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Fuzzy Soft Matrix Theory And Its Decision Making

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ABSTRACT
The purpose of this paper is to put forward the notion of fuzzy soft matrix theory and some basic results. Finally we have put forward a decision making problem using the notion of product of fuzzy soft matrices.

Keywords – **Fuzzy set, Soft set, Fuzzy soft set, Fuzzy soft matrix.**

1. INTRODUCTION
In order to deal with many complicated problems in the fields of engineering, social science, economics, medical science etc involving uncertainties, classical methods are found to be inadequate in recent times. Molodtsov **[1]** pointed out that the important existing theories viz. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. He further pointed out that the reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory. In 1999 he proposed a new mathematical tool for dealing with uncertainties which is free of the difficulties present in these theories. He introduced the novel concept of Soft Sets and established the fundamental results of the new theory. He also showed how Soft Set Theory is free from parameterization inadequacy syndrome of Fuzzy Set Theory, Rough Set Theory, Probability Theory etc. Many of the established paradigms appear as special cases of Soft Set Theory. In 2003, P.K.Maji, R.Biswas and A.R.Roy **[2]** studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union, intersection were also defined. In 2005, Pei and Miao **[3]** and Chen et al. **[4]** improved the work of Maji et al. **[2].** In 2008, M.Irfan Ali, Feng Feng, Xiaoyan Liu,Won Keun Min, M.Shabir **[5]** gave some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets along with a new notion of complement of a soft set.

In recent times, researches have contributed a lot towards fuzzification of Soft Set Theory. Maji et al. **[6]** introduced some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, DeMorgan Law etc. These results were further revised and improved by Ahmad and Kharal **[7]**. They defined arbitrary fuzzy soft union and intersection and proved De Morgan Inclusions and De Morgan Laws in Fuzzy Soft Set Theory. In 2011, Neog and Sut [**8**] put forward some more propositions regarding fuzzy soft set theory. They studied the notions of fuzzy soft union, fuzzy soft intersection, complement of a fuzzy soft set and several other properties of fuzzy soft sets along with examples and proofs of certain results.

Matrices play an important role in the broad area of science and engineering. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment. In [**9**], Yong Yang and Chenli Ji initiated a matrix representation of a fuzzy soft set and successfully applied the proposed notion of fuzzy soft matrix in certain decision making problems. In this paper, we extend the notion of fuzzy soft matrices put forward in [**9**]. In our work, we are taking fuzzy soft sets with different set of parameters whereas in [**9**], notion of fuzzy soft matrix was put forward considering a single set of parameters, which is not the case in actual practice. Throughout our work, we are using *t*- norm and *t*- conorm as intersection and union, respectively, of fuzzy sets.

2. PRELIMINARIES

Definition 2.1 [1]
A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*.
In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

Definition 2.2 [6]
A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F : A \to \tilde{P}(U)$ is a mapping from *A* into $\tilde{P}(U)$.

Definition 2.3 [7]
Let *U* be a universe and *E* a set of attributes. Then the pair (*U*, *E*) denotes the collection of all fuzzy soft sets on *U* with attributes from *E* and is called a fuzzy soft class.

Definition 2.4 [6]
A soft set (*F*, *A*) over *U* is said to be null fuzzy soft set denoted by φ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set $\overline{0}$ of *U* where $\overline{0}(x) = 0 \forall x \in U$.

Definition 2.5 [6]
A soft set (*F*, *A*) over *U* is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set $\overline{1}$ of *U* where $\overline{1}(x) = 1 \forall x \in U$

Definition 2.6 [6]
For two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a fuzzy soft class (*U*, *E*), we say that (*F*, *A*) is a fuzzy soft subset of (*G*, *B*), if

(*i*) $A \subseteq B$

(*ii*) For all $\varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$ and is written as

$(F, A) \,\tilde{\subseteq}\, (G, B)$.

Definition 2.7 [6]
Union of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

And is written as $(F, A) \,\tilde{\cup}\, (G, B) = (H, C)$.

Definition 2.8 [6]
Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same fuzzy set) and is written as $(F, A) \,\tilde{\cap}\, (G, B) = (H, C)$.

