

Nuclear Viscosity Obtained With The Solution Of The Navier Stokes Equations.

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ABSTRACT: *The solution of the Navier-Stokes 3D equation is a generalized Fermi Dirac probability function $P(x, y, z, t) = \frac{1}{1 + e^{\frac{p_0}{2\eta}t - \mu(x^2 + y^2 + z^2)^{1/2}}}$, which permit us to determine the relationship between nuclear force and incompressible fluid. The dynamic solution naturally develops on a spherical surface, corresponding to a microscopic vortex (alpha emission) which meets the theorem of the implicit function of an fixed point $f(t, r) = 0$. Logically, the coefficient μ has a correspondence with Yukawa's potential and the term $\frac{p_0}{2\eta}$ represents the dynamics of the incompressible nuclear fluid.*

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I. INTRODUCTION

The atomic nucleus is an incompressible fluid, justified by the formula of the nuclear radius, $R = 1.2A^{\frac{1}{3}}$, where it is evident that the volume of the atomic nucleus changes linearly with $A = Z + N$, giving a density constant. All incompressible fluid and especially the atomic nucleus comply with the Navier Stokes equations (1,2,3,4,5,6). We present a rigorous demonstration on the incompressibility of the atomic nucleus, which allows to write explicitly the form of the nuclear force $\mathbf{F}_N = -\frac{g\mu^2}{8\pi}(A - 1)P(1 - P)\nabla\mathbf{r}$, which facilitates the understanding of alpha decay [1,2,3,4,5].

The Navier Stokes equations are a problem of the millennium, that has not been resolved yet in a generalized manner. We present a solution that logically meets all the requirements established by the Clay Foundation. This solution coherently explains the incompressible nuclear fluid and allows calculations of the nuclear viscosity and nuclear pressure, that are key elements of nuclear cavitation.

Theory of Quantum Games. Widely used and accepted in its mathematical formalization and in its applications in Information Theory.

The alpha particle is one of the most stable. Therefore it is believed that it can exist as such in the heavy core structure. The kinetic energy typical of the alpha particles resulting from the decay is in the order of 5 MeV. Its speed is 15,000 km/s.

For our demonstrations, we will use strictly the scheme presented by Fefferman in <http://www.claymath.org/millennium-problems>, where six demonstrations are required to accept as valid a solution to the Navier-Stokes 3D equation. [5,6].

II. MODEL

The velocity defined as $\mathbf{u} = -2\nu\frac{\nabla P}{\rho}$, with a radius noted as $r = (x^2 + y^2 + z^2)^{1/2}$ where $P(x, y, z, t)$ is the logistic probability function $P(x, y, z, t) = \frac{1}{1 + e^{kt - \mu r}}$, and the expected value $E(r|r \geq 0) < C$ exist. The term P is defined in $((x, y, z) \in \mathbb{R}^3, t \geq 0)$, where constants $k > 0, \mu > 0$ and $P(x, y, z, t)$ is the general solution of the Navier-Stokes 3D equation, which has to satisfy the conditions (1) and (2), allowing us to analyze the dynamics of an incompressible fluid. [6,7,8].

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \nu \nabla^2 \mathbf{u} - \frac{\nabla p}{\rho_0} \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0)(1)$$

With, $\mathbf{u} \in \mathbb{R}^3$ an known velocity vector, ρ_0 constant density of fluid, η dynamic viscosity, ν cinematic viscosity, and pressure $p = p_0 P$ in $((x, y, z) \in \mathbb{R}^3, t \geq 0)$.

Where velocity and pressure are depending of r and t . We will write the condition of incompressibility.

$$\nabla \cdot \mathbf{u} = 0 \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \quad (2)$$

The initial conditions of fluid movement $\mathbf{u}^0(x, y, z)$, are determined for $t = 0$. Where speed \mathbf{u}^0 must be C^∞ divergence-free vector.

$$\mathbf{u}(x, y, z, 0) = \mathbf{u}^0(x, y, z) \quad ((x, y, z) \in \mathbb{R}^3) \quad (3)$$

For physically reasonable solutions, we make sure $\mathbf{u}(x, y, z, t)$ does not grow large as $r \rightarrow \infty$. We will restrict attention to initial conditions \mathbf{u}^0 that satisfy.

$$|\partial_x^\alpha \mathbf{u}^0| \leq C_{\alpha K} (1+r)^{-K} \quad \text{on } \mathbb{R}^3 \text{ for any } \alpha \text{ and } K \quad (4)$$

The Clay Institute accepts a physically reasonable solution of (1), (2) and (3), only if it satisfies:

$$p, \mathbf{u} \in C^\infty(\mathbb{R}^3 \times [0, \infty)) \quad (5)$$

and the finite energy condition [6,7,8,9].

$$\int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz \leq C \quad \text{for all } t \geq 0 \quad (\text{bounded energy}). \quad (6)$$

The problems of Mathematical Physics are solved by the Nature, guiding the understanding, the scope, the limitations and the complementary theories. These guidelines of this research were: the probabilistic elements of Quantum Mechanics, the De Broglie equation and the Heisenberg Uncertainty principle.

2.1 Definitions

Attenuation coefficient.

We will use the known attenuation formula of an incident flux I_0 , for which $I = I_0 e^{-\mu r}$. Where, I_0 initial flux and μ attenuation coefficient of energetic molecules that enter into interaction and/or resonance with the target molecules, transmitting or capturing the maximum amount of energy [5].

Growth coefficient.

We will use an equation analogous to concentration equation of Physical Chemistry $C = C_0 e^{kt}$, where $k = \frac{p_0}{2\eta}$, is growth coefficient, p_0 is the initial pressure of our fluid, η the dynamic viscosity and C_0 the initial concentration of energetic fluid molecules.

It is evident that, in equilibrium state we can write $\mu r = kt$, however, the Navier-Stokes equation precisely measures the behavior of the fluids out of equilibrium, so that: $\mu r \neq kt$.

Fortunately, there is a single solution for out-of-equilibrium fluids, using the fixed-point theorem for implicit functions, $\frac{1}{1+e^{kt-\mu r}} = \frac{2}{\mu r}$, the proof is demonstrated in Theorem 1.

Dimensional Analysis.

We will define the respective dimensional units of each one of variables and physical constants that appear in the solution of the Navier-Stokes 3D equation. [9,10,11].

Kinematic viscosity $\nu = \frac{\eta}{\rho_0}$, [$\frac{m^2}{s}$]

Dynamic viscosity η , [pa.s], where pa represents pascal pressure unit.

Initial Pressure of out of equilibrium. p_0 , [pa]

Fluid density ρ_0 , [$\frac{kg}{m^3}$], where kg is kilogram and m^3 cubic meters.

Logistic probability function, $P(x, y, z, t) = \frac{1}{1+e^{kt-\mu r}}$, it is a real number $0 \leq P \leq 1$.

Equilibrium condition, $r = \frac{k}{\mu} t = \frac{p_0}{2\rho_0\nu\mu} t = |\mathbf{u}_e| t$, [m].

Fluid velocity in equilibrium, $|\mathbf{u}_e|$, [m/s].

Fluid field velocity out of equilibrium, $\mathbf{u} = -2\nu\mu(1-P)\nabla r$. [m/s].

Position, $r = (x^2 + y^2 + z^2)^{1/2}$, [m].

Attenuation coefficient, μ , [1/m].

Growth coefficient, $k = \frac{p_0}{2\rho_0 v}$, [1/s].

Concentration $C = C_0 \frac{1-P}{p}$.

Efficient Frontier.

Spherical surface of an implicit function $f(t, r) = 0$ of time t and radius $r = (x^2 + y^2 + z^2)^{1/2}$, which represents the solution set of the Navier-Stokes 3D equation. Every moving particle or fluid has energy measured with some standard deviation $\left\langle (E - \bar{E})^2 \right\rangle^{1/2}$. By the Heisenberg principle of uncertainty we know that there exists an unavoidable uncertainty in time $\left\langle (t - \bar{t})^2 \right\rangle^{1/2}$ given by: $\left\langle (E - \bar{E})^2 \right\rangle^{1/2} \left\langle (t - \bar{t})^2 \right\rangle^{1/2} \geq h/2\pi$, where h is the Plank constant.

Theorem 1 The velocity of the fluid is given by $\mathbf{u} = -2v \frac{\nabla P}{p}$, where $P(x, y, z, t)$ is the logistic probability function $P(x, y, z, t) = \frac{1}{1 + e^{kt - \mu r}}$, and p pressure such that $p = p_0 P$, both defined on $((x, y, z) \in \mathbb{R}^3, t \geq 0)$. The function P is the general solution of the Navier Stokes equations, which satisfies conditions (1) and (2).

