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Free Convective FlowFromAn Accelerated Infinite Vertical Platewith Varying PlateTemperatureand Uniform Mass Diffusion

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ABSTRACT: *Theoretical solution of unsteady flow past an infinite uniformly accelerated plate has been presented in presence of a variable plate temperature and uniform mass diffusion. The plate temperature is raised linearly with time. The dimensionless governing equations are solved using Laplace-transform technique. The velocity profile, the concentration, Skin friction and the rate of heat transfer in terms of Nusselt Number are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number and time.*

Keywords: *Accelerated,heattransfer, MHD,massdiffusion,vertical plate*

I. INTRODUCTION

Heat and mass transfer plays an important role in manufacturing industries for design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircrafts, missiles, spacecraft design, solar energy collectors, design of chemical processing equipments, satellites and space vehicles are examples of such engineering applications. Free convective flow past a linearly accelerated plate in presence of viscous dissipative heat was studied by Gupta et al. [1] using perturbation technique. Kafousias and Raptis [2]extended the above problem and included mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated plate with variable suction and uniform heat flux in the presence of magnetic field was studied byRaptis et al. [3]. Mass transfer effects on flow past a uniformly accelerated vertical plate was studied by Soundalgekar [4]. Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh [5]. Basant Kumar Jha and RavindraPrasad [6] analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. R. Muthucumaraswamy et al. [7] investigated the effects of a flow past a uniformlyaccelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion.

This paper deals with the study of magnetic and massdiffusion effects on the free convection flow from a uniformly accelerated infinite vertical plate with variable temperature. The solutions are in terms of exponential and complementary error function.

II. FORMULATION OF THE PROBLEM

We consider unsteady, free convective, two dimensional MHD flow of an incompressible and electrically conductingviscous fluid along an infinite vertical plate. The x-axis is taken along the plate vertically upward direction and y-axis is taken normal to the plate. A magnetic field of uniform strength B_o is applied in the direction of the flow and induced magnetic field is neglected. Initially, the plate and the fluid are at the same temperature T'_∞ in a stationary condition with concentration level C'_∞ at all points. At time t'>0 the plate starts moving with a velocity $u_0 t'$. Its temperature is raised to $T_\infty + (T_\omega - T_\infty)At'$ and the concentration of the plate is raised to C'_ω. Using the Boussinesq approximation, the governing equations for the flow are given by

$$\cdot\ \frac{\partial u'}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \upsilon\frac{\partial^2 u'}{\partial y^2} - \frac{\sigma B_0^2 u'}{\rho} \quad \rightarrow (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \rightarrow (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \quad \rightarrow (3)$$

With the following initial and boundary conditions

$$\left.\begin{array}{ll} & u'=0, T=T_{\infty}, C'=C'_{\infty} \;\; for all \;\; y,t'\le 0 \\ t'>0: & u'=u_0 t', \; T=T_{\infty}+\left(T_{\omega}-T_{\infty}\right)At', \; C'=C'_{\omega} \quad at \quad y=0 \\ & u\to 0, T\to T_{\infty}, C\to C'_{\infty} \;\; as \;\; y\to\infty \end{array}\right\} \to (4)$$

Where

$$A=\left(\frac{u_0^2}{\upsilon}\right)^{1/3}$$

We introduce following non-dimensional quantities

$$\left.\begin{array}{l} U=\frac{u'}{(\upsilon u_0)^{1/3}}, t=t'\left(\frac{u_0^2}{\upsilon}\right)^{1/3}, Y=y\left(\frac{u_0}{\upsilon^2}\right)^{1/3}, \theta=\frac{T-T_{\infty}}{T_{\omega}-T_{\infty}}, G_r=\frac{g\beta(T_{\omega}-T_{\infty})}{u_0} \\ C=\frac{C'-C'_{\infty}}{C'_{\omega}-C'_{\infty}}, G_c=\frac{g\beta^*(C'_{\omega}-C'_{\infty})}{u_0}, P_r=\frac{\mu C_p}{\kappa}, S_c=\frac{\upsilon}{D}, M=\frac{\sigma B_0^2\upsilon^{1/3}}{\rho u_0^{2/3}} \end{array}\right\}\to (5)$$