Ahmad and Kharal [**7**] pointed out that generally $F(\varepsilon)$ or $G(\varepsilon)$ may not be identical. Moreover in order to avoid the degenerate case, he proposed that $A \cap B$ must be non-empty and thus revised the above definition as follows -

Definition 2.9 [7]
Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in a soft class (*U*, *E*) with $A \cap B \neq \phi$.Then Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*,*C*) where $C = A \cap B$ and $\forall \varepsilon \in C$,
$H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.
We write $(F, A) \,\tilde{\cap}\, (G, B) = (H, C)$.

Definition 2.10 [10]
The complement of a fuzzy soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to \tilde{P}(U)$ is a mapping given by $F^c(\alpha) = [F(\alpha)]^c$, $\forall \alpha \in A$.

Definition 2.11 [11]
A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous *t* - norm if * satisfies the following conditions.
(*i*) * is commutative and associative
(*ii*) * is continuous
(*iii*) $a * 1 = a \quad \forall a \in [0,1]$
(*iv*) $a * b \leq c * d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0,1]$
An example of continuous *t* - norm is $a * b = ab$.

Definition 2.12 [11]
A binary operation $\lozenge : [0,1] \times [0,1] \to [0,1]$ is continuous *t* - conorm if $\lozenge$ satisfies the following conditions:
(*i*) $\lozenge$ is commutative and associative
(*ii*) $\lozenge$ is continuous
(*iii*) $a \lozenge 0 = a \quad \forall a \in [0,1]$
(*iv*) $a \lozenge b \leq c \lozenge d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0,1]$
An example of continuous *t* - conorm is $a * b = a + b - ab$.

Definition 2.13 [9]
Let $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$ be two fuzzy soft matrix .Then *A* is a fuzzy soft sub matrix of *B*, denoted by $A \,\tilde{\subseteq}\, B$, if $a_{ij} \leq b_{ij}, \forall i, j$,

Definition 2.14 [9]
The $m \times n$ fuzzy soft matrix whose elements are all 0 is called the fuzzy soft null matrix (or zero matrix). It is usually denoted by $\tilde{0}$ or $\tilde{0}_{m \times n}$

Definition 2.15 [9]
The $m \times n$ fuzzy soft matrix whose elements are all 1 is called the fuzzy universal soft matrix. It is usually denoted by $\tilde{U}$ or $\tilde{U}_{m \times n}$

Definition 2.16 [9]
Let $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$ be two fuzzy soft matrix .Then A is equal to fuzzy soft matrix B, denoted by $A = B$, if $a_{ij} = b_{ij}, \forall i, j$,

Definition2.12 [9]
Let $A = [a_{ij}] \in FSM_{m \times n}$. Then we define
$A^{\tilde{T}} = [a_{ij}^{\tilde{T}}] \in FSM_{n \times m}$, where $a_{ij}^{\tilde{T}} = a_{ji}$.

Definition2.13 [9]
Let $A_k \in FSM_{m \times n}, k = 1,2,3,\ldots\ldots, l$, their product is $\prod_{k=1}^{l} A_k = [c_i]_{m \times l}$. Then the set
$O_s = \{ j : c_j = \max\{c_i : i = 1,2,3,\ldots\ldots, m\}\}$ is called the optimum subscript set, and the set
$O_d = \{u_j : u_j \in U.and. j \in O_s\}$ is called the optimum decision set of *U*.

3. FUZZY SOFT MATRICES

Definition 3.1
Let $U = \{c_1, c_2, c_3, \ldots\ldots\ldots\ldots c_m\}$ be the universal set and *E* be the set of parameters given by $E = \{e_1, e_2, e_3, \ldots\ldots\ldots\ldots\ldots e_n\}$. Let $A \subseteq E$ and (F, A) be a fuzzy soft set in the fuzzy soft class (U, E).Then we would represent the fuzzy soft set (F, A) in matrix form as

$A_{m \times n} = [a_{ij}]_{m \times n}$ or simply by $A = [a_{ij}]$,

$i = 1,2,3\ldots\ldots\ldots\ldots m$; $j = 1,2,3\ldots\ldots\ldots\ldots\ldots n$,

where $a_{ij} = \begin{cases} \mu_j(c_i) & \text{if } e_j \in A \\ 0 & \text{if } e_j \notin A \end{cases}$

Here $\mu_j(c_i)$ represents the membership of c_i in the fuzzy set $F(e_j)$. We would identify a fuzzy soft set with its fuzzy soft matrix and use these two concepts interchangeable. The set of all $m \times n$ fuzzy soft matrices over *U* would be denoted by $FSM_{m \times n}$.