Proof. To verify condition (2), $\nabla \cdot \mathbf{u} = 0$, we must calculate the gradients and laplacians of the radius. $\nabla r = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right)$, and $\nabla^2 r = \nabla \cdot \nabla r = \frac{(y^2+z^2)+(x^2+z^2)+(x^2+y^2)}{(x^2+y^2+z^2)^{3/2}} = \frac{2}{r}$.

$$\nabla \cdot \mathbf{u} = -2v \nabla \cdot \frac{\nabla P}{p} = -2v \mu \nabla \cdot ((1 - P) \nabla r) \quad (7)$$

Replacing the respective values for the terms: $\nabla^2 r$ and $|\nabla r|^2$ in the equation (7).

$$\begin{aligned} \nabla \cdot \mathbf{u} &= -2v \mu \nabla \cdot ((1 - P) \nabla r) \quad (8) \\ &= -2v \mu \nabla \cdot ((1 - P) \nabla r) \\ &= -2v \mu [-\mu(P - P^2) |\nabla r|^2 + (1 - P) \nabla^2 r] \end{aligned}$$

Where the gradient modulus of $\nabla P = \mu(P - P^2) \nabla r$, has the form $|\nabla P|^2 = \mu^2(P - P^2)^2 |\nabla r|^2 = \mu^2(P - P^2)^2$.

$$\nabla \cdot \mathbf{u} = -2v \mu (1 - P) \left[-\mu P + \frac{2}{r} \right] = 0 \quad (9)$$

Simplifying for $(1 - P) \neq 0$, we obtain the main result of this paper, which represents a fixed point of an implicit function $f(t, r)$ where $f(t, r) = P - \frac{2}{\mu r} = 0$. In Nuclear Physics, $r_0 < r < 1.2A^{1/3}$.

$$P = \frac{1}{1 + e^{kt - \mu(x^2+y^2+z^2)^{1/2}}} = \frac{2}{\mu(x^2+y^2+z^2)^{1/2}} \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \quad (10)$$

Equation (10) has a solution according to the fixed-point theorem of an implicit function, and it is a solution to the Navier Stokes stationary equations, which are summarized in: $\nabla^2 P = \frac{2}{\mu} \nabla^2 \left(\frac{1}{r} \right) = 0$. Furthermore, it is the typical solution of the Laplace equation for the pressure of the fluid $\nabla^2 p = p_0 \nabla^2 P = 0$. Kerson Huang (1987).

To this point, we need to verify that equation (10) is also a solution of requirement (1), $\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \nu \nabla^2 \mathbf{u} - \frac{\nabla p}{\rho_0}$. We will do the equivalence $\mathbf{u} = \nabla \theta$ after we replace in equation (1). Taking into account that $\theta = -2v \ln(P)$, and that $\nabla \theta$ is irrotational, $\nabla \times \nabla \theta = 0$, we have: $(\mathbf{u} \cdot \nabla) \mathbf{u} = (\nabla \theta \cdot \nabla) \nabla \theta = \frac{1}{2} \nabla (\nabla \theta \cdot \nabla \theta) - \nabla \theta \times (\nabla \times \nabla \theta) = \frac{1}{2} \nabla (\nabla \theta \cdot \nabla \theta)$, and $\nabla^2 \mathbf{u} = \nabla (\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u}) = \nabla (\nabla \cdot \nabla \theta) - \nabla \times (\nabla \times \nabla \theta) = \nabla (\nabla^2 \theta)$. Simplifying terms in order to replace these results in equation (1) we obtain

$$\begin{aligned} (\mathbf{u} \cdot \nabla) \mathbf{u} &= \frac{1}{2} \nabla (\nabla \theta \cdot \nabla \theta) = 2v^2 \nabla \left(\frac{|\nabla P|^2}{P^2} \right) \\ \nabla^2 \mathbf{u} &= \nabla (\nabla \cdot \mathbf{u}) = \nabla (\nabla^2 \theta) = 0 \end{aligned}$$

$$= -2\nu\nabla\left(\frac{|\nabla P|^2}{P^2} - \frac{\nabla^2 P}{P}\right) = 0$$

The explicit form of velocity is $\mathbf{u} = -2\mu\nu(1 - P)\nabla r$. Next, we need the partial derivative $\frac{\partial \mathbf{u}}{\partial t}$

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} &= -2\mu\nu k P(1 - P)\nabla r, \\ -\frac{\nabla p}{\rho_0} &= -\frac{\mu p_0}{\rho_0} P(1 - P)\nabla r. \end{aligned}$$

After replacing the last four results $(\mathbf{u} \cdot \nabla)\mathbf{u}$, $\nabla^2 \mathbf{u}$, $\frac{\partial \mathbf{u}}{\partial t}$ and $-\frac{\nabla p}{\rho_0}$ in equation (1) we obtain (11).

$$-2\mu\nu k P(1 - P)\nabla r = 2\nu^2 \nabla\left(\frac{|\nabla P|^2}{P^2}\right) - \frac{\mu p_0}{\rho_0} P(1 - P)\nabla r. \quad (11)$$

The equation (11) is equivalent to equation (1). After obtaining the term $\frac{|\nabla P|^2}{P^2}$ from the incompressibility equation $\nabla(\nabla^2 \theta) = -2\nu\nabla\left(-\frac{|\nabla P|^2}{P^2} + \frac{\nabla^2 P}{P}\right) = 0$ and replacing in equation (11).

$$-2\mu\nu k P(1 - P)\nabla r = 2\nu^2 \nabla\left(\frac{\nabla^2 P}{P}\right) - \frac{\mu p_0}{\rho_0} P(1 - P)\nabla r. \quad (12)$$

Equation (10) simultaneously fulfills requirements (1) expressed by equation (12) and requirement (2) expressed by equation (7), for a constant $k = \frac{p_0}{2\rho_0\nu} = \frac{p_0}{2\eta}$. Moreover, according to equation (10), the probability $P = \frac{2}{\mu r}$ which allows the Laplace equation to be satisfied: $\nabla^2 P = \frac{2}{\mu} \nabla^2\left(\frac{1}{r}\right) = 0$. In other words, the Navier-Stokes 3D equation system is solved.

Implicit Function.

An implicit function defined as (10), $f(t, r) = \frac{1}{1+e^{kt-\mu r}} - \frac{2}{\mu r} = 0$ has a fixed point (t, r) of $R = \{(t, r) | 0 < a \leq t \leq b, 0 < r < +\infty\}$, where m and M are constants, such as: $m \leq M$. Knowing that the partial derivative exists: $\partial_r f(t, r) = \nu P(1 - P) + \frac{2}{\mu r^2}$ we can assume that: $0 < m \leq \partial_r f(t, r) \leq M$. If, in addition, for each continuous function φ in $[a, b]$ the composite function $g(t) = f(t, \varphi(t))$ is continuous in $[a, b]$, then there is one and only one function: $r = \varphi(t)$ continuous in $[a, b]$, such that $f[t, \varphi(t)] = 0$ for all t in $[a, b]$.

Theorem 2 An implicit function defined as (10) $f(t, r) = \frac{1}{1+e^{kt-\mu r}} - \frac{2}{\mu r} = 0$ has a fixed point (t, r) of $R = \{(t, r) | 0 < a \leq t \leq b, 0 < r < +\infty\}$. In this way, the requirements (1) and (2) are fulfilled.

Proof. Let C be the linear space of continuous functions in $[a, b]$, and define an operator $T: C \rightarrow C$ by the equation:

$$T\varphi(t) = \varphi(t) - \frac{1}{M} f[t, \varphi(t)].$$

Then we prove that T is a contraction operator, so it has a unique fixed point $r = \varphi(t)$ in C . Let us construct the following distance.

$$T\varphi(t) - T\psi(t) = \varphi(t) - \psi(t) - \frac{f[t, \varphi(t)] - f[t, \psi(t)]}{M}.$$

Using the mean value theorem for derivation, we have

$$f[t, \varphi(t)] - f[t, \psi(t)] = \partial_\phi f(t, z(t))[\varphi(t) - \psi(t)].$$

Where $\phi(t)$ is situated between $\varphi(t)$ and $\psi(t)$. Therefore, the distance equation can be written as:

$$T\varphi(t) - T\psi(t) = [\varphi(t) - \psi(t)] \left[1 - \frac{\partial_\phi f(t, z(t))}{M}\right]$$

Using the hypothesis $0 < m \leq \partial_r f(t, r) \leq M$ we arrive at the following result:

$$0 \leq 1 - \frac{\partial_\phi f(t, \phi(t))}{M} \leq 1 - \frac{m}{M},$$

with which we can write the following inequality:

$$|T\varphi(t) - T\psi(t)| = |\varphi(t) - \psi(t)| \left(1 - \frac{m}{M}\right) \leq \alpha \|\varphi - \psi\|. \quad (13)$$

Where $\alpha = \left(1 - \frac{m}{M}\right)$. Since $0 < m \leq M$, we have $0 \leq \alpha < 1$. The above inequality is valid for all t of $[a, b]$. Where T is a contraction operator and the proof is complete, since for every contraction operator $T: C \rightarrow C$ there exists one and only one continuous function φ in C , such that $T(\varphi) = \varphi$. Using equation (10), which represents the fundamental solution of the Navier-Stokes 3D equation, we verify equation (2), which represents the second of the six requirements of an acceptable solution.

