

## Dynamic System Model Identification And Estimation for the Control of Solid Flow Applied To Car Dumpers

\*José Pinheiro de Moura<sup>1</sup>, \*\*Breno Linhares Pinheiro<sup>2</sup>,  
\*Patrícia Helena Moraes rego<sup>3</sup>, \*\*João Viana da Fonseca Neto<sup>4</sup>

\* Center of Technological Sciences/ State University of Maranhão, Brazil

\*\* Department of Electrical Engineering / Federal University of Maranhão /Brazil

Corresponding Author : \*José Pinheiro De Moura<sup>1</sup>

**ABSTRACT:** This paper proposes a mathematical model for dynamical systems applied to a discharge plant of solid bulk by car dumper. So that a model identified from real plant data is presented and an estimated model is proposed to be used in generalized controller implementations available in the literature.

**Keywords:** Car Dumper, Dynamic Systems, Generalized Controllers, Identified Model, Mathematical Model.

### I. INTRODUCTION

The solid flow in Car Dumper (CD) is the process that consists of emptying the supply silos by means of a Belt Feeder (FD). The control of solid flow in the industry is done empirically in most cases by varying the rotation speed of the FD, that is, based on the operator's expertise. This article presents a conception of an approximated mathematical model to the model of the plant for insertion of control in the operational life of the process. Identification and estimation of model for dynamic systems in the solid flow control applied in car dumpers is very complex and difficult to be represented by mathematical models that represent the physical aspects of the process itself. The system identification can be done through dynamic analysis of the physical system and the development of a mathematical model from the behavior of the process [1]. The identification of systems can also be done through computational analysis or empirical modelling that involves collecting data from the input-output characteristics of the system and using them to obtain a mathematical model that approximates this observed behavior [2]. A fundamental reason for obtaining a mathematical model of a dynamic system is the need for the existence of this model so that a controller can be developed for the dynamic [3]. In the estimated model validation, the following methodology is presented in this article: 1) a solution to the optimum control problem applied to the solid flow control of the Car Dumper process; 2) a Fuzzy controller, also applied to the solid flow control of the Car Dumper process; and 3) the plant transfer function.

Nonlinear systems with very general performance indices are difficult to deduce with explicit mathematical expressions for optimal control [4] Considering the extremely important special case of linear systems with quadratic performance indices, so that these performance indices can be considered as dimensional quadratic surfaces  $(n+m)$ , where  $n$  and  $m$  are the vector dimensions of state and control [5]. It is observed that very refined solutions can be given in two instances: the fixed final state situation, which leads to an open-loop control strategy and the final free state situation, which leads to a closed-loop strategy [5].

Considering that the plant to be controlled can be described by the linear equation:

$$x_{k+1} = A_k x_k + B_k u_k \quad (1)$$

with  $x_k \in R^n$  and  $u_k \in R^m$ . The associated performance index is the quadratic function given by:

$$J_i = \frac{1}{2} x_N^T S_N x_N + \frac{1}{2} \sum_{i=1}^{N-1} (x_N^T Q_k x_k + u_k^T R_k u_k) \quad (2)$$

defined the interval of time  $[i, N]$ , it is observed that both the plant and the cost-weighting matrices can, in general, be time-varying. The initial state of the plant is given as  $x_i$ . It is considered that  $Q_k$ ,  $R_k$  and  $S_k$  are symmetric positive semidefinite matrices and that  $|R_k| \neq 0$  for all  $k$  [6].

The objective is to find the control sequence  $u_i, u_{i+1}, \dots, u_{N-1}$ , which minimizes  $J_i$ . Inferring models from observations and studying their properties is really what science does. The models (“hypotheses”, “laws of nature”, “paradigms”, etc.) may be of a more or less formal character, but they have the basic characteristic of bonding/observations together in some pattern. For some systems, it is appropriate to use numerical tables and/or charts to describe their properties. These descriptions are called graphic models. Linear systems, for example, can be described solely by their impulse responses or step responses to their transfer functions. Their graphics representations are widely used for a variety of project purposes.

