\hat{d}_0' - \frac{1}{(\alpha_1+\beta_2)}\hat{d}_0\right]\right\}$$

$$-\frac{\beta_1}{(\alpha_1+\beta_2)^2}\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0 - \frac{\theta_1}{(\alpha_2+\theta_1)}\right] + \frac{\beta_1}{(\alpha_1+\beta_1)}\left[\frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0' - \frac{\gamma_2}{(\gamma_2+\alpha_1)}\hat{f}_0 + \frac{\theta_1}{(\alpha_2+\theta_1)}\right]\}$$

$$-\{\frac{\beta_2}{(\alpha_1+\beta_1)}\hat{c}_0'\hat{d}_0' - \frac{\beta_2}{(\alpha_1+\beta_2)}\hat{c}_0\left[\hat{d}_0' - \frac{1}{(\alpha_1+\beta_2)}\hat{d}_0\right]\}$$

$$-\left[1 - \frac{\beta_1\theta_1}{(\alpha_1+\beta_2)(\alpha_2+\beta_1)}\right] - \frac{\beta_2\beta_1\theta_1}{(\alpha_1+\beta_2)(\alpha_2+\beta_1)(\alpha_2+\theta_1)}\hat{c}_0\hat{d}_0\left[\frac{1}{(\alpha_1+\beta_1)} + \frac{1}{(\alpha_2+\theta_1)}\right]$$

$$\hat{a}_0 = -\alpha_1/(\alpha_1+\beta_2), \hat{b}_0 = -\alpha_2/(\alpha_2+\beta_1), \hat{c}_0 = -\alpha_2/(\alpha_1+\gamma_1) ,$$

$$\hat{d}_0 = -\alpha_1/(\alpha_1+\gamma_2), \hat{e}_0 = -\alpha_1/(\alpha_1+\theta_2), \hat{f}_0 = -\alpha_2/(\alpha_2+\theta_1) ,$$

$$\hat{a}_0' = -\hat{a}_0\left[\frac{1}{(\alpha_1+\beta_2)}+\frac{q}{\delta}\right], \hat{b}_0' = -\hat{b}_0\left[\frac{1}{(\alpha_2+\beta_1)}+\frac{q}{\delta}\right], \hat{c}_0' = -\hat{c}_0\left[\frac{(\alpha_2+2\gamma_1)}{\gamma_1(\gamma_1+\alpha_2)}+\frac{q}{\delta}\right],$$

$$\hat{d}_0' = -\hat{d}_0\left[\frac{(\alpha_1+2\gamma_2)}{\gamma_2(\gamma_2+\alpha_1)}+\frac{q}{\delta}\right], \hat{e}_0' = -\hat{e}_0\left[\frac{(\alpha_1+2\theta_2)}{\theta_2(\alpha_1+\theta_2)}+\frac{q}{\delta}\right], \hat{f}_0' = -\hat{f}_0\left[\frac{(\alpha_2+2\theta_1)}{\theta_2(\alpha_2+\theta_1)}+\frac{q}{\delta}\right],$$

7.2 The two units are similar: Let

$$F_i(t) = F(t),\ L_i(t) = L(t), G_i(t) = G(t),\ H_i(t) = H(t),$$

The transition probabilities are

$$p_{02} = p_{13} = p\int \bar{L}(t)dF(t), p_{04} = p_{15} = \int \bar{F}(t)dL(t),\ p_{06} = p_{17} = q\int \bar{L}(t)dF(t),$$

$$p_{20} = p_{31} = p\int \bar{F}(t)dG(t), p_{28} = p_{39} = \int \bar{G}(t)dF(t), p_{23}^{(8)} = p_{32}^{(9)} = p\int F(t)dG(t),$$

$$p_{27}^{(8)} = p_{36}^{(9)} = q\int F(t)dG(t), p_{40} = p_{51} = \int \bar{F}(t)dH(t), p_{4,10} = p_{5,11} = \int \bar{H}(t)dF(t),$$

$$p_{42}^{(10)} = p_{53}^{(11)} = p\int F(t)dH(t), p_{46}^{(10)} = p_{57}^{(11)} = q\int F(t)dH(t), p_{62} = p_{73} = \int dJ(t),$$