The equations (1), (2), (3) become:

$$\frac{\partial U}{\partial t}=G_r\,\theta+G_c C+\frac{\partial^2 U}{\partial Y^2}-MU \qquad \to (6)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial Y^2} \qquad \to (7)$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial Y^2} \qquad \to (8)$$

Under boundary conditions:

$$\left.\begin{array}{l} U=0, \;\; \theta=0, \;\; C=0 \;\; for\, Y,t\le 0 \\ t>0: \quad U=t, \;\; \theta=t, \;\; C=1 \;\; at \;\; Y=0 \\ \quad U\to 0;\, \theta\to 0;\, C\to 0 \; as \; Y\to\infty \end{array}\right\} \to (9)$$

III. METHOD OF SOLUTION

We solve the governing equations in an exact form by using Laplace-transformations.The Laplace Transformation of the equations (6), (7), (8) and the boundary conditions (9) are given by:

$$\frac{d^2\bar{u}}{dY^2}-(S+M)\bar{u}=-G_r\,\bar{\theta}-G_c\bar{C} \qquad \to (10)$$

$$\frac{d^2\bar{\theta}}{dY^2}-P_r s\,\bar{\theta}=0 \qquad \to (11)$$

$$\frac{d^2\bar{C}}{dY^2}-S_c\, s\bar{C}=0 \qquad \to (12)$$

Under the boundary conditions

$$\left.\begin{array}{l} \bar{U}=0, \quad \bar{\theta}=0, \quad \bar{C}=0 \quad when \;\; Y,t\le 0 \\ when \;\; t\ge 0: \bar{U}=\frac{1}{s^2}, \;\; \bar{\theta}=\frac{1}{s^2}, \;\; \bar{C}=\frac{1}{s} \qquad at \quad Y=0 \\ \quad \bar{U}\to 0, \;\; \bar{\theta}\to 0, \;\; \bar{C}\to 0 \quad as \quad Y\to 0 \end{array}\right\}\to (13)$$

Solving equations (10), (11), and (12) under the boundary conditions (13), we get

$$\bar{u}(Y,s)=\frac{\exp\left\{-\sqrt{(S+M)}Y\right\}}{s^2}+\frac{G_r}{s^2(P_r-1)\left(s-\frac{M}{P_r-1}\right)}\left[\exp\left\{-\sqrt{(S+M)}Y\right\}-\exp\left\{-\sqrt{P_r s}Y\right\}\right]$$

$$+\frac{G_c}{s(S_c-1)\left(s-\frac{M}{S_c-1}\right)}\left[\exp\left\{-\sqrt{(S+M)}Y\right\}-\exp\left\{-\sqrt{S_c s}Y\right\}\right] \quad \to (14)$$

$$\bar{\theta}(Y,s)=\frac{\exp\left\{-\sqrt{P_r s}Y\right\}}{s^2} \quad \to (15)$$

$$\bar{C}(Y,s)=\frac{\exp\left\{-\sqrt{S_c s}Y\right\}}{s} \quad \to (16)$$

Inverting equations (14), (15) and (16), we get

For $P_r \neq 1, S_c \neq 1$

$$u(Y,t)=\frac{t}{2}\left\{\left(1+\frac{\eta}{\sqrt{Mt}}\right)erfc\left(\eta+\sqrt{Mt}\right)+\left(1-\frac{\eta}{\sqrt{Mt}}\right)erfc\left(-2\eta\sqrt{Mt}\right)\right\}+\frac{G_r t}{2M}\left\{\begin{array}{l}2\left(1+2\eta^2P_r\right)erfc\left(\eta\sqrt{P_r}\right)-\frac{4\eta}{\sqrt{\pi}}\sqrt{P_r}\exp\left(-\eta^2P_r\right)\\-\left(1+\frac{\eta}{\sqrt{Mt}}\right)erfc\left(\eta+\sqrt{Mt}\right)+\left(1-\frac{\eta}{\sqrt{Mt}}\right)erfc\left(-2\eta\sqrt{Mt}\right)\end{array}\right\}$$