Kalman in 1960 published an article dealing with linear-quadratic feedback control, setting the stage for what came to be known as LQR (Linear-Quadratic-Regulator) control, which had great influence on researchers, professors and control practitioners through following decades [7]. Fuzzy logic can also be used to implement controllers, applied to different types of processes. This methodology incorporates the human way of thinking of a control system, behaving in a manner similar to deductive reasoning. The use of fuzzy rules in a control system has the following advantages: Simplification of the process model; Better treatment of inaccuracies inherent in the sensors used; Ease in the specification of control rules in language close to natural; Satisfaction of multiple control objectives and Ease of knowledge incorporation of human specialists [8].

In this work, it was observed the real data collected from the behavior of the input and output variables of the plant for the mathematical formulation of the problem and it is presented a methodology for identification of mathematical models for control of dynamic load from real data of a discharge plant from solids to bulk by car dumper using MATLAB software for computational analysis and development of an approximate model of the actual plant model under study. It is not the purpose of this work to control the plant, but to present a model of the plant to be implemented controllers, such as: Control LQR; Fuzzy controller; PID controller; etc.

The main contribution of this article is to introduce a conception methodology based on computational methods applied in industrial processes aimed at improving the optimum performance of the system. This paper is organized as follows: in Section 2 it is presented the description of the plant, in Section 3 it is presented the formulation of the problem, in Section 4 it is designed the mathematical models of the plant, in Section 5 it is presented the computational experiments and the validation of the state-space model in an LQR control system and a Fuzzy Controller, and finally in Section 6 it is presented the conclusion of the work.

## 1. PLANT DESCRIPTION

Car dumpers are used for the unloading of wagons with bulks. They rotate up to  $180^\circ$ , it is unloaded two wagons each turn, the wagons are conjugated, that is, between a wagon and another it has a fixed hitch, called fixed bar and at the ends of each pair of wagons the couplings are movable, allowing the wagons to rotate during unloading without the need to uncouple them as shown in Fig. 1.



**Figure 1:** Car dumper.

When rotating, the load of the wagons is transferred to the silos according to Fig. 2 which is directed to an operational route that is composed of the following equipment: Car Dumper; Feeders; Conveyor Belts (CB) and Stacker (ST) that performs the stacking of the load in the storage yard.

In this paper, it is presented a modelling for the implementation of controllers that optimize the dynamic load flow at silos output.

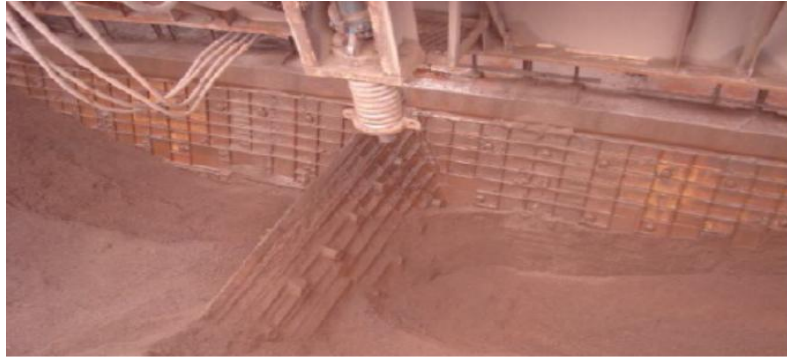


Figure 2: Silos.

An operational route consists of a source equipment (CD), intermediate equipment (FD and CB) and a destination equipment (ST). An operational route diagram is shown in Fig. 3.

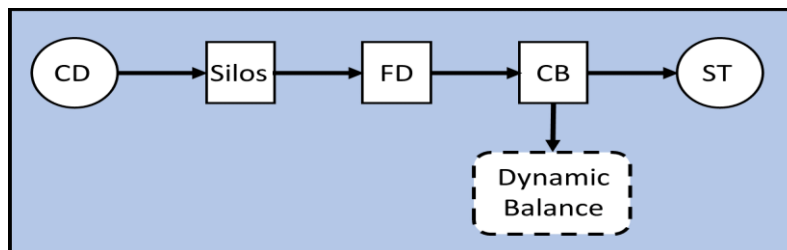


Figure 3: operational route.

