

## Effect of Addition of Zero and Pole in Different System

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**ABSTRACT:** The frequency response methods are most powerful in the conventional control system and step responses are commonly used in control systems analysis and design. In this paper, we will discuss step response and frequency response in different system after the addition of pole, zero in the transfer function. Some applications numerical are provided. A number of illustrated figures are presented to validate the concepts.

**Keywords:** Transfer Function, Step response, Frequency response, Mat-lab

### I. INTRODUCTION

The relation between input and output of a system is given by Transfer function. The transfer function  $G(s)$  is given by

$$G(s) = \frac{L_o}{L_i} \quad (1)$$

where  $L$  denotes the Laplace transform. The frequency response function and the transfer function are interchangeable by the substitution  $s = j\omega$  [1].

The frequency response function  $G(j\omega)$  is

$$G(j\omega) = \frac{F_o}{F_i} \quad (2)$$

where  $F$  denotes the Fourier transform.

TF has been used in many applications. One important application among them is monitoring the mechanical integrity of transformer windings (during testing and while in service). Mechanical deformation arise mainly due to short circuit forces, unskilled handling and rough transportation. Information related to winding deformation is embedded in the TF. Hence the first step should be correct interpretation of TF [2]. TF can be used to describe a variety of filter or to express solution of linear differential equation accurately [3]. The TF of system is analyzed and response curves are simulated [4]. The location of poles and zeros gives idea regarding response characteristics of a system.

### II. SYSTEM STABILITY

If poles are in LHP, the system is stable; if poles are in RHP, the system is unstable and poles on imaginary axis then system is marginally stable or limitedly stable [5,6,7].

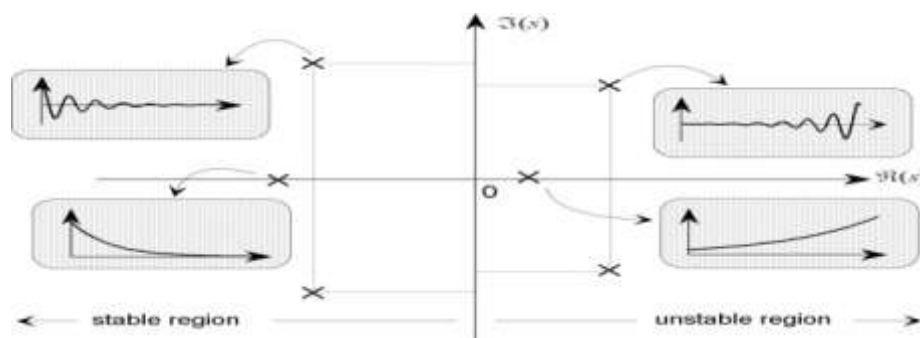


Fig.1. Stable and unstable region according to pole position

### III. STEP RESPONSE AND FREQUENCY RESPONSE ANALYSIS

Step signals and their responses are commonly used in control systems analysis and design.[8,9] Consider TF then adding zero and pole in it and observing changes in step response and frequency response of these functions and discuss different cases of systems in following numerical.

Numerical Example 1: If the transfer function is (1)  $\frac{8}{s^2 + 4s + 8}$  then (2) we added one zero ( $z=1$ ) in RHP and (3) added one pole ( $p=-1$ ) in LHP to first TF, its step response and frequency response are demonstrated in figure 2, figure 3 respectively..

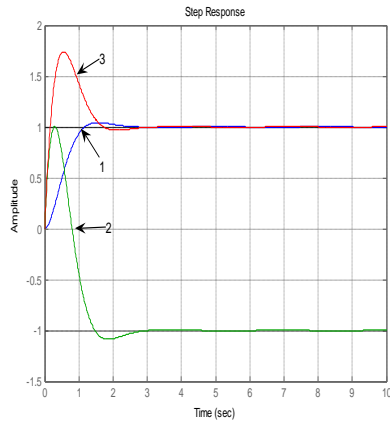


Fig. 2 Step response of Numerical 1

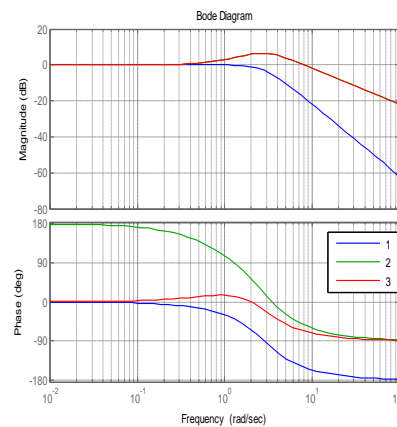


Fig. 3 Frequency response of Numerical 1

If we add zero in RHP to the TF, step response slower and it make undershoot. On adding pole in LHP to the TF, the step response slower. Magnitude of all TF is coincide with 0 db line and changes with frequency but magnitude of case 2 and 3 are same. Phase angle of 1 and 3 are coincide with 0 degree and decreases to -180 degree and -90degree respectively but in non-minimum phase system i.e. case 2, phase changes from 180 deg to -90degree.

Numerical Example 2: If the transfer function is (1)  $\frac{8}{s^2 + 4s + 8}$  then (2) we added one zero ( $z=1$ ) in RHP and (3) we moves from left to right, added one zero ( $z=2$ ) in RHP to first TF, its step response and frequency response are demonstrated in figure 4 and figure 5 respectively.