Example 3.1
Let $U = \{c_1, c_2, c_3, c_4\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, e_4, e_5\}$.
Let $P = \{e_1, e_2, e_4\} \subseteq E$ and (F, P) is the fuzzy soft set

$$\begin{aligned}(F,P) \quad &= \{F(e_1) = \{(c_1,0.7),(c_2,0.6),(c_3,0.7),(c_4,0.5)\},\\ &F(e_2) = \{(c_1,0.8),(c_2,0.6),(c_3,0.1),(c_4,0.5)\},\\ &F(e_4) = \{(c_1,0.1),(c_2,0.4),(c_3,0.7),(c_4,0.3)\}\}\end{aligned}$$

The fuzzy soft matrix representing this fuzzy soft set would be represented in our notation as

$$A = \begin{bmatrix} 0.7 & 0.8 & 0.0 & 0.1 & 0.0 \\ 0.6 & 0.6 & 0.0 & 0.4 & 0.0 \\ 0.7 & 0.1 & 0.0 & 0.7 & 0.0 \\ 0.5 & 0.5 & 0.0 & 0.3 & 0.0 \end{bmatrix}_{4\times 5}$$

Proposition 3.1
Let $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$, $C = [c_{ij}]_{m\times n}$ be three fuzzy soft matrix. Then

$(i) \tilde{0} \mathrel{\tilde{\subseteq}} A$

$(ii) A \mathrel{\tilde{\subseteq}} \tilde{U}$

$(iii) A \mathrel{\tilde{\subseteq}} A$

$(iv) A \mathrel{\tilde{\subseteq}} B,\ B \mathrel{\tilde{\subseteq}} C \implies A \mathrel{\tilde{\subseteq}} C$

Proof: The proof is straight forward and follows from definition.

Definition 3.2
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = \mu_j(c_i)$. If $m \neq n$, then A is called a fuzzy soft rectangular matrix.

Definition 3.3
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = \mu_j(c_i)$. If $m = n$, then A is called a fuzzy soft square matrix.

Definition 3.4
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = \mu_j(c_i)$. If $m = 1$, then A is called a fuzzy soft row matrix.

Definition 3.5
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = \mu_j(c_i)$. If $n = 1$, then A is called a fuzzy soft column matrix.

Definition 3.6
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = \mu_j(c_i)$. Then A is called fuzzy soft diagonal matrix if $m = n$ and $a_{ij} = 0$ for all $i \neq j$.

Definition 3.7
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = \mu_j(c_i)$. Then A is called fuzzy soft scalar matrix if $m = n, a_{ij} = 0$ for all $i \neq j$ and $a_{ij} = \lambda \in [0,1]\ \forall\, i = j$.

Definition 3.8
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = \mu_j(c_i)$. Then A is called fuzzy soft upper triangular matrix if $m = n$, $a_{ij} = 0$ for all $i > j$.

Definition 3.9
Let $A = [a_{ij}] \in FSM_{m\times n}$, where $a_{ij} = \mu_j(c_i)$. Then A is called fuzzy soft lower triangular matrix if $m = n$, $a_{ij} = 0$ for all $i < j$.
A fuzzy soft matrix is said to be triangular if it is either fuzzy soft lower or fuzzy soft upper triangular matrix.

Definition 3.10
Let $A = [a_{ij}] \in FSM_{m\times m}$, where $a_{ij} = \mu_j(c_i)$, then the elements $a_{11}, a_{12}, \ldots\ldots\ldots, a_{mm}$ are called the diagonal elements and the line along which they lie is called the principal diagonal of the fuzzy soft matrix.