## II. PROBLEM FORMULATION

In general, we denote the input and output of the system at time  $t$  by  $u(t)$  and  $y(t)$ , respectively. One of the most basic relations between input and output is given by the equation:

$$y(t) + a_1 t(t-1) + \dots + a_n y(t-n) = b_1 u(t-1) + \dots + b_m u(t-m) \quad (3)$$

In this work, it was chosen to represent, first, the system in discrete time, mainly because the observed data are always collected by sampling. Thus, it is easier to relate observed data to discrete time models. In Equation (3), it was assumed that a sampling interval is a unit of time. This is not essential, but it facilitates notation.

A pragmatic and useful way of looking at Equation (3) is to observe the next output value given earlier observations, which is given by:

$$y(t) - a_1 t(t-1) - \dots - a_n y(t-n) = b_1 u(t-1) + \dots + b_m u(t-m) \quad (4)$$

For a more compact notation, the vectors are introduced:

$$\theta = [a_1 \dots a_n \ b_1 \dots b_m]^T, \quad (5)$$

$$\phi(t) = [-y(t-1) \dots -y(t-n) \ u(t-1) \dots u(t-m)]^T \quad (6)$$

The Equation (4) can be rewritten as:

$$y(t) = \phi^T(t)\theta \quad (7)$$

It should be noted that the calculation of  $y(t)$  of data passed from Equation (4) in fact depends on the parameters  $\theta$ , this calculated value is called  $\hat{y}(t|\theta)$  and is written by:

$$\hat{y}(t|\theta) = \phi^T(t)\theta \quad (8)$$

The wagon unloading process consists of an equipment (CD) which discharges it, two supply silos and two conveyor feeders (FD) which control the outflow of the bulk solid from the silos:

- $v(t)$  Speed reference of the FD;
- $y(t)$  Measured load on scale;

In this paper, the model developed  $y(t)$  depends on  $v(t)$ . The velocity variation of the FD is directly proportional to the load measured on the scale. The average charge is  $v(t)$ . This suggests the following model:

$$y(t) = y(t-1) + \alpha v(t-1) \quad (9)$$

That fits in the form given by:

$$y(t) = \theta_1 y(t-1) + \theta_2 v(t-1) \quad (10)$$

This is a model of an input ( $v$ ) and an output and corresponds to the choice:

$$\phi(t) = [-y(t-1) \quad v(t-1)]^T \quad (11)$$

### III. PLANT MATHEMATICAL MODEL CONCEPTION

The mathematical model was designed using real data collected from the plant. For the development of the mathematical model, the speed reference of the FD was used as an input signal of the plant and, as an output signal, the load flow. With this data processed and simulated in MATLAB software, an estimated model of the plant process was generated. In Equations (12) and (13), it is the representation of the plant in state space in discrete time.

$$x(t+Ts) = Ax(t) + Bu(t) + Ke(t) \quad (12)$$

$$Y(t) = Cx(t) + Du(t) + e(t) \quad (13)$$

$$A = \begin{bmatrix} 0.9962 & 0.005764 & 0.0002512 \\ -0.003626 & 0.9998 & 0.008117 \\ -0.01034 & -0.005678 & 0.9924 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.0005299 \\ 0.0008351 \\ -0.001915 \end{bmatrix}$$

$$C = [-379.9 \quad -8.186 \quad -0.6594]$$

$$k = \begin{bmatrix} 0.0003371 \\ 0.005759 \\ -2.209 \end{bmatrix}$$

$$D = [0]$$

The plant was also modeled in state space in continuous time, according to Equations (14) and (15).

$$\frac{dx}{dt} = Ax(t) + Bu(t) + Ke(t) \quad (14)$$

$$y(t) = Cx(t) + Du(t) + e(t) \quad (15)$$

The presentation of the identified second order dynamic model of the plant in continuous time is represented by:

$$A_1 = \begin{bmatrix} -0.3964 & -0.7646 \\ -0.2801 & -0.007378 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -2.211 \times 10^8 \\ -3.0289 \times 10^6 \end{bmatrix}$$

$$C_1 = [-0.1174 \quad 22.48]$$

$$D_1 = [0]$$

The presentation of the identified third order dynamic model of the plant in continuous time is represented by:

$$A_2 = \begin{bmatrix} -0.3795 & 0.5922 & 0.01945 \\ -0.4426 & -0.004761 & -0.7432 \\ 0.9988 & 0.621 & -0.77 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -0.04912 \\ 0.06278 \\ 0.1976 \end{bmatrix}$$

$$C_2 = [-0.8459 \quad 0.01044 \quad -0.001332]$$

$$k_2 = [0 \quad 0 \quad 0]$$

$$D_2 = [0]$$

Note: The estimated third order continuous time state space model was used to validate the LQR control.

