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Stability of System matrix via Gerschgorin circles

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ABSTRACT
In this paper the stability of the system can be analyzed graphically using Gerschgorin circle theorem. Analytically it has been proved that if the left Gerschgorin bound are very much greater than the right Gerschgorin bound and the trace of the matrix is equal to length of left Gerschgorin bound then there is no eigenvalues on the RHS of s-plane.

Keywords - Eigenvalues,* Gerschgorin *bound Gerschgorin circles, stability, and trace

I. INTRODUCTION

The concept of stability plays very important role in the analysis of the system. In literature there is various methods to find the stability of the system. Given a characteristic polynomial when all the co-efficient of the characteristic polynomial are positive by Routh stability criterion [1] we have to construct the Routh table and in the first column of the Routh array, if there exist change in sign then the system is said to be unstable. This requires the computation of the characteristic polynomial from the system matrix which takes lots of computation and constructing the Routh table requires computation. In this paper attempts have been made to find whether the system is stable graphically. Given a system matrix of order (nxn), than we draw the Gerschgorin circles [2] of the matrix. If the length of the Gerschgorin bound is more on the left hand side of the s-plane and if the trace of the matrix is equal to length left bound, than using Gerschgorin theorem [2] it has proved that the system is stable . This approach does not require any computation.

II. MATHEMATICAL ANALYSIS:

Given a = left bound.

T = trace.

b = right bound.

Also a >> b a = T is known

To prove that: There exists no eigenvalues on the right half of the of s-plane which implies that the system is stable.

Determination of the bounds of the using Gerschgorin's theorem:

Consider a system matrix $[A]_{nxn} \in R^{nxn}$

For Row wise circles:-

$$L_{rk} = a_{kk} - \sum_{\substack{k=1 \\ i \neq k}}^{n} a_{ik} \quad k = 1, 2, 3, ..., n \quad \rightarrow (1)$$

$$L_r = \min(L_{rk}) \quad k = 1, 2, 3, ..., n \quad \rightarrow (2)$$

$$R_{rk} = a_{kk} - \sum_{\substack{k=1 \\ i \neq k}}^{n} a_{ik} \quad k = 1, 2, 3, ..., n \quad \rightarrow (3)$$

$$R_c = \max(R_{ck}) \quad k = 1, 2, 3, ..., n \quad \rightarrow (4)$$

L_{rk} = is the left bound for each row-wise circle

L_r = extreme left bound for row-wise circle.

R_{rk} = is the right bound for each row-wise circle.

R_c = extreme right bound for row-wise circle.

For Column wise circles:-

$$L_{ck} = a_{kk} - \sum_{\substack{k=1 \\ j \neq k}}^{n} a_{kj} \quad k = 1, 2, 3, ..., n \quad \rightarrow (5)$$

$$L_c = \min R_{ck} \quad k = 1, 2, 3, ..., n \quad \rightarrow (6)$$

$$R_{ck} = a_{kk} - \sum_{\substack{k=1 \\ j \neq k}}^{n} a_{kj} \quad k = 1, 2, 3, ..., n \quad \rightarrow (7)$$

$$R_{r=} \max R_{rk} \quad k = 1, 2, 3, ..., n \quad \rightarrow (8)$$

L_{ck} = is the left bound for each column wise circle.

L_c = extreme left bound for column-wise circle.

R_{ck} = is the right bound for each column-wise circle.

R_c = extreme right bound for column -wise circle.

To obtain the left bound:

If $L_c, L_r < 0$ then the left bound

$a = \max(L_c, L_r)$

f $L_c, L_r > 0$ then the left bound

$a = \min(L_c, L_r)$

If $L_c \leq 0$, $L_r > 0$ then the left bound $a = L_c$

If $L_r \leq 0$, $L_c > 0$ then the left bound

$a = L_r$

To obtain the right bound:

If $R_c , R_r < 0$ then the left bound $b = \max\,(R_c, R_r)$

If $R_c, R_r > 0$ then the left bound

$b = \min\,(R_c, R_r)$

If $R_c \leq 0 , R_c > 0$ then the left bound

$b = R_r$

If $R_r \leq 0 , R_c > 0$ then the left bound

$b = R_c$

Since a>> b , let $L_c , L_r < 0$ then the left bound

$a = \max(L_c, L_r)$

Let us suppose that $a = L_c$ -Extreme left bound of the column wise circles.

If $R_c, R_r > 0$ then the left bound

$b = \min\,(R_c, R_r)$

Let us suppose that $b = R_c$- Extreme right bound of the column wise circles.

Given a >> b, and a = T

$$\text{trace} = \sum_{i=1}^{k} a_{ii} - sum\ of\ \ principle\ diagonal\ elements$$

$$\text{trace} = \sum_{i=1}^{k} \lambda_{ii} - sum\ of\ \ eigenvalues$$

Since $a = L_c$ $b = R_c$ Also given that a >> b

$L_c \gg R_c$ and $L_c = trace < 0$

$$\min\,(a_{kk} - \sum_{\substack{k=1 \\ i \neq k}}^{n} a_{ik}) < 0, k = 1,2,3,....,n$$

- ➔ All centers of the circles are on the left half of the s-plane. (Since a >> b)
- ➔ There exist no circles with the centre on the RHS s-plane. Hence there exists no eigenvalues on the RHS of s-plane.

III. EXAMPLE

$$\begin{pmatrix} -3 & 1 & 1 \\ 3 & -1 & -1 \\ 1 & 1 & -1 \end{pmatrix} \qquad \rightarrow (9)$$

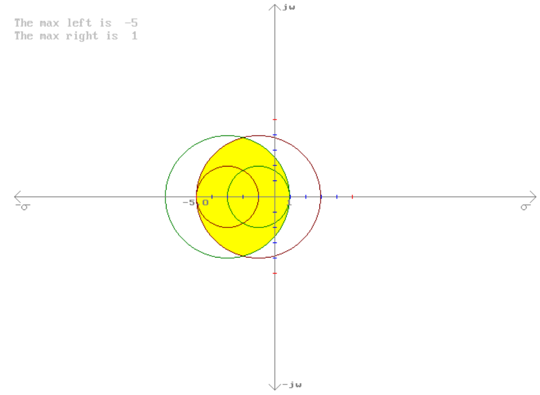
Gerschgorin circles of the above matrix is

Eigenvalues of the above matrix are

λ = 0

λ = -1

λ = -4



Gerschgorin bound [- 6, 1]

Conclusion:

Hence given a system matrix A of any order , with the condition that the left bound is very much greater than the right bound and also the trace of the matrix is equal to the length of the left bound , than the system does not contain any eigenvalues on the right hand side of s-plane. This is applicable only for the few class of matrices. The advantage of this graphical approach is it requires no computation. Also there exist no eigenvalue on the right half of s-plane. By observing the Gerschgorin circles the stability can be identified.

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