The mean sojourn times are

$$\mu_0 = \mu_1 = \int \bar{F}(t)\bar{L}(t)dt,\ \mu_2 = \mu_3 = \int \bar{G}(t)\bar{F}(t)dt,\ \mu_4 = \mu_5 = \int \bar{F}(t)\bar{H}(t)dt,$$

$$\mu_6 = \mu_7 = \int \bar{J}(t)dt,\ \mu_8 = \mu_9 = \int \bar{G}(t)dt,\ \mu_{10} = \mu_{11} = \int \bar{H}(t)dt.$$

The mean time to system failure with starting state s_0 is $MTSF = \hat{\hat{N}}_0 / \hat{\hat{D}}_0$,

$$\hat{\hat{N}} = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2,\ \hat{\hat{D}}_0 = 1 - (p_{01}p_{10} + p_{01}p_{20}).$$

The steady state availability of the system is $\hat{\hat{A}}_0(\infty) = \hat{\hat{N}}_1 / \hat{\hat{D}}_1$,

$$\hat{\hat{N}}_1 = (1 - p_{11}^{(4)} - p_{11}^{(4)})\mu_0 + \left[p_{01} + p_{02}(p_{21}^{(5)} + p_{23}^{(5)}) + p_{03}\right]\mu_1 + p_{02}(1 - p_{11}^{(4)} - p_{11}^{(4)})\mu_2,$$

$$\hat{\hat{D}}_1 = p_{10}\mu_0 + (1 - p_{02}p_{20})\mu_1 + p_{02}p_{10}\mu_2 + (p_{13}^{(4)} + p_{02}p_{10}p_{23}^{(5)} + p_{10}p_{03} - p_{02}p_{20}p_{13}^{(4)})\mu_3.$$

The busy period analysis $B_0^*(s) = N_2(s) / D_1(s)$,

$$\hat{\hat{N}}_2(s) = V_1^*\left[q_{01}^* + q_{02}^*(q_{21}^{(5)*} + q_{23}^{(5)*}q_{31}^*) + q_{03}^*q_{31}^*\right]$$
$$+V_2^*\left[q_{01}^*(1 - q_{11}^{(4)*} - q_{31}^*q_{13}^{(4)*})\right] ,$$
$$+V_3^*\left[q_{01}^*q_{13}^{(4)*} + q_{02}^*(1 - q_{11}^{(4)*})q_{01}^{(5)*}\right.$$
$$\left.+q_{02}^*q_{21}^{(5)*}q_{13}^{(4)*} + q_{03}^*(1 - q_{11}^{(4)*})\right]$$
$$\hat{\hat{D}}_1(s) = 1 - q_{11}^{(4)*} - q_{13}^{(4)*}q_{31}^* - q_{10}^*q_{01}^* - q_{10}^*q_{02}^*(q_{21}^{(5)*} + q_{23}^{(5)*}q_{31}^*)$$
$$-q_{03}^*q_{31}^*q_{10}^* - q_{20}^*q_{02}^*(1 - q_{11}^{(4)*}) + q_{20}^*q_{02}^*q_{31}^*q_{13}^{(4)*} .$$

7.3. The two unite are similar with exponential distributions : Let

$$F_i(t) = 1 - e^{-\alpha\ t}, L_i(t) = 1 - e^{-\beta\ t}, G_i(t) = 1 - e^{-\gamma\ t} ,$$
$$H_i(t) = 1 - e^{-\theta\ t}, J_i(t) = 1 - e^{-\delta\ t} .$$