Proposition 3 Requirement (3). *The initial velocity can be obtained from: $\mathbf{u}(x, y, z, 0) = -2\nu \frac{\nabla P}{\rho}$, where each of the components u_x, u_y and u_z are infinitely derivable.*

$$\mathbf{u}(x, y, z, 0) = \mathbf{u}^0(x, y, z) = -2\nu\mu(1 - P_0) \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right) \quad ((x, y, z) \in \mathbb{R}^3) \quad (14)$$

$$P_0 = \frac{1}{1 + e^{-\mu r_0}}$$

Proof. Taking the partial derivatives of $\partial_x^n \left(\frac{x}{r}\right), \partial_y^n \left(\frac{x}{r}\right)$ and $\partial_z^n \left(\frac{x}{r}\right)$.

$$\partial_x^n \left(\frac{x}{r}\right) = n \partial_x^{n-1} \left(\frac{1}{r}\right) + x \partial_x^n \left(\frac{1}{r}\right) \quad (15)$$

$$\partial_y^n \left(\frac{x}{r}\right) = n \partial_y^{n-1} \left(\frac{1}{r}\right) + y \partial_y^n \left(\frac{1}{r}\right)$$

$$\partial_z^n \left(\frac{x}{r}\right) = n \partial_z^{n-1} \left(\frac{1}{r}\right) + z \partial_z^n \left(\frac{1}{r}\right)$$

Recalling the derivatives of special functions (Legendre), it is verified that there exists the derivative C^∞ .

$$\partial_x^n \left(\frac{1}{r}\right) = (-1)^n n! (x^2 + y^2 + z^2)^{-\frac{(n+1)}{2}} P_n \left(\frac{x}{(x^2 + y^2 + z^2)^{1/2}}\right) \quad (16)$$

$$\partial_y^n \left(\frac{1}{r}\right) = (-1)^n n! (x^2 + y^2 + z^2)^{-\frac{(n+1)}{2}} P_n \left(\frac{y}{(x^2 + y^2 + z^2)^{1/2}}\right)$$

$$\partial_z^n \left(\frac{1}{r}\right) = (-1)^n n! (x^2 + y^2 + z^2)^{-\frac{(n+1)}{2}} P_n \left(\frac{z}{(x^2 + y^2 + z^2)^{1/2}}\right)$$

Physically, this solution is valid for the initial velocity, indicated by Eq. (4), where the components of the initial velocity are infinitely differentiable, and make it possible to guarantee that the velocity of the fluid is zero when $r \rightarrow \infty$. [6,8,9]

Proposition 4 Requirement (4). *Using the initial velocity of a moving fluid given by $\mathbf{u}(x, y, z, 0) = \mathbf{u}^0(x, y, z) = -2\nu\mu(1 - P_0) \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$, it is evident that*

$$|\partial_x^\alpha \mathbf{u}^0| \leq C_{\alpha K} (1 + r)^{-K} \quad \text{on } \mathbb{R}^3 \text{ for any } \alpha \text{ and } K$$

Proof. Using the initial velocity of a moving fluid given by $\mathbf{u}^0(x, y, z) = -2\nu\mu(1 - P_0) \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$, we can find each of the components: $\partial_x^\alpha u_x^0, \partial_y^\alpha u_y^0$ and $\partial_z^\alpha u_z^0$.

$$\left(\partial_x^\alpha \frac{x}{r}\right)^2 = \left(\alpha \partial_x^{\alpha-1} \left(\frac{1}{r}\right) + x \partial_x^\alpha \left(\frac{1}{r}\right)\right) \left(\alpha \partial_x^{\alpha-1} \left(\frac{1}{r}\right) + x \partial_x^\alpha \left(\frac{1}{r}\right)\right)$$

For the three components x, y, z the results of the partial derivatives are as follows:

$$\left(\partial_x^\alpha \frac{x}{r}\right)^2 = \alpha^2 \left(\partial_x^{\alpha-1} \frac{1}{r}\right)^2 + 2\alpha x \partial_x^{\alpha-1} \frac{1}{r} \partial_x^\alpha \frac{1}{r} + x^2 \left(\partial_x^\alpha \frac{1}{r}\right)^2 \quad (17)$$

$$\left(\partial_y^\alpha \frac{y}{r}\right)^2 = \alpha^2 \left(\partial_y^{\alpha-1} \frac{1}{r}\right)^2 + 2\alpha y \partial_y^{\alpha-1} \frac{1}{r} \partial_y^\alpha \frac{1}{r} + y^2 \left(\partial_y^\alpha \frac{1}{r}\right)^2$$

$$\left(\partial_z^\alpha \frac{z}{r}\right)^2 = \alpha^2 \left(\partial_z^{\alpha-1} \frac{1}{r}\right)^2 + 2\alpha z \partial_z^{\alpha-1} \frac{1}{r} \partial_z^\alpha \frac{1}{r} + z^2 \left(\partial_z^\alpha \frac{1}{r}\right)^2$$

Replacing equation (17) with the explanatory form of the Legendre polynomials, for the following terms $\partial_x^{\alpha-1} \frac{1}{r}$ and $\partial_x^\alpha \frac{1}{r}$.

$$\begin{aligned} \partial_x^\alpha \frac{1}{r} &= (-1)^\alpha \alpha! (x^2 + y^2 + z^2)^{-\frac{(\alpha+1)}{2}} P_\alpha\left(\frac{x}{(x^2+y^2+z^2)^{1/2}}\right) (18) \\ \partial_x^{\alpha-1} \frac{1}{r} &= (-1)^{\alpha-1} (\alpha-1)! (x^2 + y^2 + z^2)^{-\frac{\alpha}{2}} P_{\alpha-1}\left(\frac{x}{(x^2+y^2+z^2)^{1/2}}\right) \end{aligned}$$

Also, knowing that for each $\alpha \geq 0$, the maximum value of $P_\alpha(1) = 1$. We can write the following inequality

$$\begin{aligned} x^2 \left(\partial_x^\alpha \left(\frac{1}{r} \right) \right)^2 &\leq x^2 (\alpha!)^2 r^{-2(\alpha+1)} (19) \\ 2\alpha x \partial_x^{\alpha-1} \frac{1}{r} \partial_x^\alpha \frac{1}{r} &\leq 2\alpha x (\alpha!) (\alpha-1)! (-1)^{2\alpha-1} r^{-2\alpha-1} \\ \alpha^2 \left(\partial_x^{\alpha-1} \left(\frac{1}{r} \right) \right)^2 &\leq \alpha^2 ((\alpha-1)!)^2 r^{-2\alpha} \end{aligned}$$

Grouping terms for $\left(\partial_x^\alpha \frac{x}{r}\right)^2$, $\left(\partial_y^\alpha \frac{y}{r}\right)^2$ and $\left(\partial_z^\alpha \frac{z}{r}\right)^2$ we have the next expressions.

$$\begin{aligned} \left(\partial_x^\alpha \frac{x}{r}\right)^2 &\leq r^{-2\alpha} \left[\frac{x^2(\alpha!)^2}{r^2} - \frac{2x(\alpha!)^2}{r} + \alpha^2((\alpha-1)!)^2 \right] (20) \\ \left(\partial_y^\alpha \frac{y}{r}\right)^2 &\leq r^{-2\alpha} \left[\frac{y^2(\alpha!)^2}{r^2} - \frac{2y(\alpha!)^2}{r} + \alpha^2((\alpha-1)!)^2 \right] \\ \left(\partial_z^\alpha \frac{z}{r}\right)^2 &\leq r^{-2\alpha} \left[\frac{z^2(\alpha!)^2}{r^2} - \frac{2z(\alpha!)^2}{r} + \alpha^2((\alpha-1)!)^2 \right] \end{aligned}$$