The presentation of the identified fourth order dynamic model of the plant in continuous time is represented by:

$$A_3 = \begin{bmatrix} -70.83 & 0.7335 & 7.015 & -203.7 \\ -3.527 & 0.04019 & -0.4515 & -11.17 \\ -1.364 & 0.5283 & 0.7083 & -0.7962 \\ -166.9 & 3.293 & 16.46 & -482.8 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} -106.5 \\ -5.838 \\ -0.7682 \\ -252.1 \end{bmatrix}$$

$$C_3 = [-337 \quad 1.08 \quad -0.9114 \quad 142.3]$$

$$K_3 = \begin{bmatrix} -0.7528 \\ -0.02099 \\ -0.007008 \\ -0.805 \end{bmatrix}$$

$$D_3 = [0]$$

Table 1 shows the performance of Estimated Predicted Value (EPV), Final Prediction Error (FPE) and Mean Square Error (MSE) of the estimated models for the process of the **Bulk Solid Flow by Car Dumpers Plant** in State Space Models – N4SID of 2<sup>nd</sup> order, 3<sup>rd</sup> order and 4<sup>th</sup> order.

**Table 1:** Comparison between 2<sup>nd</sup> order, 3<sup>rd</sup> order and 4<sup>th</sup> order models

	2 <sup>ND</sup> ORDER	3 <sup>RD</sup> ORDER	4 <sup>TH</sup> ORDER
EPV	99.99%	100%	99.92%
FPE	$1.278 \times 10^{-7}$	$2.472 \times 10^{-26}$	$8.999 \times 10^{-22}$
MSE	$1.277 \times 10^{-7}$	$2.465 \times 10^{-26}$	$5.175 \times 10^{-6}$

The mathematical representation of the model identified in State Space Models - N4SID of 3<sup>rd</sup> order, according to the data collected from the plant, is given by the transfer function:

$$G_p(s) = \frac{18.74s^2 - 1.41s + 30.38}{s^3 + 1.154s^2 + 1.002s + 0.8232} \quad (16)$$

In Equation (17), it is the representation of the plant in Polynomial Models - ARX (autoregressive with exogenous entry):

$$A(z)y(t) = B(z)u(t) + e(t) \quad (17)$$

The estimated model of the plant in Polynomial Models - ARX (autoregressive with exogenous entry) is given by:

$$A(z) = 1 - 1.992z^{-1} + 1.977z^{-3} - 0.984z^{-4} \quad (18)$$

$$B(z) = 6.029 \times 10^{-5} z^{-1} \quad (19)$$

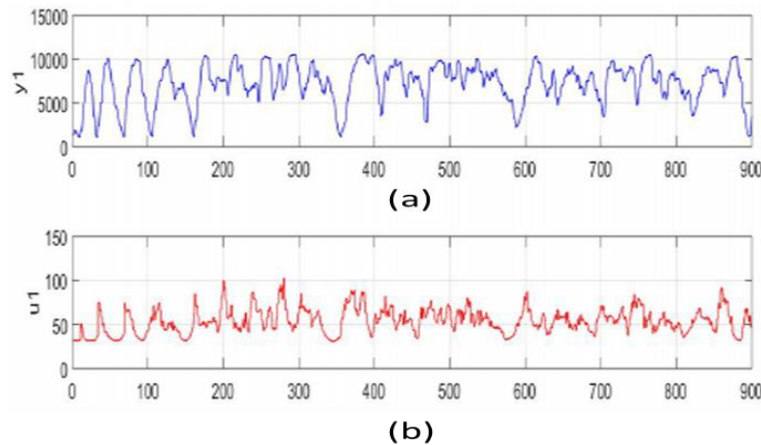
Table 2 shows the performance comparison of the EPV, FPE and MSE of the model identified in State Space Models – N4SID of the 3<sup>rd</sup> order with the identified model of Polynomial Models – ARX.