$$+\frac{G_r(P_r-1)}{2M^2}\exp\left(\frac{Mt}{P_r-1}\right)\left[\exp\left(2\eta\sqrt{\frac{MP_rt}{P_r-1}}\right)\left\{erfc\left(\eta+\sqrt{\frac{MP_rt}{P_r-1}}\right)-erfc\left(\eta+\sqrt{\frac{Mt}{P_r-1}}\right)\right\}+\exp\left(-2\eta\sqrt{\frac{MP_rt}{P_r-1}}\right)\left\{erfc\left(\eta-\sqrt{\frac{MP_rt}{P_r-1}}\right)-erfc\left(\eta-\sqrt{\frac{Mt}{P_r-1}}\right)\right\}\right]$$

$$+\frac{G_c}{2M}\exp\left(\frac{Mt}{S_c-1}\right)\left[\exp\left(2\eta\sqrt{\frac{MS_ct}{S_c-1}}\right)\left\{erfc\left(\eta+\sqrt{\frac{MS_ct}{S_c-1}}\right)-erfc\left(\eta+\sqrt{\frac{Mt}{S_c-1}}\right)\right\}\right.$$

$$\left.+\exp\left(-2\eta\sqrt{\frac{MS_ct}{S_c-1}}\right)\left\{erfc\left(\eta-\sqrt{\frac{MS_ct}{S_c-1}}\right)-erfc\left(\eta-\sqrt{\frac{Mt}{S_c-1}}\right)\right\}\right]+\frac{G_c}{2M}\left\{2erfc\left(\eta\sqrt{S_c}\right)\right.$$

$$\left.-\exp\left(2\eta\sqrt{Mt}\right)erfc\left(\eta+\sqrt{Mt}\right)-\exp\left(-2\eta\sqrt{Mt}\right)erfc\left(\eta-\sqrt{Mt}\right)\right\}\to(17)$$

$$\theta(Y,t)=t\left[\left(1+2\eta^2P_r\right)erfc\left(\eta\sqrt{P_r}\right)-\frac{2\eta\sqrt{P_r}}{\sqrt{\pi}}\exp\left(-\eta^2P_r\right)\right] \quad \to(18) \qquad C(Y,t)=erfc\left(\eta\sqrt{S_c}\right) \quad \to(19)$$

And for $P_r = S_c = 1$

$$u(Y,t)=\frac{t}{2}\left[\left(1+\frac{\eta}{\sqrt{Mt}}\right)\exp\left(2\eta\sqrt{Mt}\right)erfc\left(\eta+\sqrt{Mt}\right)+\left(1-\frac{\eta}{\sqrt{Mt}}\right)\exp\left(-2\eta\sqrt{Mt}\right)erfc\left(\eta-\sqrt{Mt}\right)+\right.$$

$$\left.\frac{G_r}{M}\left\{2\left(1+2\eta^2\right)erfc(\eta)-\frac{4\eta}{\sqrt{\pi}}\exp\left(-\eta^2\right)-\left(1+\frac{\eta}{\sqrt{Mt}}\right)\exp\left(2\eta\sqrt{Mt}\right)erfc\left(\eta+\sqrt{Mt}\right)-\left(1-\frac{\eta}{\sqrt{Mt}}\right)\exp\left(-2\eta\sqrt{Mt}\right)erfc\left(\eta-\sqrt{Mt}\right)\right\}\right]+$$

$$\frac{G_c}{2M}\left[2\,erfc(\eta)-\exp\left(2\eta\sqrt{Mt}\right)erfc\left(\eta+\sqrt{Mt}\right)\right. \quad \to(20)$$

$$\theta(Y,t)=t\left[\left(1+2\eta^2\right)erfc(\eta)-\frac{2\eta}{\sqrt{\pi}}\exp\left(-\eta^2\right)\right] \quad \to(21) \qquad C(Y,t)=erfc(\eta) \quad \to(22)$$