The module of $|\partial_x^\alpha \mathbf{u}^0|$ is given by $|\partial_x^\alpha \mathbf{u}^0| = \left(\left(\partial_x^\alpha \frac{x}{r}\right)^2 + \left(\partial_y^\alpha \frac{y}{r}\right)^2 + \left(\partial_z^\alpha \frac{z}{r}\right)^2 \right)^{1/2}$. Simplifying and placing the terms of equation (20) we have

$$|\partial_x^\alpha \mathbf{u}^0| \leq r^{-2\alpha} \left[3(\alpha!)^2 + \alpha^2((\alpha-1)!)^2 - \frac{2(x+y+z)(\alpha!)^2}{r} \right]$$

Taking into consideration that $\left|\frac{x}{r}\right| \leq 1$, $\left|\frac{y}{r}\right| \leq 1$, $\left|\frac{z}{r}\right| \leq 1$ the last term $|\partial_x^\alpha \mathbf{u}^0|$ can be easily written that.

$$\begin{aligned} |\partial_x^\alpha \mathbf{u}^0| &\leq \frac{2(\alpha!)^2}{r^{2\alpha}} \left[2 + \left|\frac{x}{r}\right| + \left|\frac{y}{r}\right| + \left|\frac{z}{r}\right| \right] \\ |\partial_x^\alpha \mathbf{u}^0| &\leq \frac{10(\alpha!)^2}{r^{2\alpha}} \end{aligned}$$

It is verified that there exists $C_\alpha = 10(\alpha!)^2$ such that if $r \rightarrow 0$, then $|\partial_x^\alpha \mathbf{u}^0| \rightarrow 0$. Thus, we proved requirement (4).

According to Mathematics, and giving an integral physical structure to the study, we need to prove that there are the spatial and temporal derivatives of the velocity and pressure components, satisfying the requirement (5).

Proposition 5 Requirement (5). The velocity can be obtained from: $\mathbf{u}(x, y, z, t) = -2v \frac{\nabla P}{P}$ and each of the components u_x, u_y and u_z are infinitely derivable.

$$\begin{aligned} \mathbf{u}(x, y, z, t) &= 2v^2 \left(\frac{x}{r^2}, \frac{y}{r^2}, \frac{z}{r^2} \right) \quad ((x, y, z) \in \mathbb{R}^3) (21) \\ P(x, y, z, t) &= \frac{1}{1 + e^{\frac{p_0 t}{2\eta} - \mu r}} = \frac{2}{\mu r} \end{aligned}$$

Proof. Taking partial derivatives for $\partial_x^n \left(\frac{x}{r^2}\right)$, $\partial_y^n \left(\frac{x}{r^2}\right)$ and $\partial_z^n \left(\frac{x}{r^2}\right)$.

$$\begin{aligned} \partial_x^n \left(\frac{x}{r^2}\right) &= n \partial_x^{n-1} \left(\frac{1}{r^2}\right) + x \partial_x^n \left(\frac{1}{r^2}\right) (22) \\ \partial_y^n \left(\frac{x}{r^2}\right) &= n \partial_y^{n-1} \left(\frac{1}{r^2}\right) + y \partial_y^n \left(\frac{1}{r^2}\right) \\ \partial_z^n \left(\frac{x}{r^2}\right) &= n \partial_z^{n-1} \left(\frac{1}{r^2}\right) + z \partial_z^n \left(\frac{1}{r^2}\right) \end{aligned}$$

Recalling the derivatives of special functions, it is verified that the derivative C^∞ exists. These derivatives appear as a function of the Legendre polynomials $P_n(\cdot)$.

$$\begin{aligned} \partial_x^n \left(\frac{1}{r^2}\right) &= (-1)^n n! (x^2 + y^2 + z^2)^{-(n+1)} P_n \left(\frac{x}{(x^2+y^2+z^2)}\right) (23) \\ \partial_y^n \left(\frac{1}{r^2}\right) &= (-1)^n n! (x^2 + y^2 + z^2)^{-(n+1)} P_n \left(\frac{y}{(x^2+y^2+z^2)}\right) \\ \partial_z^n \left(\frac{1}{r^2}\right) &= (-1)^n n! (x^2 + y^2 + z^2)^{-(n+1)} P_n \left(\frac{z}{(x^2+y^2+z^2)}\right) \end{aligned}$$

There are the spatial derivatives n and the time derivative which is similar to equations (25).

Proposition 6 Requirement (5). The pressure is totally defined by the equivalence $p(x, y, z, t) = p_0 P(x, y, z, t)$ and is infinitely differentiable in each of its components.

$$p(x, y, z, t) = p_0 P(x, y, z, t) \quad ((x, y, z) \in \mathbb{R}^3) (24)$$

Proof. Taking partial derivatives for $\partial_x^n \left(\frac{1}{r}\right)$, $\partial_y^n \left(\frac{1}{r}\right)$ and $\partial_z^n \left(\frac{1}{r}\right)$, recalling the derivatives of special functions of equation (16), it is shown that the derivative C^∞ . We only have to find the time derivatives: $\partial_t^n (p_0 P) = p_0 \partial_t^n (P)$. Using equation (21) for P , we have.

$$\begin{aligned} \partial_t P &= (-k)P(1 - P) (25) \\ \partial_t^2 P &= (-k)^2 (1 - 2P)P(1 - P) \\ \partial_t^3 P &= (-k)^3 (1 - 6P + 6P^2)P(1 - P) \\ \partial_t^4 P &= (-k)^4 (1 - 14P + 36P^2 - 24P^3)P(1 - P) \\ \partial_t^5 P &= (-k)^5 (1 - 30P + 150P^2 - 240P^3 + 120P^4)P(1 - P) \\ \partial_t^n (P) &= \partial_t (\partial_t^{n-1} (P)) \end{aligned}$$

It is always possible to find the derivative $\partial_t^n (P)$ as a function of the previous derivative, since the resulting polynomial of each derivative $n - 1$ is of degree n .

Proposition 7 Requirement (6). The energy must be limited in a defined volume and fundamentally it must converge at any time, such that $t \geq 0$.

$$\int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz \leq C \quad \text{for all } t \geq 0 \quad (\text{bounded energy}).$$

Proof. We will use the explicit form of velocity given in equation (21) $\mathbf{u}(x, y, z, t) = 2\nu\mu(1 - P)\nabla r$, to obtain the vector module: $|\mathbf{u}|^2 = 4\nu^2\mu^2(1 - P)^2$. Rewriting equation (21), and applying a change of variable in: $dx dy dz = 4\pi r^2 dr$.

$$\int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz = 16\pi\nu^2\mu^2 \int_{r_0}^{\infty} r^2 (1 - P)^2 dr (26)$$

Making another change of variable $dP = \mu P(1 - P)dr$. Using (10), replacing $r^2 = \left(\frac{2}{\mu P}\right)^2$ we have

$$\begin{aligned} \int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz &= 16\pi\nu^2\mu^2 \int_{P_0}^{P_\infty} \left(\frac{2}{\mu P}\right)^2 (1 - P)^2 \frac{dP}{\mu P(1 - P)} (27) \\ &= \frac{64\pi\nu^2}{\mu} \int_{P_0}^{P_\infty} \frac{1 - P}{P^3} dP \end{aligned}$$

Where radius $r \rightarrow \infty$, when $t \geq 0$, we have $\lim_{r \rightarrow \infty} P = \lim_{r \rightarrow \infty} \frac{1}{1 + \frac{\exp(kt)}{\exp(\mu r)}} = P_\infty = 1$. Moreover, physically if $r \rightarrow r_0 \approx 0$ then $t \rightarrow 0$ we have $\lim_{r \rightarrow 0} P = \lim_{r \rightarrow 0} \frac{1}{1 + \frac{\exp(kt)}{\exp(\mu r)}} = P_0 = \frac{1}{2}$. Here, a probability $\frac{1}{2}$ represents maximum entropy.

$$\int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz = \frac{64\pi v^2}{\mu} \int_{1/2}^1 \frac{1-P}{P^3} dP = \frac{64\pi v^2}{\mu} \left[\frac{2P-1}{2P^2} \right]_{1/2}^1 \quad (28)$$

$$\int_{\mathbb{R}^3} |\mathbf{u}|^2 dx dy dz \leq \frac{32\pi v^2}{\mu} \quad \text{for all } t \geq 0$$

In this way the value of the constant C is $C = \frac{32\pi v^2}{\mu}$. Verifying the proposition (6) completely. In general, equation (10) can be written $f(t, r + r_0) = \frac{1}{1 + e^{kt - \mu(r+r_0)}} - \frac{2}{\mu(r+r_0)} = 0$ and in this way discontinuities are avoided when $r \rightarrow 0$, but this problem does not occur since in the atomic nucleus $r_0 < r < 1.2A^{1/3}$ is satisfied.