The transition probabilities are

$$p_{02} = p_{13} = p\alpha/(\alpha+\beta), p_{04} = p_{15} = \beta/(\alpha+\beta), p_{06} = p_{17} = q\alpha/(\alpha+\beta),$$
$$p_{20} = p_{31} = \gamma/(\gamma+\beta), p_{28} = p_{39} = \alpha/(\gamma+\alpha), p_{23}^{(8)} = p_{32}^{(9)} = p\alpha/(\alpha+\gamma),$$
$$p_{27}^{(8)} = p_{36}^{(9)} = q\alpha/(\alpha+\gamma), p_{40} = p_{51} = \theta/(\alpha+\theta), p_{4.10} = p_{5,11} = \alpha/(\alpha+\theta)$$
$$p_{42}^{(10)} = p_{53}^{(11)} = p\alpha/(\alpha+\theta), p_{46}^{(10)} = p_{57}^{(11)} = q\alpha/(\alpha+\theta)\ p_{62} = p_{73} = 1 .$$

The mean sojourn times are

$$\mu_0 = \mu_1 = 1/(\alpha+\beta)\ , \mu_2 = \mu_3 = 1/(\alpha+\gamma)\ , \mu_4 = \mu_5 = 1/(\alpha+\theta) ,$$
$$\mu_6 = \mu_7 = 1/\delta\ , \mu_8 = \mu_9 = 1/\gamma\ , \mu_{10} = \mu_{11} = 1/\theta ,$$
$$\mu_{02} = \mu_{13} = p\alpha/(\alpha+\beta)^2, \mu_{04} = \mu_{15} = \beta/(\alpha+\beta)^2\ \mu_{06} = \mu_{17} = q\alpha/(\alpha+\beta)^2,$$
$$\mu_{20} = \mu_{31} = \gamma/(\gamma+\alpha)^2, \mu_{28} = \mu_{39} = \alpha/(\gamma+\alpha)^2 ,$$
$$\mu_{23}^{(8)} = \mu_{32}^{(9)} = p\alpha/(\alpha+2\gamma)/\gamma(\gamma+\alpha)^2, \mu_{27}^{(8)} = \mu_{36}^{(9)} = q\alpha/(\alpha+2\gamma)/\gamma(\gamma+\alpha)^2$$
$$\mu_{40} = \mu_{51} = \theta/(\alpha+\theta)^2, \mu_{4,10} = \mu_{5,11} = \alpha/(\alpha+\theta)^2 ,$$
$$\mu_{42}^{(10)} = \mu_{53}^{(11)} = p\alpha/(\alpha+2\theta)/\theta(\alpha+\theta)^2,$$
$$\mu_{46}^{(10)} = \mu_{57}^{(11)} = q\alpha/(\alpha+2\theta)/\theta(\alpha+\theta)^2, \mu_{62} + \mu_{73} = 1/\delta .$$

The mean time to system failure with starting state s_0 is $MTSF = \tilde{N}_0/\tilde{D}_0$

$$\tilde{N}_0 = \frac{1}{(\alpha+\beta)}\left[1+\frac{p\alpha}{(\alpha+\gamma)}+\frac{\beta}{(\alpha+\theta)}\right], \tilde{D}_0 = 1-\frac{1}{(\alpha+\beta)}\left[\frac{p\alpha\gamma}{(\alpha+\gamma)}+\frac{\beta\theta}{(\alpha+\theta)}\right].$$

The steady state availability of the system is . $\tilde{A}_0(\infty) = \tilde{N}_1 / \tilde{D}_1$

$$\tilde{N}_1 = \frac{1}{(\alpha+\beta)}\left[1+\frac{\beta}{(\alpha+\beta)}\right],$$

$$\tilde{D}_1 = \frac{p\alpha}{(\alpha+\beta)^2}+\frac{1}{(\alpha+\gamma)}+\frac{q\alpha}{\delta(\alpha+\gamma)}\left[1+\frac{\gamma}{(\alpha+\beta)}\right] + \frac{\beta(q\alpha+\delta)(\gamma-\theta)}{\delta(\alpha+\beta)(\alpha+\gamma)(\alpha+\theta)}.$$