SKIN-FRICTION

We know study skin-friction from velocity field. It is given by

$$\tau=-\mu\left.\frac{\partial u'}{\partial y'}\right|_{y'=0}$$

This reduces to

$$\tau=-\left(\frac{\partial u}{\partial Y}\right)_{Y=0}$$

Now for $P_r \neq S_c \neq 1$

$$\tau=t\left[\left(\frac{1}{2t\sqrt{M}}+\sqrt{M}\right)erf\left(\sqrt{Mt}\right)+\frac{1}{\sqrt{\pi t}}\exp(-Mt)\right]+$$

$$\frac{G_r t}{M}\left[2\sqrt{\frac{P_r}{\pi t}}-\frac{1}{\sqrt{\pi t}}-\left(\frac{1}{2t\sqrt{M}}+\sqrt{M}\right)erf\left(\sqrt{Mt}\right)\right]+$$
$$\frac{G_r\left(P_r-1\right)}{M^2}\left[\sqrt{\frac{P_r}{\pi t}}-\frac{1}{\sqrt{\pi t}}-\sqrt{M}\,erf\left(\sqrt{Mt}\right)\right]+$$
$$\frac{G_c}{M}\left[\sqrt{\frac{S_c}{\pi t}}-\frac{1}{\sqrt{\pi t}}-\sqrt{M}\,erf\left(\sqrt{Mt}\right)\right]+$$
$$\frac{G_r\left(P_r-1\right)}{M^2}\sqrt{\frac{MP_r}{P_r-1}}\exp\left(\frac{Mt}{P_r-1}\right)\left[erf\left(\sqrt{\frac{MP_r t}{P_r-1}}\right)-erf\left(\sqrt{\frac{Mt}{P_r-1}}\right)\right]$$
$$+\frac{G_c}{M}\sqrt{\frac{MS_c}{S_c-1}}\exp\left(\frac{Mt}{S_c-1}\right)\left[erf\left(\sqrt{\frac{MS_c t}{S_c-1}}\right)-erf\left(\sqrt{\frac{Mt}{S_c-1}}\right)\right]\quad\rightarrow(23)$$

And when $P_r = S_c = 1$

$$\tau=\left(\frac{2G_r+G_c}{M}\right)\left[\sqrt{M}\ erf\left(\sqrt{Mt}\right)+\frac{1}{\sqrt{\pi t}}\exp(-Mt)\right]-$$
$$\left(\frac{4G_r+G_c}{M}\right)\frac{1}{\sqrt{\pi t}}+\frac{G_r}{M\sqrt{Mt}}erf\left(\sqrt{Mt}\right)$$
$$-t\left[\left(\sqrt{M}+\frac{1}{2t\sqrt{M}}\right)erf\left(\sqrt{Mt}\right)+\frac{1}{\sqrt{\pi t}}\exp(-Mt)\right]\quad\rightarrow(24)$$

Nusselt Number

In non-dimensional form the rate of heat transfer is given by

$$Nu=-\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0}$$
$$Nu=2\sqrt{\frac{P_r t}{\pi}}\qquad\rightarrow(25)$$

IV. RESULTS AND DISCUSSION

In order to point out the effects of various parameters on the flow characteristics, the following discussion is set out. The values of the Prandtl number are chosen such that they represent air (P_r=0.71) and water (P_r=7). The values of Schmidt number are chosen to represent the presence of species of hydrogen (S_c=0.22), water vapour (S_c=0.6) and ammonia (S_c=0.78). The numerical values of the velocity, temperature and concentration are computed for different physical parameters like Prandtl number, Schmidt number, thermal Grashof number, mass Grashof number and time and are represented graphically.The velocity profiles for variation of time t, Schmidt number S_c, Grashof numbers G_r and G_c, and the Hartmann number M are studied and presented in fig.1 to fig.4Fig. 1 displays the effect of time t on the velocity field for the fluid when P_r=0.71, S_c=0.6 and G_r=G_c=5. Here we see that in case of cooling of the plate (as G_r>0), the velocity near the plate increases owing to the presence of water vapour in the flow field.In Fig. 2 we notice that although there is a rise in the velocity due to the presence of water vapour and ammonia, but it is not so high in the presence of Hydrogen. Fig. 3 reveals the velocity variations with G_r, G_c and P_r. It is observed that the greater cooling of the surface (increase in G_r) and increase in G_c results in an increase in the velocity for air (P_r=0.71). It is due to the fact that an increase in the