**Table 2:** Comparison of the State Space Models – N4SID of 3<sup>rd</sup> order with the model of Polynomial Models – ARX.

	STATE SPACE MODELS	POLYNOMIAL MODELS – ARX
EPV	100%	100%
FPE	$2.472 \times 10^{-26}$	$2.246 \times 10^{-28}$
MSE	$2.465 \times 10^{-26}$	$1.317 \times 10^{-25}$

#### IV. COMPUTATIONAL EXPERIMENTS

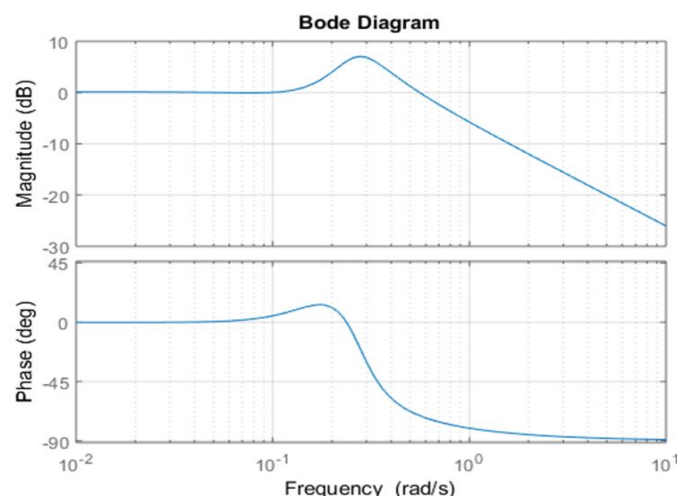
The reference values of frequency inverter of the FD were compared with the load flow measured by a dynamic scale installed on a conveyor belt where the load was being transported. The comparison of the behavior of these data showed a very large correlation between them, as shown in Fig. 4, in part (a) of Fig. 4 is the behavior of the load flow measured by the scale and in part (b) of Fig. 4 is the behavior of the speed reference of the FD measured by the frequency inverter.



**Figure 4:** Input/output model without disturbance

As the behavior of the system input (FD velocity reference) closely resembled the output (load flow measured by the scale), the system was modeled to operate as follows: the system input is the FD speed reference of the CD  $u(t)$  and the output  $y(t)$  is the load flow measured by the scale.

Fig. 5 shows the Bode plot for the transfer function of the study plant given by Equation (16).



**Figure 5:** Bode diagram



Fig. 6 shows the behavior of the unit step response curve for the plant transfer function given by Equation (16).

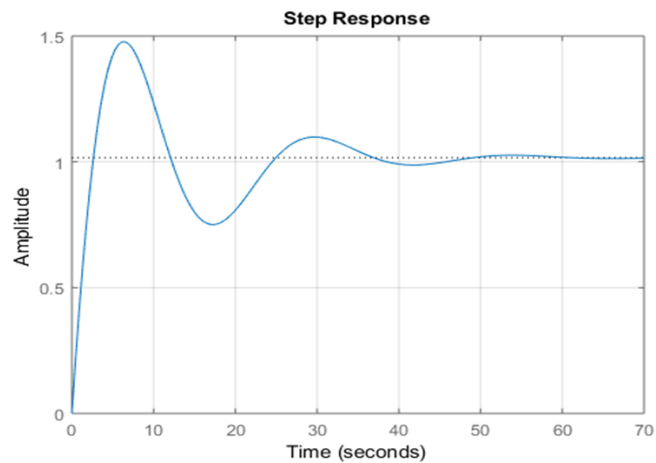


Figure 6: unit step response

Fig. 7 shows the behavior of the impulse response curve for the plant transfer function given by Equation (16).