Definition 3.11
Let $A = [a_{ij}], B = [b_{ij}] \in FSM_{m\times n}$. Then union of A, B is defined by $A_{m\times n} \mathbin{\tilde{\cup}} B_{m\times n} = C_{m\times n} = [c_{ij}]_{m\times n}$, where $c_{ij} = a_{ij} \Diamond b_{ij} = a_{ij} + b_{ij} - a_{ij} b_{ij}$ for all i and j.

Example 3.2

$$\text{Let } A = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.2 & 0.5 & 0.7 \\ 0.0 & 1.0 & 0.3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.5 & 0.2 & 0.6 \\ 0.1 & 0.3 & 0.2 \\ 0.2 & 0.7 & 0.0 \end{bmatrix}.$$

Then

$$A_{3\times 3} \mathbin{\tilde{\cup}} B_{3\times 3} = C_{3\times 3} = \begin{bmatrix} 0.55 & 0.36 & 0.72 \\ 0.28 & 0.65 & 0.76 \\ 0.20 & 1.00 & 0.30 \end{bmatrix}$$

Definition 3.12
Let $A = [a_{ij}], B = [b_{ij}] \in FSM_{m\times n}$ Then intersection of A, B is defined by $A_{m\times n} \mathbin{\tilde{\cap}} B_{m\times n} = C_{m\times n} = [c_{ij}]_{m\times n}$, where $c_{ij} = a_{ij} * b_{ij} = a_{ij} b_{ij}$ for all i and j.

Example 3.3
For the fuzzy soft matrices cited in **example 3.2**, we have

$$A_{3\times 3} \mathbin{\tilde{\cap}} B_{3\times 3} = C_{3\times 3} = \begin{bmatrix} 0.05 & 0.04 & 0.18 \\ 0.02 & 0.15 & 0.14 \\ 0.00 & 0.70 & 0.00 \end{bmatrix}$$

Proposition 3.2

Let $A, B \in FSM_{m\times n}$ .Then

(i) $A \tilde{\cup} \tilde{0} = A$

(ii) $A \tilde{\cup} \tilde{U} = \tilde{U}$

(iii) $A \tilde{\cup} B = B \tilde{\cup} A$

(iv) $(A \tilde{\cup} B) \tilde{\cup} C = A \tilde{\cup} (B \tilde{\cup} C)$

Proof: Let $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$, $C = [c_{ij}]_{m\times n}$ be three fuzzy soft matrices .

$(i)\ A \tilde{\cup} \tilde{0} \quad = [a_{ij} + 0 - a_{ij} \times o] = [a_{ij}] = A$

$(ii)\ A \tilde{\cup} \tilde{U} \quad = [a_{ij} + 1 - a_{ij} \times 1] = [1] = \tilde{U}$

$$\begin{aligned}(iii)\ A \tilde{\cup} B &= [a_{ij} + b_{ij} - a_{ij} \times b_{ij}] \\ &= [b_{ij} + a_{ij} - b_{ij} \times a_{ij}] \\ &= B \tilde{\cup} A\end{aligned}$$

$$\begin{aligned}(iv)\ (A \tilde{\cup} B) \tilde{\cup} C &= [a_{ij} + b_{ij} - a_{ij} \times b_{ij}] \tilde{\cup} [c_{ij}] \\ &= [(a_{ij} + b_{ij} - a_{ij} \times b_{ij}) + c_{ij} \\ &\quad - (a_{ij} + b_{ij} - a_{ij} \times b_{ij}) c_{ij}] \\ &= [a_{ij} + b_{ij} + c_{ij} - a_{ij} \times b_{ij} \\ &\quad - a_{ij} \times c_{ij} - b_{ij} \times c_{ij} + a_{ij} \times b_{ij} \times c_{ij}]\end{aligned}$$

$$\begin{aligned}A \tilde{\cup} (B \tilde{\cup} C) &= [a_{ij}] \tilde{\cup} [b_{ij} + c_{ij} - b_{ij} \times c_{ij}] \\ &= [a_{ij} + (b_{ij} + c_{ij} - b_{ij} \times c_{ij}) \\ &\quad - a_{ij} \times (b_{ij} + c_{ij} - b_{ij} \times c_{ij})] \\ &= [a_{ij} + b_{ij} + c_{ij} - a_{ij} \times b_{ij} \\ &\quad - a_{ij} \times c_{ij} - b_{ij} \times c_{ij} + a_{ij} \times b_{ij} \times c_{ij}]\end{aligned}$$

Hence $(A \tilde{\cup} B) \tilde{\cup} C = A \tilde{\cup} (B \tilde{\cup} C)$.