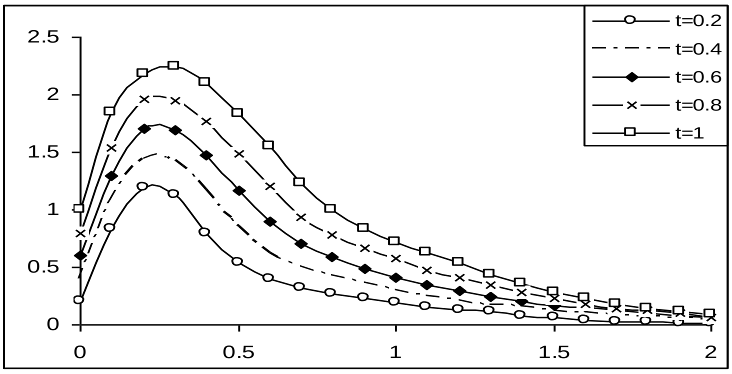

Fig. 1 Velocity profile for different values of t

Values of G_r and G_c has the tendency to increase the thermal and mass buoyancy effect.

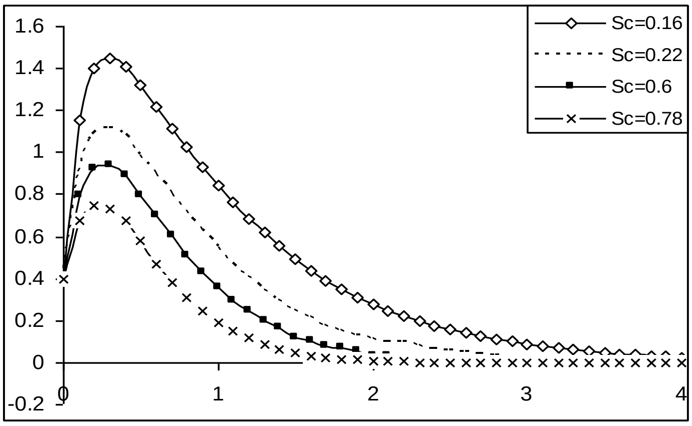

Fig. 2 Velocity profile for different values of S_c

We also observe that the velocity near the plate is greater than at the plate. The maximum velocity attains near the plate and is in the neighbourhood of the point $\eta = 0.2$. After $\eta > 0.2$, the velocity decreases and tend to zero as $\eta \to \infty$.

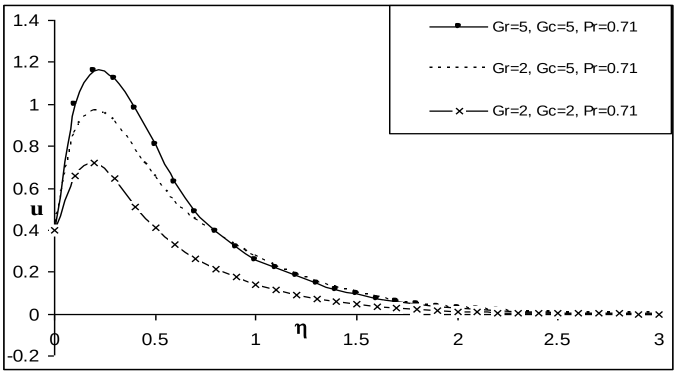

Fig 3: Velocity profiles for different values of G_r, G_c (t= 0.4, M=2.0, S_c=0.6)

It is also found in Fig.4 that the velocity decreases with the increase in the magnetic parameter M. It is because of the fact that when a transverse magnetic field is applied, a resistive type of force (known as Lorentz force) get produced which tends to resist the fluid motion and thus reducing the velocity