7.4. The two units are dissimilar with a cold standby : In this case

$$L_i(t) = H_i(t) = 0 \; , \bar{L}_i(t) = \bar{H}_i(t) = 1 .$$

The transition probabilities are :

$$\hat{p}_{02} = \hat{p}_{13} = p \; , \; \hat{p}_{06} = \hat{p}_{17} = q ,$$

$$\hat{p}_{04} = \hat{p}_{15} = \hat{p}_{40} = \hat{p}_{51} = \hat{p}_{42}^{(10)} = \hat{p}_{53}^{(11)} = \hat{p}_{46}^{(10)} = \hat{p}_{57}^{(11)} = 0 ,$$

$$\hat{p}_{4,10} = \hat{p}_{5,11} = \hat{p}_{62} = \hat{p}_{73} = 1, \hat{p}_{20} = \int \bar{F}_2(t)dG_1(t), \hat{p}_{28} = \int \bar{G}_1(t)dF_2(t) ,$$

$$\hat{p}_{23}^{(8)} = \int \bar{F}_2(t)dG_1(t), \hat{p}_{27}^{(8)} = q\int \bar{F}_2(t)dG_1(t) \; \hat{p}_{31} = \int \bar{F}_2(t)dG_2(t),$$

$$\hat{p}_{39} = \int \bar{G}_2(t)dF_1(t) , \; \hat{p}_{32}^{(9)} = p\int F_1(t)dG_2(t) \; , \hat{p}_{36}^{(9)} = q\int F_1(t)dG_2(t) .$$

The mean sojourn time are

$$\hat{\mu}_0 = \int \bar{F}_1(t)dt , \quad \hat{\mu}_1 = \int \bar{F}_2(t)dt , \; \hat{\mu}_2 = \int \bar{G}_1(t)\bar{F}_2(t)dt ,$$

$$\hat{\mu}_3 = \int \bar{G}_2(t)\bar{F}_1(t)dt , \hat{\mu}_4 = \int \bar{F}_1(t)\bar{H}_2(t)dt , \hat{\mu}_5 = \int \bar{F}_2(t)\bar{H}_1(t)dt ,$$

$$\hat{\mu}_6 = \hat{\mu}_7 \int \bar{J}(t)dt \; , \; \hat{\mu}_8 = \int \bar{G}_1(t)dt , \hat{\mu}_9 = \int \bar{G}_2(t)dt .$$

The mean time to system failure with starting state s_0 is $MTSF = \hat{\hat{N}}_0 / \hat{\hat{D}}_0$,

$$\hat{\hat{N}}_0 = p(\hat{\mu}_{13}\hat{p}_{31} + p\hat{\mu}_{13}) + q(1-p\hat{p}_{31}), \quad \hat{\hat{D}}_0 = (1-\hat{p}_{13}\hat{p}_{31}) ,$$

In this case $\hat{M}_i(t)$ are $\hat{M}_0(t) = \hat{M}_4(t) = \bar{F}_1(t), \hat{M}_1(t) = \hat{M}_5(t) = \bar{F}_2(t),$

$$\hat{M}_2(t) = \bar{G}_1(t)\bar{F}_2(t) \; , \; \hat{M}_3(t) = \bar{G}_2(t)\bar{F}_1(t) .$$