Remark

Let $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$ be two fuzzy soft matrices. If we consider $C_{m\times n} = A_{m\times n} \tilde{\cup} B_{m\times n} = [c_{ij}]_{m\times n}$, where $c_{ij} = \max\{a_{ij}, b_{ij}\}$, for all *i* and *j*, then $A \tilde{\cup} A = A$.

Proposition 3.3

Let $A, B \in FSM_{m\times n}$ .Then

(i) $A \tilde{\cap} \tilde{0} = A$

(ii) $A \tilde{\cap} \tilde{U} = A$

(iii) $A \tilde{\cap} B = B \tilde{\cap} A$

(iv) $(A \tilde{\cap} B) \tilde{\cap} C = A \tilde{\cap} (B \tilde{\cap} C)$

Proof: Let $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$, $C = [c_{ij}]_{m\times n}$ be three fuzzy soft matrices, for all *i* and *j*.

$(i)\ A \tilde{\cap} \tilde{0} \quad = [a_{ij} \times o] = [0] = \tilde{0}$

$(ii)\ A \tilde{\cap} \tilde{U} \quad = [a_{ij} \times 1] = [a_{ij}] = A$

$$\begin{aligned}(iii)\ A \tilde{\cap} B &= [a_{ij} \times b_{ij}] \\ &= [b_{ij} \times a_{ij}] \\ &= B \tilde{\cap} A\end{aligned}$$

$$\begin{aligned}(iv)(A \tilde{\cap} B) \tilde{\cap} C &= [a_{ij} \times b_{ij}] \tilde{\cap} [c_{ij}] \\ &= [(a_{ij} \times b_{ij}) \times c_{ij}] \\ &= [a_{ij} \times (b_{ij} \times c_{ij})] \\ &= [a_{ij}] \tilde{\cap} [b_{ij} \times c_{ij}] \\ &= A \tilde{\cap} (B \tilde{\cap} C)\end{aligned}$$

Remark

Let $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$ be two fuzzy soft matrices. If we consider $C_{m\times n} = A_{m\times n} \tilde{\cap} B_{m\times n} = [c_{ij}]_{m\times n}$, where $c_{ij} = \min\{a_{ij}, b_{ij}\}$, for all *i* and *j*, then $A \tilde{\cap} A = A$.

Definition 3.13

Let $A = [a_{ij}]_{m\times n}$, then complement of A is denoted by $A^C = [c_{ij}]$, where $c_{ij} = 1 - a_{ij}$, for all *i* and *j*.

Example 3.4

Let $A = \begin{bmatrix} 0.9 & 0.8 & 0.0 \\ 0.8 & 0.5 & 0.0 \\ 1.0 & 0.0 & 0.0 \end{bmatrix}$

Then

$A^C = \begin{bmatrix} 0.1 & 0.2 & 1.0 \\ 0.2 & 0.5 & 1.0 \\ 0.0 & 1.0 & 1.0 \end{bmatrix}$

Proposition 3.4

(i) $(A^C)^C = A$

(ii) $(\tilde{0})^C = \tilde{U}$

Proof

(i) Let $A = [a_{ij}]_{m\times n}$ be a fuzzy soft matrix. Then

$A^C = [1 - a_{ij}]$, for all i and j

$\therefore (A^C)^C = [1 - (1 - a_{ij})] = [a_{ij}] = A$

(ii) $(\tilde{0})^C = [1 - 0] = [1] = \tilde{U}$

Proposition 3.5

Let $A=\lfloor a_{ij}\rfloor_{m\times n}$, $B=\lfloor b_{ij}\rfloor_{m\times n}$ be two fuzzy soft matrices. Then De Morgan`s Laws are valid, for all *i* and *j*.