Lemma 8 *The irrotational field represented by the logistic probability function $P(x, y, z, t)$ associated with the velocity $\mathbf{u} = -2v \frac{\nabla P}{P}$, can produce vortices, due to the stochastic behavior of the physical variables p_0, η, μ . These stochastic variations are in orders lower than the minimum experimental value.*

Proof. The implicit function representing the solution of the Navier-Stokes 3D equation, $\frac{1}{1 + e^{-\mu(r - \frac{k}{\mu}t)}} = \frac{2}{\mu r}$ depends on the values of initial pressure p_0 , viscosity v and attenuation coefficient μ . Due to Heisenberg uncertainty principle, these parameters have a variation when we measure and use them, as is the case of the estimate of $\xi = r - \phi(t, k, \mu)$. The function $\phi(t, k, \mu) = \frac{k}{\mu}t$, expressly incorporates these results, when $-\infty < \xi < +\infty$. The physical and mathematical realities are mutually conditioned and allow for these surprising results. For a definite t there exist infinities (x, y, z) that hold the relationship $r = (x^2 + y^2 + z^2)^{1/2}$. Moreover, for a definite r there are infinities t that respect the fixed-point theorem and create spherical trajectories. When the physical variables k, μ vary, even at levels of 1/100 or 1/1000, they remain below the minimum variation of the experimental value. We could try to avoid the existence of trajectories on the spherical surface, for which we must assume that the fluid is at rest or it is stationary, which contradicts the Navier-Stokes 3D equation, where all fluid is in accelerated motion $\frac{\partial \mathbf{u}}{\partial t} \neq 0$. In short, if there are trajectories in the sphere as long as it is probabilistically possible, this is reduced to showing that the expected value of the radius $E[r|r \geq 0]$ exists and it is finite.

Derivation of $E(r|r \geq 0)$.

The logistic density function for ξ when $E(\xi) = 0$ and $Var(\xi) = \sigma^2$ is defined by

$h(\xi) = \frac{\mu \exp(-\xi)}{[1 + \exp(-\xi)]^2}$, where $\frac{1}{\mu} = \sigma\sqrt{3}/\pi$ is a scale parameter. Given that $r = \phi(t, p_0, \eta, \mu) + \xi$ function for r is then $f(r) = \frac{\exp[-r - \phi(\bullet)/\tau]}{\tau[1 + \exp(-(r - \phi(\bullet))/\tau)]^2}$, to facilitate the calculations we put $\phi(\bullet) = \phi(t, k, \mu) = \frac{k}{\mu}t$. By definition, the truncated density for r when $r \geq 0$ is given by $f(r|r \geq 0) = \frac{f(r)}{P(r \geq 0)}$ for $r \geq 0$. Given that the cumulative distribution function for r is given by $F(r) = \frac{1}{1 + \exp(kt - \mu r)}$, it follows that $P(r \geq 0) = 1 - F(0) = \frac{\exp(\phi(\bullet))}{1 + \exp(\phi(\bullet))} = \frac{1}{1 + \exp(-\phi(\bullet))}$. The derivation of $E(r|r \geq 0)$ then proceeds as follows:

$$E(r|r \geq 0) = \int_0^\infty \mu r f(r|r \geq 0) dr = \frac{1}{P(r \geq 0)} \int_0^\infty \mu r \frac{\exp[kt - \mu r]}{[1 + \exp[kt - \mu r]]^2} dr \quad (29)$$

$$E(r|r \geq 0) = \frac{1}{P(r \geq 0)} \int_{1/2}^1 \frac{2}{\mu P} (\mu P(1 - P)) \frac{dP}{P(1 - P)}$$

We replaced in equation (29) $dP = \mu P(1 - P)dr$ and $r^2 = \left(\frac{2}{\mu P}\right)^2$ of this manner we obtain

$$E(r|r \geq 0) = \frac{1}{P(r \geq 0)} \int_{1/2}^1 \frac{2}{\mu P} (\mu P(1 - P)) \frac{dP}{P(1 - P)} = \frac{1}{P(r \geq 0)} \frac{2}{\mu} \log(2) \quad (30)$$

where we have used the fact that

$$P(r \geq 0) = \frac{\exp kt}{1 + \exp kt}$$

$$E(r|r \geq 0) = \frac{1}{P(r \geq 0)} \frac{2}{\mu} \log(2) \leq \frac{2}{\mu} \log(2) \quad \text{for all } t \geq 0 \quad (31)$$

where the last equality follows from an application of the L'Hopital's rule $P(r \geq 0) = \lim_{t \rightarrow \infty} \frac{\exp kt}{1 + \exp kt} = 1$.

2.2 The nuclear force and the Navier Stokes force are the same.

Firstly, we will use the concepts of Classic Mechanics and the formulation of the Yukawa potential, $\Phi(r) = \frac{g}{4\pi r} (A - 1) e^{-\mu r}$ to find the nuclear force exerted on each nucleon at interior of the atomic core $\mathbf{F}_N = -\nabla\Phi(r)$. Also, replace the terms of the potential $e^{-\mu r} = \frac{1-P}{P}$ and $\frac{1}{r} = \frac{\mu}{2} P$ by the respective terms already obtained in equation (10).

$$\Phi(r) = \frac{g(A-1)}{4\pi r} e^{-\mu r} = \frac{g\mu(A-1)}{8\pi} (1 - P) \quad (32)$$

The general form of the equation (32), is a function of (x, y, z, t) .

$$\Phi(r, t) = \frac{g(A-1)}{4\pi r} e^{kt - \mu r} = \frac{g\mu(A-1)}{8\pi} (1 - P(r, t)) \quad (33)$$

Secondly, we will obtain the Navier Stokes force equation given by:

$$\frac{d\mathbf{u}}{dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \frac{\partial \mathbf{u}}{\partial t} + 2\nu^2 \nabla \left(\frac{|\nabla P|^2}{P^2} \right) = -2\mu\nu k P (1 - P) \nabla \mathbf{r} \quad (34)$$

Theorem 9 *The Nuclear Force and Navier Stokes Force are proportional inside the atomic nucleus $\mathbf{F}_N = C \mathbf{F}_{NS}$.*

Proof. Equation (34) rigorously demonstrated by theorems and propositions 1 through 8, represent the acceleration of a particle within the atomic nucleus. According to Classical Mechanics the force of Navier Stokes applied to a particle of mass m , would have the form:

$$\mathbf{F}_{NS} = m \frac{d\mathbf{u}}{dt} = -2m\mu\nu k P (1 - P) \nabla \mathbf{r}. \quad (35)$$

Proof. The nuclear force on its part would be calculated as follows $\mathbf{F}_N = -\nabla\Phi(r)$.

$$\mathbf{F}_N = -\nabla\Phi(r) = -\frac{g\mu}{8\pi} (A - 1) \nabla P. \quad (36)$$

Replacing the term $\nabla P = \mu(P - P^2) \nabla r$ of equation (7), we obtain

$$\mathbf{F}_N = -\nabla\Phi(r) = -\frac{g\mu^2}{8\pi} (A - 1) P (1 - P) \nabla \mathbf{r}. \quad (37)$$

It is possible to write nuclear force as a function of speed.

$$\mathbf{F}_N = -\nabla\Phi(r) = -\frac{g\mu^2}{8\pi} (A - 1) P (1 - P) \nabla \mathbf{r}.$$

Finally, we can show that the nuclear force and force of Navier Stokes differ at most in a constant C . Equating (35) and (37), we find the value g as a function of the parameters nuclear viscosity ν , attenuation μ and growth coefficient of the nuclear reaction k , nucleon mass m and $C \neq 1$.

$$g = \frac{16m\pi\nu k}{\mu(A-1)} C \quad (38)$$

Nuclear stability.