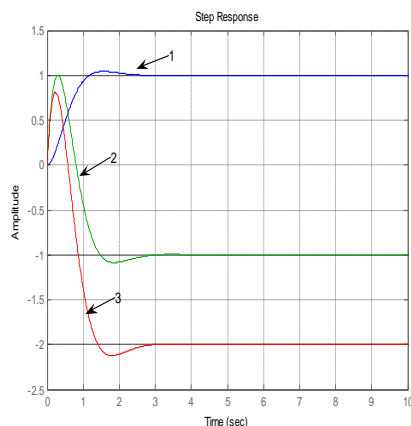


Fig. 4 Step response of Numerical 2

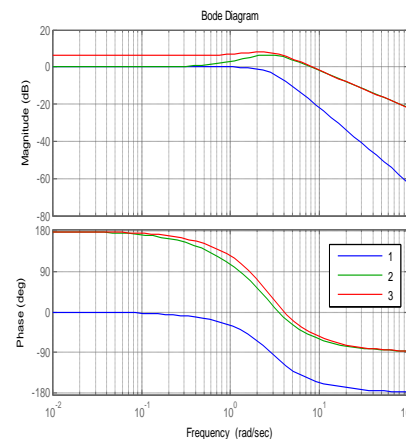


Fig. 5 Frequency response of Numerical 2

If we add zero in RHP to the TF, step response slower and it make undershoot. Magnitude of case 1 and 2 are coincide with 0 db line but in case 3, magnitude is slightly above 0db line. Phase of non-minimum system coincide with 0 db line and decreases with negative slopes and again coincide at -90degree.

Numerical Example 3: If the transfer function is (1)  $\frac{8}{s^2 + 4s + 8}$  then (2) we added one zero ( $z = -1$ ) in LHP and (3) we move from right to left, added one zero ( $z = -2$ ) in LHP to first TF, its step response and frequency response are demonstrated in figure 6 and figure 7 respectively..

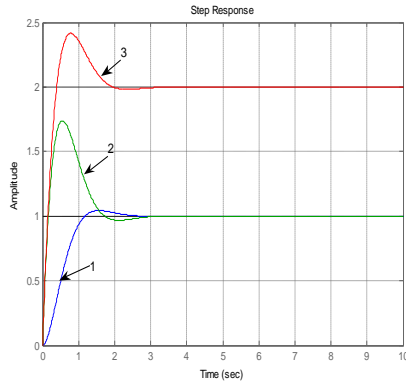


Fig. 6 Step response of Numerical 3

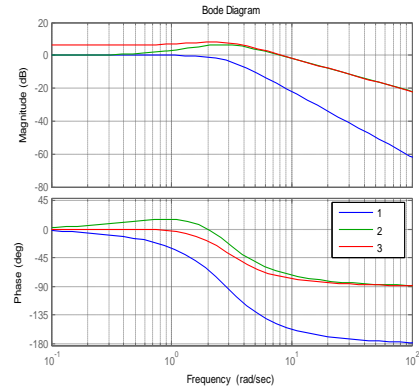


Fig. 7 Frequency response of Numerical 3

In minimum phase system, on adding zero in LHP to the transfer function makes the step response faster, decreases the rise time and the peak time and increases the overshoot. Magnitude of minimum phase system after adding zero to TF are slightly different but phase angle decreases to -90 degrees.

Numerical Example 4: If the transfer function is (1)  $\frac{8}{s^2 + 4s + 8}$  then (2) we added one zero ( $z = -1$ ) in LHP and (3) added one pole ( $p = -1$ ) in LHP to first TF, its step response and frequency response are demonstrated in figure 8 and figure 9 respectively.

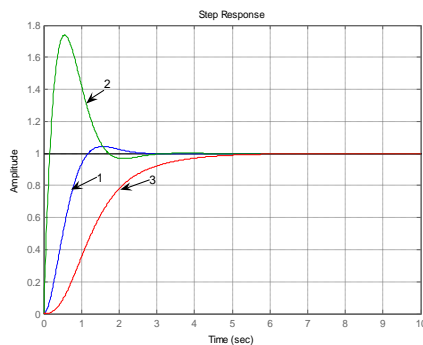


Fig. 8 Step response of Numerical 4

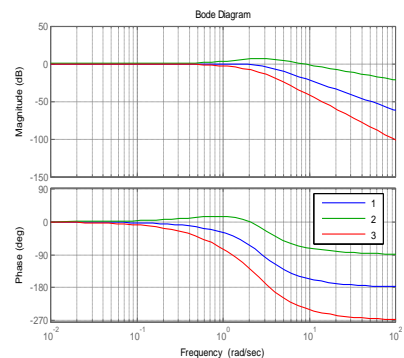


Fig. 9 Frequency response of Numerical 4

In minimum phase system, adding zero to LHP makes step response faster as compared to adding pole to LHP. In Frequency response, magnitude of all cases coincide with 0 dB and decreases with different slopes with increasing frequency and phase angle again coincide with 0 degrees and decreases to -180, -90, -270 degrees respectively to cases 1, 2, 3. Poles in the RHP will cause the system response to exponentially increase resulting in instability, but zeros do not cause this. It is the controller design that can cause the zeros to have an effect on system stability [10].

#### IV. CONCLUSION

On observing step response and frequency response after adding zero and pole to LHP and RHP i.e. minimum phase and non-minimum phase system. On adding zero in LHP to the TF, the step response is faster, overshoot increases and rise time, peak time decrease. If we add zero in RHP to the TF, step response is faster and it makes undershoot. On adding pole in LHP to the TF, the step response is slower. Also observe changes in frequency responses. Magnitude of minimum phase system is coincident with 0 dB line then decreases with different slopes. And phase angle also coincides with 0 degree line and decreases with different slopes.

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