(i) $(A\tilde{\cup} B)^C = A^C \tilde{\cap} B^C$

(ii) $(A\tilde{\cap} B)^C = A^C \tilde{\cup} B^C$

Proof:

(i) $$\begin{aligned}(A\tilde{\cup} B)^C &= \left(\lfloor a_{ij}\rfloor \tilde{\cup} \lfloor b_{ij}\rfloor\right)^C\\ &= \left[a_{ij}+b_{ij}-a_{ij}b_{ij}\right]^C\\ &= \lfloor 1-a_{ij}-b_{ij}+a_{ij}b_{ij}\rfloor\\ &= \lfloor (1-a_{ij})(1-b_{ij})\rfloor\\ &= \left[1-a_{ij}\right]\tilde{\cap}\left[1-b_{ij}\right]\\ &= A^C \tilde{\cap} B^C\end{aligned}$$

(ii) $$\begin{aligned}(A\tilde{\cap} B)^C &= \left(\lfloor a_{ij}\rfloor \tilde{\cap} \lfloor b_{ij}\rfloor\right)^C\\ &= \left[a_{ij}b_{ij}\right]^C\\ &= \lfloor 1-a_{ij}b_{ij}\rfloor\end{aligned}$$

$$\begin{aligned}A^C \tilde{\cup} B^C &= \lfloor 1-a_{ij}\rfloor \tilde{\cup} \lfloor 1-b_{ij}\rfloor\\ &= \lfloor 1-a_{ij}+1-b_{ij}-(1-a_{ij})(1-b_{ij})\rfloor\\ &= \lfloor 1-a_{ij}b_{ij}\rfloor\end{aligned}$$

Hence $(A\tilde{\cap} B)^C = A^C \tilde{\cup} B^C$

Definition 3.14

Let $A=\lfloor a_{ij}\rfloor \in FSM_{m\times n}$ and k, $0\le k\le 1$ any number called scalar. The scalar multiple of A by k is denoted by $kA=\lfloor ka_{ij}\rfloor_{m\times n}$.

Example 3.5

Let $A=\begin{bmatrix}0.1 & 0.2 & 0.3\\ 0.2 & 0.5 & 0.7\\ 0.0 & 1.0 & 0.3\end{bmatrix}$

$\therefore 0.5A=\begin{bmatrix}0.05 & 0.10 & 0.15\\ 0.10 & 0.25 & 0.35\\ 0.00 & 0.50 & 0.15\end{bmatrix}$

Proposition 3.6

If s and t are two scalars such that $0\le s,t\le 1$ and $A=\lfloor a_{ij}\rfloor_{m\times n}$ is any fuzzy soft matrix, then

(i) $s(tA)=(st)A$

(ii) $s\le t \Rightarrow sA\subseteq tA$

(iii) $A\subseteq B \Rightarrow sA\subseteq sB$

Proof

We only prove (i) and others follow the similar lines.

Let $A = \lfloor a_{ij}\rfloor_{m\times n}$

$$\begin{aligned}s(tA) &= s\left(t\lfloor a_{ij}\rfloor_{m\times n}\right)\\ &= s\lfloor ta_{ij}\rfloor_{m\times n}\\ &= \lfloor s(ta_{ij})\rfloor_{m\times n}\\ &= \lfloor (st)a_{ij}\rfloor_{m\times n}\\ &= (st)\lfloor a_{ij}\rfloor_{m\times n} = (st)A\end{aligned}$$

Definition 3.15

Let $A=\lfloor a_{ij}\rfloor_{m\times m}$, be a fuzzy soft square matrix. Then

$$\tilde{tr}A=\sum_{i=1}^{m} a_{ii} = a_{11}+a_{22}+a_{33}+\ldots\ldots\ldots\ldots+a_{nn}$$

Example 3.6

Let $A=\begin{bmatrix}0.1 & 0.2 & 0.3\\ 0.2 & 0.5 & 0.7\\ 0.0 & 1.0 & 0.3\end{bmatrix}$