First we will rewrite the probability that solves the Navier Stokes equation, meets the potential of Yukawa and represents the probability distribution law of Fermi Dirac. This equation is in the reference system of the atomic nucleus.

$$P = \frac{1}{1 + e^{kt - \mu r}} = \frac{2}{\mu r}$$

Using the classical definition of half-life in the laboratory reference system, we have $C = C_0 e^{-\lambda\tau}$, where $\frac{C_0}{2} = C_0 e^{-\lambda\tau}$

$$\tau_{1/2} = \frac{\ln(2)}{\lambda}$$

According to special relativity, the relation of time τ in the laboratory system, with respect to the time measured in the reference system of the atomic nucleus is $t = \frac{\tau_{1/2}}{\sqrt{1 - (\frac{u}{c})^2}}$.

The concentration is expressed as a function of time and space in the following way $C = C_0 \frac{P}{1-P}$. To give a systematic consistency to the whole theory is necessary $P = 1/3$. Redefining own average life time $t_{1/2}$ we must also redefine the radius of the alpha particle in the case of alpha decay $r_{1/2}$ of this manner $\frac{C_0}{2} = C_0 \frac{1-P}{P}$ where $P = 1/3$.

$$\begin{aligned} kt_{1/2} - \mu r_{1/2} &= \ln(2) \\ \mu r_{1/2} &= 3 \end{aligned}$$

2.3 Cross Section and Golden Ratio $\left(\frac{\sigma_1}{\sigma_2}\right)$.

According to NIST and GEANT4, current tabulations of $\frac{\mu}{\rho}$ rely heavily on theoretical values for the total cross section per atom, σ_{tot} , which is related to $\frac{\mu}{\rho}$ by the following equation:

$$\frac{\mu}{\rho} = \frac{\sigma_{tot}}{uA} \quad (39)$$

In (eq. 3), $u (= 1.6605402 \times 10^{-24} \text{ gr})$ is the atomic mass unit (1/12 of the mass of an atom of the nuclide 12C) ⁴.

The attenuation coefficient, photon interaction cross sections and related quantities are functions of the photon energy. The total cross section can be written as the sum over contributions from the principal photon interactions $\sigma_{tot} = \sigma_{pe} + \sigma_{coh} + \sigma_{incoh} + \sigma_{trip} + \sigma_{ph.n}$ (40)

Where σ_{pe} is the atomic photo effect cross section, σ_{coh} coh and σ_{incoh} are the coherent (Rayleigh) and the incoherent (Compton) scattering cross sections, respectively, σ_{pair} and σ_{trip} are the cross sections for electron-positron production in the fields of the nucleus and of the atomic electrons, respectively, and $\sigma_{ph.n}$ is the photonuclear cross section^{3,4}.

We use data of NIST and simulations with GEANT4 for elements $Z=11$ to $Z=92$ and photon energies 1.0721E-03 MeV to 1.16E-01 MeV, and have been calculated according to:

$$\frac{\mu}{\rho} = (\sigma_{pe} + \sigma_{coh} + \sigma_{incoh} + \sigma_{trip} + \sigma_{ph.n}) / \mu A \quad (41)$$

The attenuation coefficient μ of a low energy electron beam [10,100]eV will essentially have the elastic and inelastic components. It despises Bremsstrahlung emission and Positron annihilation.

$$\sigma_{tot} = \sigma_{coh} + \sigma_{incoh} \quad (42)$$

Resonance region for photons.

A resonance region is created in a natural way at the K-shell between the nucleus and the electrons at S-level. The condition for the photons to enter in the resonance region is given by $r_a \geq r_n + \lambda$. This resonance region gives us a new way to understand the photoelectric effect. There is experimental evidence of the existence of resonance at K-level due to photoelectric effect, represented by the resonance cross section provided by NIST and calculated with GEANT4 for each atom. In the present work we focus on the resonance effects but not on the origin of resonance region.

The resonance section is responsible for large and / or anomalous variations in absorbed radiation $(I_2 - I_1)$.

$$\frac{I_2 - I_1}{\frac{I_2 + I_1}{2}} = -\frac{\rho r}{\mu A} (\sigma_2 - \sigma_1) \quad (43)$$

Theorem 10 *Resonance region. The resonance cross section is produced by interference between the atomic nucleus and the incoming X-rays inside the resonance region, where the boundaries are the surface of the atomic nucleus and K-shell.*

The cross section of the atomic nucleus is given by:

$$\sigma_{r_n} = 4\pi r_n^2 = 4\pi A^{2/3} r_n^2 \quad (44)$$

The photon cross section at K-shell depends on the wave length and the shape of the atomic nucleus:

$$\sigma_{r_n+\lambda} = 4\pi(r_n + \lambda)^2 \quad (45)$$

Subtracting the cross sections (13) and (14) we have:

$$\sigma_\lambda = \sigma_{r_n+\lambda} - \sigma_{r_n} = 4\pi(2r_n\lambda + \lambda^2) = 4\pi(2r_p\lambda + 2(r_n - r_p)\lambda + \lambda^2) \quad (46)$$

The resonance is produced by interactions between the X-rays, the K-shell electrons and the atomic nucleus. The cross sections corresponding to the nucleus is weighted by probability pn and should have a simple dependence of an interference term. This last depends on the proton radius rp or the difference between the nucleus and proton radius ($r_n - r_p$) according to the following relation

$$\text{Max}(\sigma_2 - \sigma_1) = \left(\frac{\sigma_1}{\sigma_2}\right)^{-b} (\sigma_2 - \sigma_1) = 4\pi(2r_p\lambda) \quad (47)$$

We note that left hand side of equations (46) and (47) should have a factor larger than one due to resonance. The unique factor that holds this requirement is $\left(\frac{\sigma_1}{\sigma_2}\right)^b$ Where a, b constants.

$$\frac{8\pi\bar{r}\lambda}{(\sigma_2 - \sigma_1)} = a \left(\frac{\sigma_1}{\sigma_2}\right)^b \quad (48)$$

After performing some simulations it is shown that the thermal a represents the dimensionless Rydberg constant $a = R_\infty = 1.0973731568539 * 10^7$.

$$\frac{8000\pi\bar{r}\lambda}{(\sigma_2 - \sigma_1)} = R_\infty \left(\frac{\sigma_1}{\sigma_2}\right)^{2.5031} \quad (49)$$

The radius of the neutron can be obtained using equation (49) in the following way.

$$\bar{r} = \frac{R_\infty}{8000\pi a} \quad (49)$$

$$r_N = \frac{A}{N}\bar{r} + \frac{Z}{N}r_p$$

2.4 Navier Stokes Equation and Cross Section in Nuclear Physics.

The speed needs to be defined as $\mathbf{u} = -2v\frac{\nabla P}{P}$, where $P(x, y, z, t)$ is the logistic probability function $P(x, y, x, t) = \frac{1}{1+e^{kt-\mu}}$, $r = (x^2 + y^2 + z^2)^{1/2}$ defined in $((x, y, z) \in \mathbb{R}^3, t \geq 0)$ This P is the general solution of the Navier Stokes 3D equations, which satisfies the conditions (50) and (51), allowing to analyze the dynamics of an incompressible fluid.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\nabla p}{\rho_0} \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \quad (50)$$

Where, $\mathbf{u} \in \mathbb{R}^3$ an known velocity vector, ρ_0 constant density of fluid and pressure $p = p_0 P \in \mathbb{R}$.

With speed and pressure dependent on r and t . We will write the condition of incompressibility as follows.

$$\nabla \cdot \mathbf{u} = 0 \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \quad (51)$$

Theorem 11 The velocity of the fluid given by: $\mathbf{u} = -2v\frac{\nabla P}{P}$, where $P(x, y, z, t)$ is the logistic probability function $P(x, y, x, t) = \frac{1}{1+e^{kt-\mu(x^2+y^2+z^2)^{1/2}}}$, defined in $((x, y, z) \in \mathbb{R}^3, t \geq 0)$ is the general solution of the Navier Stokes equations, which satisfies conditions (50) and (51).