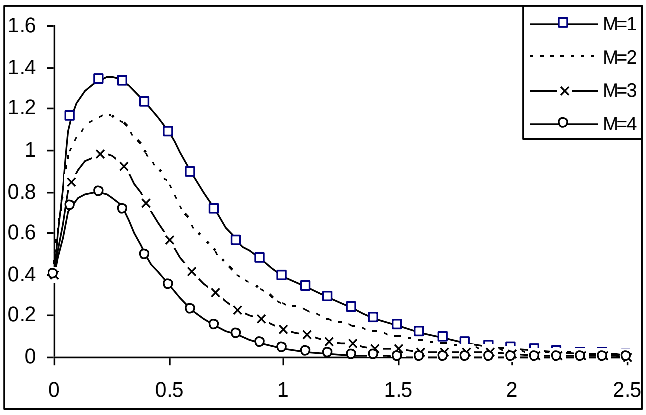

Fig. 4: Velocity profiles for variation of M (t=0.4, P_r=0.71, S_c=0.6, G_r=5, G_c=5)

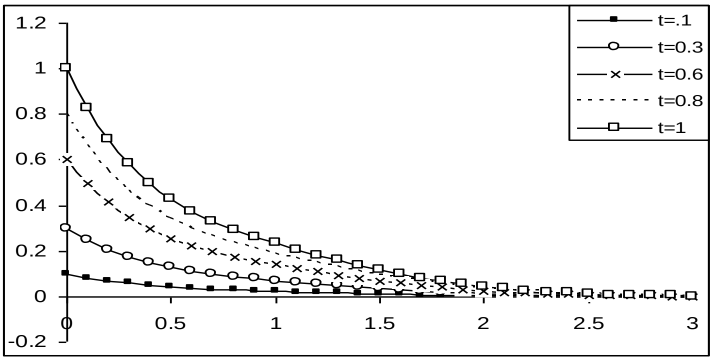

Fig. 5: Temperature profiles for variation of t (P_r=0.71, S_c=0.6)

Fig. 5 and Fig. 6 depict the temperature profiles against η for variation of time and the Prandtl number. In fig. 5, we notice that the magnitude of temperature is the maximum at the plate and decays to zero asymptotically.

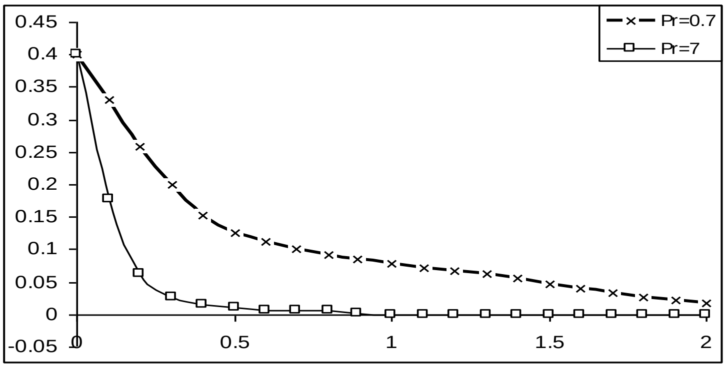

Fig.6: Temperature profiles for variation of P_r (t=0.4, S_c=0.6)

Fig. 7 demonstrates the effect of Schmidt number (S_c) on the concentration. Like temperature, the concentration is maximum at the surface and falls exponentially. The concentration decreases with an increase in S_c. Further it is seen that the concentration falls slowly and steadily for Hydrogen (S_c=0.22) as compared to other gases.

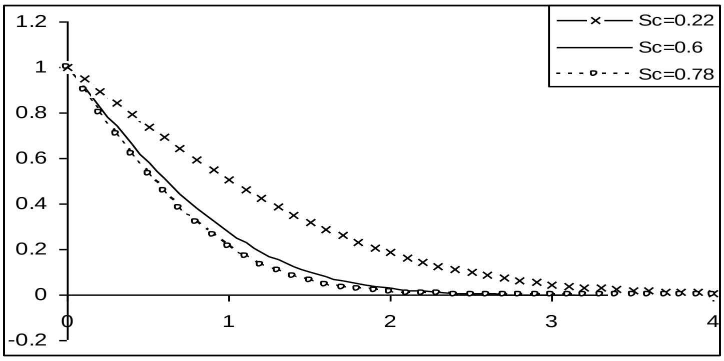

Fig. 7: Concentration profiles for variation of S_c (P_r=0.71)

Fig. 8 depicts the nature of skin-friction against time t for different values of the parameters G_r, G_c, M, S_c and P_r. Here we notice that Skin-friction increases with an increase of S_c. Further we observe that it increases with M. This is due to the fact that Lorentz force gets enhanced with increasing M, which imports additional momentum in the boundary layer. On the other hand we see that skin-friction decreases with the increase of G_r and G_c.