The steady state availability of the system is $\hat{\hat{A}}_0(\infty) = \hat{\hat{N}}_1 / \hat{\hat{D}}_1$,

$$\hat{\hat{N}}_1 = \hat{\mu}_0\hat{\hat{I}}_0 - \hat{\mu}_1\hat{\hat{I}}_1 + \hat{\mu}_2\hat{\hat{I}}_2 - \hat{\mu}_3\hat{\hat{I}}_3 + \hat{\mu}_4\hat{\hat{I}}_4 - \hat{\mu}_5\hat{\hat{I}}_5 ,$$

$$\hat{\hat{D}}_1 = \hat{\hat{I}}'_0 + \hat{\mu}_{20}\hat{\hat{I}}_1 - \hat{p}_{20}\hat{\hat{I}}'_2 + \hat{\mu}_{40}\hat{\hat{I}}_4 - \hat{p}_{40}\hat{\hat{I}}'_4 ,$$

$$\hat{\hat{I}}_0 = 1 - \hat{\hat{b}}_0\hat{p}_{31} - \hat{c}_0\hat{\hat{d}}_0 , \hat{\hat{I}}_1 = -\hat{p}_{31} - \hat{c}_0\hat{\hat{a}}_0 , \hat{\hat{I}}_2 = -[1+\hat{\hat{b}}_0\hat{p}_{31}]\hat{\hat{a}}_0,$$

$$\hat{\hat{I}}_3 = -\hat{c}_0\hat{\hat{a}}_0 , \hat{\hat{I}}_3 = \hat{\hat{I}}_5 = 0 , \hat{\hat{I}}'_0 = -\hat{\hat{b}}'_0\hat{p}_{31} - \hat{\hat{b}}_0\hat{\mu}_{31} - (\hat{c}_0\hat{\hat{d}}'_0 - \hat{c}'_0\hat{\hat{d}}_0) ,$$

$$\hat{\hat{I}}'_2 = \hat{\hat{a}}_0(\hat{\hat{b}}'_0\hat{\mu}_{31} - \hat{\hat{b}}'_0\hat{p}_{31}) - \hat{\hat{a}}'_0(1+\hat{\hat{b}}'\hat{p}_{31}) , \hat{\hat{I}}'_4 = 0 ,$$

$$\hat{c}_0 = -(\hat{p}_{23}^{(8)} + \hat{p}_{27}^{(8)}) , \hat{\hat{d}}_0 = -(\hat{p}_{32}^{(9)} + \hat{p}_{36}^{(9)}) ,$$

$$\hat{\hat{a}}'_0 = \hat{\mu}_{02} + q\,\hat{\mu}_{62} + \hat{\mu}_{06}, \hat{\hat{b}}'_0 = \hat{\mu}_{13} + q\,\hat{\mu}_{73} + \hat{\mu}_{17} ,$$

$$\hat{c}'_0 = \hat{\mu}_{23}^{(8)} + \hat{p}_{27}^{(8)}\hat{\mu}_{73} + \hat{\mu}_{27}^{(8)}, \hat{\hat{d}}'_0 = \hat{\mu}_{32}^{(9)} + \hat{p}_{36}^{(9)}\hat{\mu}_{62} + \hat{p}_{36}^{(9)} ,$$

7.5. Numerical example:

Let the two units are similar with exponential distributions and the switch is perfect, i . e.

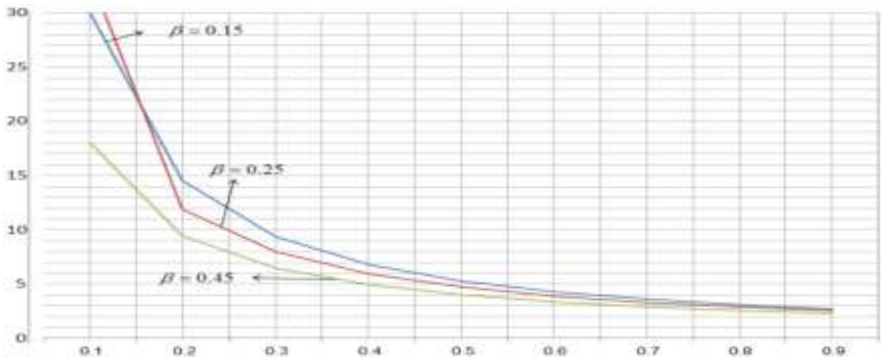
$$f(t) = \alpha e^{-\alpha t} ; a > 0 , \quad l(t) = \beta e^{-\beta t} ; \quad \beta > 0 ,$$