Proof. Firstly, we will make the equivalence $\mathbf{u} = \nabla\theta$ and replace it in equation (50). Taking into account that $\nabla\theta$ is irrotational, $\nabla \times \nabla\theta = 0$, we have.

$$(\mathbf{u} \cdot \nabla)\mathbf{u} = (\nabla\theta \cdot \nabla)\nabla\theta = \frac{1}{2}\nabla(\nabla\theta \cdot \nabla\theta) - \nabla\theta \times (\nabla \times \nabla\theta) = \frac{1}{2}\nabla(\nabla\theta \cdot \nabla\theta),$$

We can write,

$$\nabla \left(\frac{\partial \theta}{\partial t} + \frac{1}{2} (\nabla \theta \cdot \nabla \theta) \right) = \nabla(-p)$$

It is equivalent to,

$$\frac{\partial \theta}{\partial t} + \frac{1}{2} (\nabla \theta \cdot \nabla \theta) = -\frac{\Delta p}{\rho_0}$$

where Δp is the difference between the actual pressure p and certain reference pressure p_0 . Now, replacing $\theta = -2\nu \ln(P)$, Navier Stokes equation becomes.

$$\frac{\partial P}{\partial t} = \frac{\Delta p}{\rho_0} P \quad (52)$$

The external force is zero, so that there is only a constant force F due to the variation of the pressure on a cross section σ . Where $.1in.1in$ is the total cross section of all events that occurs in the nuclear surface including: scattering, absorption, or transformation to another species.

$$F = p\sigma_2 = p_0\sigma_1 \quad (53)$$

$$\Delta p = p - p_0 = \left(\frac{\sigma_1}{\sigma_2} - 1 \right) p_0 = -(1 - P)p_0$$

putting (52) in (53) we have

$$\frac{\partial P}{\partial t} = -\mu k(1 - P)P \quad (54)$$

In order to verify equation (51), $\nabla \cdot \mathbf{u} = 0$, we need to obtain

$$\nabla r = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right), \nabla^2 r = \nabla \cdot \nabla r = \frac{(y^2+z^2)+(x^2+z^2)+(x^2+y^2)}{(x^2+y^2+z^2)^{3/2}} = \frac{2}{r}$$

$$\nabla \cdot \mathbf{u} = -2\nu \nabla \cdot \frac{\nabla P}{P} = -2\nu \mu \nabla \cdot ((1 - P)\nabla r) \quad (55)$$

Replacing the respective values for the terms: $\nabla^2 P$ and $|\nabla P|^2$ of equation (55). The Laplacian of P can be written as follows.

$$\begin{aligned} \nabla^2 P &= \mu(1 - 2P)\nabla P \cdot \nabla r + \mu(P - P^2)\nabla^2 r \quad (56) \\ &= \mu^2(1 - 2P)(P - P^2)|\nabla r|^2 + \mu(P - P^2)\nabla^2 r \\ &= \mu^2(1 - 2P)(P - P^2) + \mu(P - P^2)\frac{2}{r} \end{aligned}$$

Using gradient $\nabla P = \mu(P - P^2)\nabla r$, modulus $|\nabla P|^2 = \mu^2(P - P^2)^2|\nabla r|^2$ and $\nabla^2 P$ in (56).

$$\left[\frac{\nabla^2 P}{P} - \frac{|\nabla P|^2}{P^2} \right] = 0 \quad (57)$$

Replacing equations (55) and (56) in (57) we obtain the main result of the Navier Stokes equations, the solution represents a fixed point of an implicit function $f(t, r)$ where $f(t, r) = P - \frac{2}{\mu r} = 0$.

$$P = \frac{1}{1 + e^{kt - \mu(x^2 + y^2 + z^2)^{1/2}}} = \frac{2}{\mu(x^2 + y^2 + z^2)^{1/2}} \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \quad (58)$$

III. DISCUSSION OF RESULTS.

Spherical surfaces does not imply spacetime curve

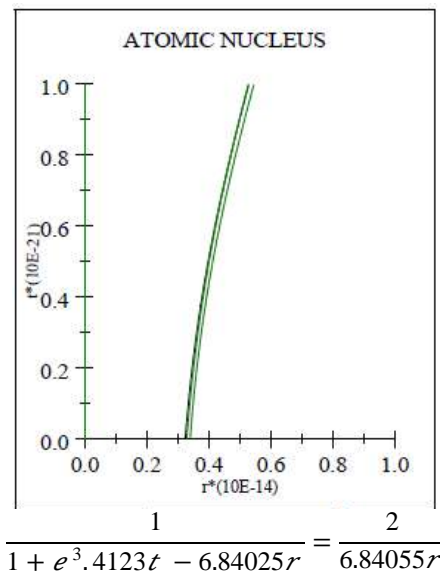
In Fefferman (2017), "In particular, the singular set \mathbf{u} cannot contain a spacetime curve of the form $\{(x, t) \in \mathbb{R}^3 \times \mathbb{R} : x = \phi(t)\}$. This is the best partial regularity theorem known so far the Navier-Stokes equation. It appears to be very hard to go further"

Using equation (10), which is an implicit function of a fixed point $\frac{1}{1 + e^{\frac{p_0}{2\eta}t - \mu r}} = \frac{2}{\mu r}$ it is possible to obtain spherical trajectories, although this does not necessarily imply a spacetime curve. For this case we must clearly rewrite $r = \sqrt{x^2 + y^2 + z^2}$ and $\{(x, y, z, t) \in \mathbb{R}^3 \times \mathbb{R}^+ : r = \phi(t)\}$. Where a natural constant $\frac{p_0}{2\eta\mu}$ appears, which in the

equilibrium satisfies $r = \frac{p_0}{2\eta\mu} t$ and has a representation similar to the Reynolds number.

Solution algorithm

1. We define a value of the initial pressure p_0 and of viscosity ν in atomic nucleus. These variables are determined by instruments with a certain degree of accuracy. However, the measurement of a physical variable always has an uncertainty independent of the accuracy of the instruments. Intrinsic uncertainty is determined by Heisenberg’s uncertainty principle.
2. The function that solves the Navier-Stokes 3D equation is a logistic probability density (Fermi Dirac probability function). The attenuation coefficient of incident molecules is μ which has a positive value, while the coefficient that weighs the evolution in time depends on pressure and dynamic viscosity $\frac{p_0}{2\eta}$. $P(x, y, z, t) = \frac{1}{1 + e^{\frac{p_0}{2\eta}t - \mu r}}$
3. For each time value t there exists a fixed point $r(x, y, z, t)$ that allows to fully comply with the Navier-Stokes 3D equation. Where $-1 \leq \frac{x}{r} \leq +1, -1 \leq \frac{y}{r} \leq +1$ and $-1 \leq \frac{z}{r} \leq +1$.
4. If we determine experimentally the value of variation of the parameters p_0, ν, μ , we find the average value of r and the standard deviation of r , determining a spherical cap in which the Navier-Stokes 3D equation are satisfied.
5. We calculate probabilities $P(x, y, z, t)$, concentrations $C(x, y, z, t)$, pressures $p = p_0 P$ and the velocity field $\mathbf{u} = -2\nu \frac{\nabla p}{\rho}$ of fluids. Also, when we apply the respective algorithm, we clearly see how for a definite radius r that we find an interval with infinite time values $t \in \left[\bar{t} - \left((t - \bar{t})^2 \right)^{1/2} \leq t \leq \bar{t} + \left((t - \bar{t})^2 \right)^{1/2} \right]$, even for variations of less than 1% in the parameters $\mu = 0.2, \nu = 0.02, p_0 = 1$. In the following figure: green color indicates minimal experimental variations in parameters superior to 1%, while red color indicates variations parameters less than 1%.



3.1 Nuclear Viscosity

It is created by the friction between the layers of protons and the sphere of neutrons that make up the atomic nucleus.

Theorem 12 *An action on the nuclear surface produces a reaction in the nuclear volume and vice versa.*

Proof. The volume and the nuclear surface are connected through the Gaussian divergence theorem and the Navier Stokes equations. For an incompressible fluid, whose velocity field $\vec{u}(x, y, z)$ is given, $\vec{\nabla} \cdot \vec{u} = 0$ is fulfilled. Logically, the integral of this term remains zero, that is:

$$\int \int \int \vec{\nabla} \cdot \vec{u} \, dx \, dy \, dz = 0$$

Writing the Divergence theorem. $\int \int \vec{u} \cdot \vec{n} \, dS = \int \int \int \vec{\nabla} \cdot \vec{u} \, dx \, dy \, dz = 0$, the first term must be equal to

zero, that is:

$$\int \int \vec{u} \cdot \vec{n} \, dS = \int \int \|\vec{u}\| \|\vec{n}\| \cos(\alpha) \, dS = 0 \rightarrow \alpha = \frac{\pi}{2}$$

The only possible trajectory is circular, because in this case the vector \vec{n} is perpendicular to the surface of the sphere. In this way the equation of the outer sphere corresponding to the surface is: $x^2 + y^2 + z^2 = 1.2A^{1/3}$. Within the nuclear fluid there are layers of nucleons that move in spherical trajectories.