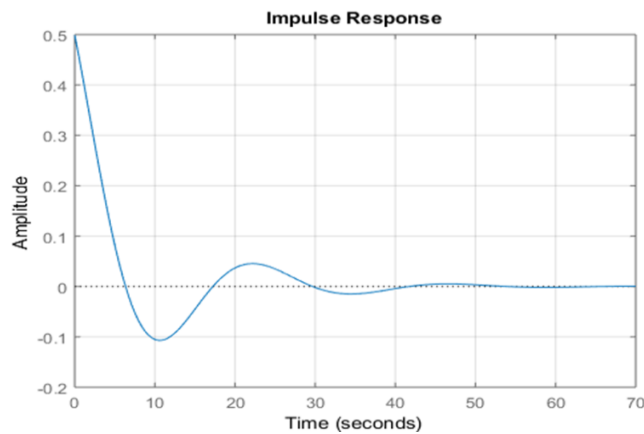


Figure 7: Impulse response

Using the System Identification tool of the MATLAB Software, a model was estimated for the study plant.

Fig. 7 shows the flow behavior measured by the plant scale indicated by the blue curve and the estimated flow behavior of the plant indicated by the red curve. In this model, the external interferences of the system, such as noise and disturbance, were disregarded.

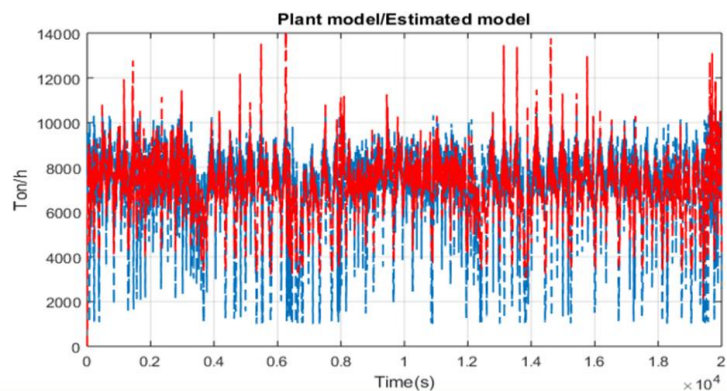


Figure 7: Plant model/Estimated model

The purpose of this work is to present a methodology for dynamic systems modelling based on the computational analysis in industrial applications.

### 5.1 LQR CONTROL

The optimal control theory concerns the operation of a dynamic system at minimal cost. The case in which the system dynamics is described by a set of linear differential equations and the cost is described by a quadratic function, which is called the LQ problem. One of the main results in the theory is that the solution is provided by the linear-quadratic regulator (LQR) [9], [10], [11].

For a continuous time system the fed-state control law  $u = -Kx$  minimizes the cost function given by:

$$J(u) = \int_0^{\infty} (x^T Q x + u^T R u) dt \tag{20}$$

Subject to the dynamics of the system given by:

$$\dot{x} = Ax + Bu \tag{21}$$

The matrix  $Q$  and the scalar  $R$  weigh the effects of the minimization on the states and the input, respectively.  $diag(Q) > R : \min \|x\|$  prevails over  $\min |u|$  (control effort).

In addition to the return gain of state  $K$ , LQR returns the solution  $S$  of the associated Riccati equation:

$$A^T S + SA - (SB + N)R^{-1}(B^T S + N^T) + Q = 0,$$

$$K_{LQR} = 10^4 [-0.2830 \quad 1.0066 \quad -0.7660]$$

$$S = 10^8 \begin{bmatrix} 0.1266 & -0.4511 & -0.1117 \\ -0.4511 & 1.6074 & 0.3981 \\ -0.1117 & 0.3981 & 0.0991 \end{bmatrix}$$

$$e = 10^3 \begin{bmatrix} -0.0057 \\ -0.0003 \\ -0.0003 \end{bmatrix}$$

### 5.2 FUZZY CONTROLLER

The first logic notions of the concepts were developed by a Polish logician [12] in 1920 who introduced sets with degrees of pertinence being 0, ½ and 1 and, later, it expanded to an infinite number of values between 0 and 1. The first publication on fuzzy logic dates from 1965, when it received this name. Its author was *Loffi Asker Zadeh*, a professor at Berkeley, University of California. *Zadeh* created the fuzzy logic by combining the concepts of classical logic and *Lukasiewicz's* sets, defining degrees of pertinence.