$$g(t) = \gamma e^{-\gamma t} ; \gamma > 0 , h(t) = \theta e^{-\theta t} ; \theta > 0,$$

$$j(t) = \delta e^{-\delta t} ; \delta > 0$$

Let $p = 1 , q = 0 , \gamma = 0.7 , \theta = 0.3 , \delta = 0.8$,

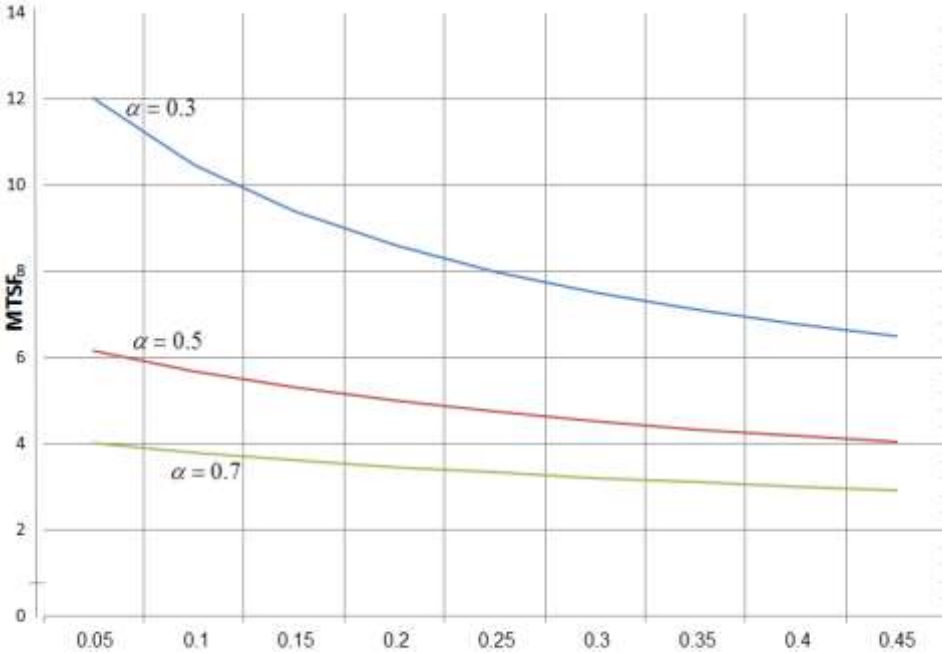
a	MTSF		
	β = 0.15	β = 0.25	β = 0.45
0.1	30.00000	33.33333	18.00000
0.2	14.57446	11.92307	9.45544
0.3	9.39393	7.98449	6.50793
0.4	6.82584	5.96846	4.98387
0.5	5.31034	4.74285	4.04255
0.6	4.31972	3.92100	3.40000
0.7	3.62637	3.33333	2.93233
0.8	3.11651	2.89342	2.57636
0.9	2.72727	2.55255	2.29629



Failure rate of the operative unit from N – mode to F – mode
Fig. 2.

Relation between the failure rate of the operative unit from N – mode to F – mode and the MTSF of the system
.

β	MTSF		
	a = 0.3	a = 0.5	a = 0.7
0.05	12.02898	6.17391	4.02597
0.10	10.47619	5.69230	3.80952
0.15	9.39393	5.31034	3.62637
0.20	8.59649	5.00000	3.46938
0.25	7.98449	4.74285	3.33333
0.30	7.50000	4.52631	3.21428
0.35	7.10691	4.34146	3.10926
0.40	6.78160	4.18181	3.01587
0.45	6.50793	4.04255	2.93233



Failure rate of the standby unit from N – mode to F – mode
Fig. 3.

Relation between the failure rate of the standby unit from N– mode to F – mode and the MTSF of the system.

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