$\therefore \tilde{tr}A = 0.1+0.5+0.3=0.9$

Proposition3.8

Let A and B be two fuzzy soft square matrices of order m and k be a scalar. Then $\tilde{tr}(kA)=k\tilde{tr}A$

Proof: Let $A=\lfloor a_{ij}\rfloor_{m\times m}$

We have, $kA=\lfloor ka_{ij}\rfloor_{m\times m}$

$$\therefore \tilde{tr}(kA)=\sum_{i=1}^{m} ka_{ii} = k\sum_{i=1}^{m} a_{ij} = k\tilde{tr}A$$

Proposition 3.7

If A be a fuzzy soft matrix of order $m\times n$, then $(kA)^{\tilde{T}} = kA^{\tilde{T}}$, k being any scalar.

Proof

Let $A=\lfloor a_{ij}\rfloor_{m\times n}$ be fuzzy soft matrix.

We have $kA=\lfloor ka_{ij}\rfloor_{m\times n}$

$\therefore (kA)^{\tilde{T}} = \left[ka_{ji}\right]_{n\times m} = k\left[a_{ji}\right]_{n\times m} = kA^{\tilde{T}}$

Proposition 3.8

Let $A,B\in FSM_{m\times n}$ .Then

(i) $(A\tilde{\cup} B)^{\tilde{T}} = A^{\tilde{T}} \tilde{\cup} B^{\tilde{T}}$

(ii) $(A\tilde{\cap} B)^{\tilde{T}} = A^{\tilde{T}} \tilde{\cap} B^{\tilde{T}}$

(iii) $\left(A^C\right)^{\tilde{T}} = (A^{\tilde{T}})^C$

Remark

Let $A,B\in FSM_{m\times n}$ .Then the following distributive laws are valid for *max* and *min* operations only.

(i) $A\tilde{\cup}(B\tilde{\cap} C)=(A\tilde{\cup} B)\tilde{\cap}(A\tilde{\cup} C)$

(ii) $A\tilde{\cap}(B\tilde{\cup} C)=(A\tilde{\cap} B)\tilde{\cup}(A\tilde{\cap} C)$

4. *T* - PRODUCT OF FUZZY SOFT MATRICES

Definition 4.1

Let $A_k = [a^k{}_{ij}] \in FSM_{m\times n}, k = 1,2,3,\ldots\ldots,l$.

Then the T - product of fuzzy soft matrices, denoted by $\prod_{k=1}^{l} A_k = A_1 \times A_2 \times A_3 \times \ldots\ldots \times A_l$, is defined by $\prod_{k=1}^{l} A_k = [c_i]_{m\times 1}$, where

$$c_i = \sum_{j=1}^{n} \underset{k=1}{\overset{l}{T}} a_{ij}{}^k, i = 1,2,3\ldots\ldots\ldots\ldots m$$

While defining T-Product in [**9**], c_i is calculated by the formula -

$$c_i = \frac{1}{n} \sum_{j=1}^{n} \underset{k=1}{\overset{l}{T}} a_{ij}{}^k, i = 1,2,3\ldots\ldots\ldots\ldots m$$

This requires more computational time. Our method requires less computational time and we obtain the same result as was obtained in [**9**]. In our work, we will take T = * or $T = \lozenge$ according to the type of the problems.

Example 4.1

We assume that $A_1, A_2, A_3 \in FSM_{4\times 2}$ are given as follows

$$A_1 = \begin{bmatrix} 0.2 & 0.7 \\ 0.6 & 0.4 \\ 0.3 & 0.6 \\ 0.1 & 0.9 \end{bmatrix}, A_2 = \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.6 \\ 0.8 & 0.5 \\ 0.8 & 0.2 \end{bmatrix}, A_3 = \begin{bmatrix} 0.4 & 0.3 \\ 0.3 & 0.1 \\ 0.3 & 0.2 \\ 0.5 & 0.7 \end{bmatrix}.$$