Remark 13 *The information in the volume of the inner layer that contains the neutrons $\int \int \int \vec{V} \cdot \vec{u} \, dx \, dy \, dz$, is equal to the information of the outer layer that contains the protons $\int \int \vec{u} \cdot \vec{n} \, dS$.*

Theorem 14 *In the atomic nucleus, the angular moments of the neutron sphere and the proton layer are canceled $I_n w_n - I_p w_p = 0$.*

The angular momentum in a classical way is given by $L = I_n w_n - I_p w_p$, where $I_n = \frac{2}{5}(1.2)^2 N^{5/3} m_n$ is the angular momentum of the neutron sphere with its respective angular velocity w_n while for protons we have $I_p = \frac{2}{5} m_p (1.2)^2 [(N+Z)^{5/3} - N^{5/3}]$. Clearing $\frac{w_n}{w_p}$ of the equation $I_n w_n - I_p w_p = 0$ you get a theoretical result given by

$$\frac{w_n}{w_p} = \left(\left(\frac{N+Z}{N} \right)^{5/3} - 1 \right) \frac{m_p}{m_n}$$

It is easy to obtain an experimental result based on the measured energies of the protons and neutrons that leave the atomic nucleus, where: $\frac{w_n}{w_p} = \sqrt{\frac{S_n m_p}{S_p m_n}}$. In this relation $\frac{w_n}{w_p}$, we take into consideration that the protons and neutrons that are in contact leave the same place because of the action of the nuclear viscosity and the centrifugal force.

Remark 15 *Protons, neutrons and alpha particles leave the atomic nucleus by joint action of nuclear viscosity and the centrifugal force. The escape energy of protons S_p and the neutrons S_n are respectively the maximum allowed by the nuclear viscosity, in such a way that when an alpha particle comes out, it must have an energy functionally defined by the energies of protons and neutrons.*

3.2 Maximum number of protons Z^* , on the nuclear surface.

The radius of the proton appears naturally, when all the elements of the periodic table are analyzed, under the model of two fully established physical layers, subject to the following restriction.

$$1.2A^{1/3} \geq 1.2N^{1/3} + r_p$$

The nuclear surface houses all the protons, which are at a maximum distance from each other, fulfilling the following inequality.

$$4\pi(1.2A^{1/3})^2 \geq Z\pi(r_p)^2$$

When the previous inequalities are transformed into equalities, we obtain the maximum number of protons on the nuclear surface and the average radius of the proton. The study of the whole periodic table shows that $3.4 < \frac{Z^*}{Z} < 4.5$, for $Z \geq 20$.

Proposition 16 *Minimum energy implies maximum entropy in the probability of spins.*

Proof. Be x the probability associated with the spin $s = 1/2$ and $(1-x)$ the probability associated to the spin $s = -1/2$ we can define entropy as $H = x \ln(1/x) + (1-x) \ln(1/(1-x))$ the maximum value $\max(H) = \ln(2)$ has to be calculated for $x = 1/2$. Which indicates that the spins in pairs are always annulled independently for protons and neutrons.

Nuclide	Z	N	A	α, β	Product	$t_{1/2}$ [s]	λ [1/s]	E_α (MeV)	u_α [m/s]	$P = v/\eta_0$	v (m ² /s) = $u_\alpha u_\alpha$	ν (m ² /s)	F_α [N]	F_{vis} [N]	$C = F_\alpha/F_{vis}$
Th ²³²	90	142	232	α	Ra ²²⁸	4.4790E+17	1.5475E-18	4.0810	13961140	0.9	9.7728E-08	9.773E+22	2424	1.2989E-37	1.87E+40
Ra ²²⁶	88	144	232	β^-	Ac ²²⁶	1.7729E+08	3.9006E-09	0.0460	1462233.1	0.9	1.0370E-06	1.038E+22	2362	3.4839E-29	6.78E+31
Ac ²²⁸	89	139	228	β^-	Th ²²⁸	2.1960E+04	3.1564E-05	2.1240	10071980	0.9	7.0504E-06	7.05E+22	2321	1.9113E-24	1.21E+27
Th ²³⁰	90	138	228	α	Ra ²²⁶	5.9098E+07	1.1729E-08	5.5200	16237050	0.9	1.1866E-07	1.137E+23	2321	1.1449E-27	2.03E+30
Ra ²²⁴	88	136	224	α	Rn ²²⁰	3.1104E+05	2.2285E-06	5.7890	16627979	0.9	1.164E-07	1.164E+23	2279	2.2278E-25	1.02E+28
Rn ²²⁰	86	134	220	α	Po ²¹⁶	5.5000E+01	1.2603E-02	6.4040	17488930	0.9	1.2242E-07	1.224E+23	2238	1.3251E-21	1.69E+24
Po ²¹⁶	84	132	216	α	Pb ²¹²	1.4000E-01	4.9511E+00	6.9060	5217650.9	0.9	3.6524E-08	3.652E+22	2196	1.5531E-19	1.41E+22
Pb ²¹²	82	130	212	β^-	Bi ²¹²	3.8160E+04	1.8164E-05	0.5700	10371027	0.9	7.2597E-08	7.26E+22	2153	1.1320E-24	1.90E+27
Bi ²¹²	83	129	212	β^- (64.06%)	Po ²¹²	3.6600E+03	1.8938E-04	2.2520	17219219	0.9	1.2053E-07	1.205E+23	2155	1.9605E-23	1.10E+26
Bi ²¹²	83	129	212	α (35.94%)	Tl ²⁰⁸	3.6600E+03	1.8938E-04	6.2080	17219219	0.9	1.2053E-07	1.205E+23	2113	1.9605E-23	1.08E+26
Tl ²⁰⁸	81	127	208	α	Pb ²⁰⁴	1.8000E+02	3.7260E-03	8.9550	20680940	0.9	1.4477E-07	1.448E+23	2155	4.6334E-22	4.63E+24
Po ²¹²	84	128	212	α	Pb ²⁰⁸	3.0000E-07	2.3105E+06	4.9990	15451803	0.9	1.0916E-07	1.082E+23	2155	2.1464E-13	1.00E+16
Pb ²⁰⁸	82	126	208	stable

Table 1.- Obtaining the nuclear viscosity from the speed of the alpha particles in the disintegration of Thorium.

IV. CONCLUSIONS

turbulent flows and vortices in atomic nucleus.

The necessary condition for the existence of turbulent flows occurs when the velocity of the nuclear fluid $|\mathbf{u}| > |\mathbf{u}_e| = \frac{p_0}{2\eta\mu}$ is greater than the equilibrium velocity $|\mathbf{u}_e|$, obtained as a function of the parameters of the medium such as: initial pressure p_0 , dynamic viscosity η and attenuation coefficient μ .

The sufficient condition for the existence of vortices is given by the fixed-point theorem in implicit functions and by the expected value theorem of the logistic density function, which complemented the requirement (6).

- The statistical equilibrium of a physical system such as a fluid is obtained for the maximum entropy corresponding to the probability value of $P = \frac{1}{2}$, and physically it is equivalent to $k\epsilon - \mu r = 0$.
- The function $r = \phi(t)$ that appears in the fixed-point theorem and in the expected value theorem needs to be generalized to respect the Heisenberg's theorem, which involves the intrinsic variation of x, y, z, t , and energy, as follows: $r = \phi(t, k, \mu)$.

Stochastic fluctuations of the parameters $k = \frac{p_0}{2\eta}$ and μ

The evolution of pressure and velocity also depends on the stochastic fluctuations of the parameters, which are local and not global.

We obtain a dynamic and probabilistic solution of the Navier-Stokes 3D equation, which represents a fixed point. This function corresponds to a spherical boundary, defined for the solution of the Navier Stokes equations. This spherical surface is the macroscopic solution domain, and it is a vortex.

- At the macroscopic level, turbulence centers are also resonance centers where energy is efficiently deposited or captured. Turbulence is where and when all random effects cancel out and only cooperative effects of order and apparent coordination are manifested, creating minimal entropy.

Quantum Mechanics.

De Broglie's law is the bridge between Quantum Mechanics and Navier-Stokes 3D equation, by virtue of this complementarity:

- Fluid molecules interact as incompressible spheres between them, similar to a pool game between experts and produce curved displacements and vortices. All collisions that produce random effects cancel each other out and do not produce vortices, whereas coordinated and correlated collisions can produce quantum vortices.

- Viscosity represents the velocity of the De Broglie wave packet. Can be understood as quantum adherence.

- Minimal entropy governs turbulence sustained at atomic nucleus level and at macroscopic level, where the effects are cooperative, causal and long range. The cooperation is manned by order and by the viscosity, causing a chain of events unfortunate for humans, which at the level of physics are simply processes of minimal entropy.

- A way to eliminate the destructive turbulences in vortices, could be by maximization of entropy.

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