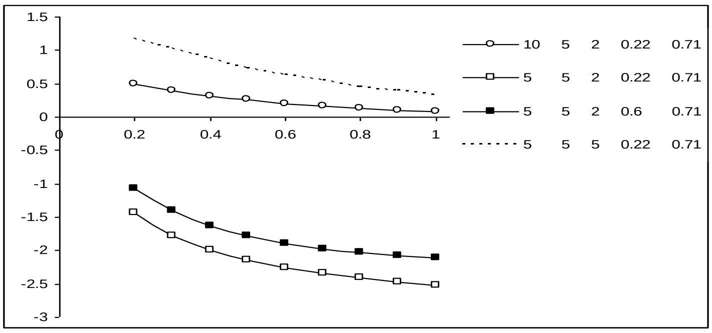

Fig. 8: Skin-friction

Fig. 9 exhibits the behavioure of co-efficient of rate of heat transfer (Nusselt number) against time. Here we observe that Nusselt number decreases with time. Also we notice that it is higher for P_r=7 than that for Pr=0.71. This is due to the fact that P_r is small means thermal conductivity is more, which results more rapid diffusion of heat from the plate and hence rate of heat transfer get reduced.

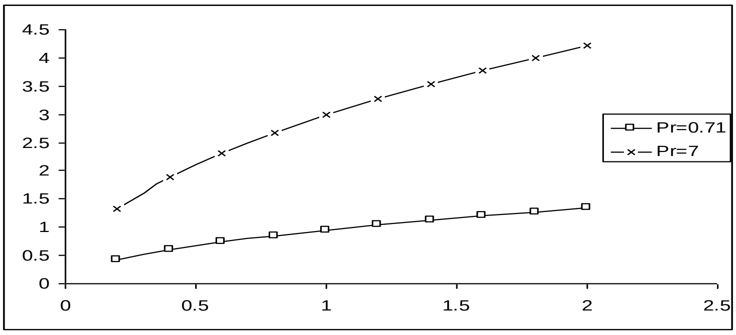

Fig.9: Nusselt number

V. CONCLUSION

From the above discussions, the following conclusions are set out:

1) The velocity decreases with an increase in magnetic parameter, Schmidt number and Prandtl number. On the other hand, it increases with an increase in the values of thermal Grashof number, mass Grashof number and time.
2) The concentration decreases with an increase in Schmidt number.
3) The skin-friction increases with the increase in magnetic parameter, Schmidt number and Prandtl number, while it decreases with an increase of thermal Grashof number, mass Grashof number and time.
4) Nusselt number increases with the increase in Prandtl number.

APPENDIX

A. Nomenclature

A constant

C' Species concentration in the fluid

C Dimensionless concentration

C_p Specific heat at constant pressure

D Mass diffusion coefficient

G_c Mass Grashof number

G_r Thermal Grashof number

g Acceleration due to gravity

κ Thermal conductivity

P_r Prandtl number

S_c Schmidt number
T Temperature of the fluid near the fluid
t' Time
t Dimensionless time
u Velocity of the fluid in x-direction
u_0 Velocity of the plate
U Dimensionless velocity
x Spatial co-ordinate along the plate
y' Co-ordinate axis normal to the plate
y Dimensionless co-ordinate axis normal to the plate

B.Greek Symbols

β Volumetric coefficient of thermal expansion
β^* Volumetric coefficient of thermal expansion with Concentration
μ Coefficient of viscosity
υ Kinematic viscosity
ρ Density of the fluid
τ Dimensionless skin-friction
θ Dimensionless temperature
η Similarity parameter
erf Error function
$erfc$ Complementary error function

C. Subscripts

ω Conditions at the plate
∞ Conditions in the free stream

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