Then the * product is

$$\prod_{k=1}^{3} A_k = A_1 \times A_2 \times A_3 = \begin{bmatrix} 0.2*0.6*0.4+0.7*0.2*0.3 \\ 0.6*0.1*0.3+0.4*0.6*0.1 \\ 0.3*0.8*0.3+0.6*0.5*0.2 \\ 0.1*0.8*0.5+0.9*0.2*0.7 \end{bmatrix} = \begin{bmatrix} 0.090 \\ 0.042 \\ 0.132 \\ 0.166 \end{bmatrix}$$

Proposition 4.1

Let $A, B \in FSM_{m\times n}$ .Then

(i) $A \times B = B \times A$

(ii) $(A\times B)\times C = A\times(B\times C)$

Proof

(i) Let $A = \lfloor a_{ij} \rfloor$, $B = \lfloor b_{ij} \rfloor$ be two fuzzy soft matrices. Then

$$A\times B = [\sum_{j=1}^{n} a_{ij} T b_{ij}] = [\sum_{j=1}^{n} b_{ij} T a_{ij}] = B\times A, \text{ where } T = * \text{ or } \lozenge$$

(ii) The proof is similar to that of (i).

Proposition 4.2

Let $A, B, C \in FSM_{m\times n}, B \subseteq C$. Then $A\times B \subseteq A\times C$

Proof: The proof follows similar lines as above.

5. FUZZY SOFT MATRICES IN DECISION MAKING

In this section, we put forward a fuzzy soft matrix decision making method by using fuzzy soft "*" product.

ALGORITHIM

Input: Fuzzy soft sets with *m* objects, each of which has *n* parameters.

Output: An optimum set.

Step1: Choose the set of parameters

Step2: Construct the fuzzy soft matrices for each set of parameters.

Step3: Compute "*" product of the fuzzy soft matrices

Step4: Find the optimum subscript set O_s

Step5: Find the optimum decision set O_d

Suppose $U = \{c_1, c_2, c_3, c_4, c_5\}$ be the five candidates appearing in an interview for appointment in managerial level in a company and

$$E = \{e_1(\text{enterprising}), e_2(\text{confident}), e_3(\text{wiling to take risk})\}$$

be the set of parameters. Suppose three experts, Mr. *A*, Mr. *B* and Mr. *C* take interview of the five candidates and the following fuzzy soft matrices are constructed accordingly.

$$A = \begin{bmatrix} 0.3 & 0.2 & 0.1 \\ 0.5 & 0.4 & 0.2 \\ 0.6 & 0.5 & 0.7 \\ 0.4 & 0.6 & 0.8 \\ 0.8 & 0.6 & 0.3 \end{bmatrix}, B = \begin{bmatrix} 0.7 & 0.2 & 0.5 \\ 0.6 & 0.4 & 0.9 \\ 0.7 & 0.8 & 0.6 \\ 0.5 & 0.6 & 1.0 \\ 0.4 & 0.5 & 0.7 \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} 0.5 & 0.4 & 0.6 \\ 0.4 & 0.7 & 0.6 \\ 0.6 & 0.5 & 0.5 \\ 0.8 & 0.6 & 0.4 \\ 0.5 & 0.6 & 0.5 \end{bmatrix}$$

We have,

$$A \times B \times C$$

$$= \begin{bmatrix} 0.3*0.7*0.5+0.2*0.2*0.4+0.1*0.5*0.6 \\ 0.5*0.6*0.4+0.4*0.4*0.7+0.2*0.9*0.6 \\ 0.6*0.7*0.6+0.5*0.8*0.5+0.7*0.6*0.5 \\ 0.4*0.5*0.8+0.6*0.6*0.6+0.8*1.0*0.4 \\ 0.8*0.4*0.5+0.6*0.5*0.6+0.3*0.7*0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.151 \\ 0.340 \\ 0.662 \\ 0.696 \\ 0.445 \end{bmatrix}$$

It is clear that the maximum score is 0.696, scored by c_4 and the decision is in favor of selecting c_4.

6. CONCLUSION

In this work, we have put forward the notions related to fuzzy soft matrices. Our work is in fact an attempt to extend the notion of fuzzy soft matrices put forward in [**9**]. Future work in this regard would be required to study whether the notions put forward in this paper yield a fruitful result

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