Between 1970 and 1980, the industrial applications of fuzzy logic became more important in Europe and, after 1980, Japan began its use with applications in industry. By 1990, fuzzy logic had attracted more interest in US companies. Due to the development and numerous practical possibilities of fuzzy systems and the great commercial success of its applications, fuzzy logic is nowadays considered a standard technique and has a wide acceptance in the area of industrial processes control.

The fuzzy controller was designed to act on the difference between the measured value and the estimated value, according to Fig. 8.

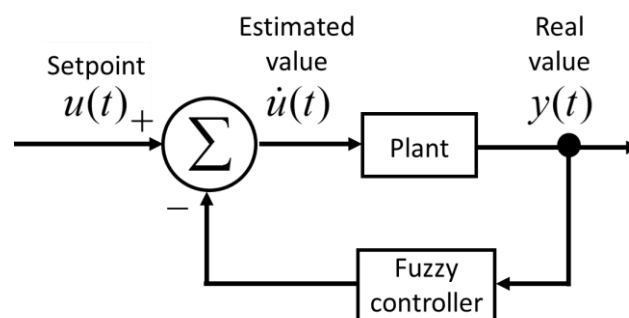


Figure 8: fuzzy controller coupled in plant



The system error is the difference between the measured value and the estimated value (setpoint) that is given by:

$$E = V_M - V_E \quad (22)$$

where  $V_M$  is the measured value and  $V_E$  is the estimated value. The rules were constructed based on the behavior of the estimated model as follows:

- Measured value higher than the estimated value, the  $E > 0$ :
  - IF  $E > 0$ , High Flow, THEN decrease FD speed.
- Measured value equals to the estimated value, the  $E = 0$ :
  - IF  $E = 0$ , Normal Flow, THEN remain constant the FD speed.
- Measured value lower than the estimated value, the  $E < 0$ :
  - IF  $E < 0$ , Low Flow, THEN increase FD speed.

In Fig. 9, it is the graphical illustration of the fuzzy controller applied to the control of the process of unloading wagons.

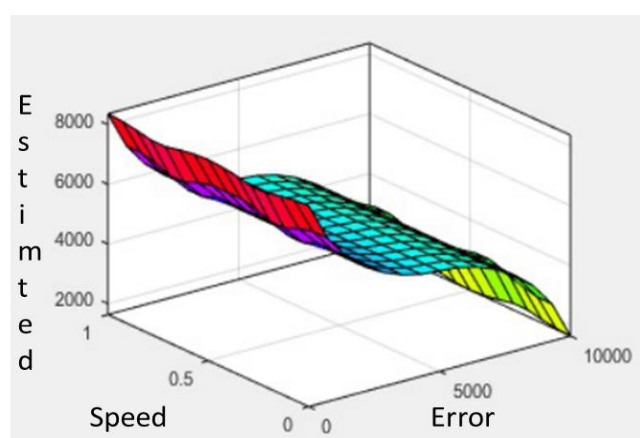


Figure 9: fuzzy controller behavior.

## V. CONCLUSIONS

The identification of dynamic systems models for real plants is not a trivial task because, in most cases, developing a mathematical model that represents the process of a real plant is very complex, however, methodologies and techniques are being increasingly improved as very important tools in designing approximate models of real models. In this work, the MATLAB System Identification Toolbox was used for the development of the presented models. For the N4SID model in Space of States, three models were estimated, being: one of 2<sup>nd</sup> order; one of the 3<sup>rd</sup> order and one of a 4<sup>th</sup> order, as presented in Table 1; and for Polynomial Models, and ARX model was identified according to Table 2.

It was observed that the 3<sup>rd</sup> order model had the best performance among the N4SID models in state space and comparing the 3<sup>rd</sup> order model with the ARX Polynomial Model, the results were very similar, the two models reached 100% EPV, the Polynomial ARX model obtained slightly better FPE and worse MSE than model N4SID in 3<sup>rd</sup> order State Space. For the validation of the N4SID model in 3<sup>rd</sup> order States Space, an LQR control, a Fuzzy controller and the plant transfer function were designed. For the ARX Polynomial Model, no controller has been designed, but it can be used in PID Controller system. Therefore, it is concluded that the models identified and estimated presented in this paper approximate the model of the actual plant under study, being ready for the implementation of control systems to improve its performance according to what was exposed in the presented computational experiments.

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