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Stability analysis and solution of head lice growth model

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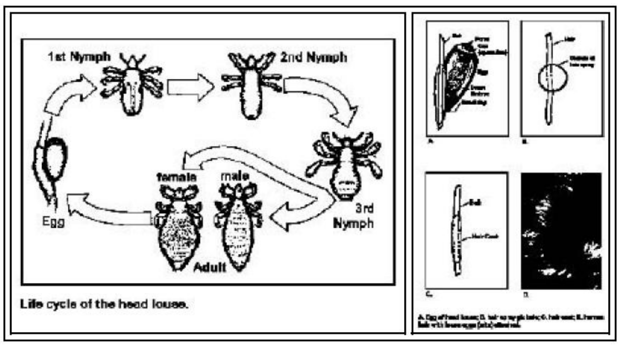
ABSTRACT
In this paper, we discuss the basic introduction on head lice problem and find the differential equations related to our assumption. Solve the differential equation by using equilibrium points and checked the stability of differential system. Finally, we will find the solution of differential system.

***Keywords*: equilibrium points, eigenvalues, lice model, stability.**

I. INTRODUCTION
Head lice are small wingless worm, which are ectoparasites specific to humans. They nourish on human blood 3-4 times a day and cannot survive for more than 2 days away from the host. They lay eggs close to the scalp on hair shafts where the temperature is Standard for incubation. Usually temperatures between 75 and 98.6°F are optimum for lice. The eggs are glued to the hair shaft and it is difficult to dislodge them. Lice insert their mouthparts into human skin to draw blood and inject their mucus that prevents blood from clotting and also causes irritation and subsequent causes itching. Excessive scratching can lead to secondary infections and Burning. Head louse can spend its whole life on human head. The details of life cycle of lice are as follows




Highly magnified

Front view of an actual louse on a hair shaft (highly magnified)

Stage 1 – Eggs
The head louse begins life as an egg, commonly referred to as a nit. Nits are laid by the adult female. The nits are attached to the hair shaft by a glue-like substance produced by the louse. Nits range in color from white to yellow to tan to grayish in color, depending upon the stage of development and whether or not they have hatched or been killed by treatment. Nits are oval or tear-drop shaped, smooth and very small (0.5-0.8 mm). Nits are hard to see and often are confused for dandruff, hair spray droplets or other debris.

Stage 2 –Nymphs
The nits incubate for about seven to 14 days, then they hatch to release a nymph. The nit shell that remains after hatching then becomes a dull yellow or translucent white and may have a wrinkled look. The lymph looks like an adult louse, but is only about the size of a pinhead. The nymph begins eating within one hour of hatching. During the next seven to 10 days, the nymph continues to grow and mature, going through three molts, until it becomes a full adult louse.

Stage 3 –Adults
The adult louse is about the size of a sesame seed. The life span of an adult louse is about 30 days. The female is usually larger than the male and can lay three to five nits per day, or up to 150 nits during her life span (only nits that are fertilized will develop and hatch). The live, adult louse needs to feed on blood every three to six hours. Without blood meals, or once away from the human host, the adult louse can usually survive for no longer than 24 to 36 hours.

II. PROBLEM DESCRIPTIONS
Lice can spend their whole lifetime as ectoparasites feeding on an infested person's blood. They suck blood from skin capillaries painlessly. Usually people get it treated because of itching, secondary infections and inflammation. But the amount of itching caused varies from person to person depending on their sensitivity. A person might have been infested with lice months before any itching is noticed. In cases where no itching is observed it is left untreated. In India in many rural areas people don't get any treatment done for lice infestation. The children in such families may also be malnourished and on the borderline of becoming anemic.

III. ASSUMPTIONS
- Male and Female is only die by natural death and transmission and fall off.
- I am assuming natural death only after the completion of life cycle.
- The rate of transmission and fall off depends on two factors:

- The total number of lice per unit area at a particular point of time.

- Density and length of hair. People with long and dense hair can support more lice and therefore transmission rate is low in such cases.

- Based on the above considerations I tried to calculate the number of lice with time for about 120 days. To start with I assumed that one adult female louse which is ready to lay eggs (on its 19th day) infects the person on the first day. Initially the data was generated considering only natural death and neglecting the effect of transmission to other persons or rate at which the lice fall off while grooming hair etc. and also the unproductiveness of the eggs i.e. it was assumed that all eggs will hatch to produce nymphs.

IV. DIFFERENTIAL EQUATIONS

$\frac{dF}{dt} = h_e(1 - m_p)E - n_d F - t_f F^2$

$\frac{dM}{dt} = h_e m_p E - n_d M - t_f M^2$

$\frac{dE}{dt} = g_e F - e_m E - h_e E$ ………… (1)

Where F, M and E are the number of female, male and eggs respectively.

The model $\frac{dF}{dt} = h_e(1 - m_p)E - n_d F - t_f F^2$

- $\frac{dF}{dt}$, the growth rate of female population defined by three different terms.
- It is positively influenced by current female population size, as shown by the term $h_e(1 - m_p)E$.
- It is negatively influenced by natural death rate of female, as shown by the term $-n_d F$.
- It is negatively influenced by death rate of female due to transmission and fall off, as shown by the term $-t_f F^2$.

The model $\frac{dM}{dt} = h_e m_p E - n_d M - t_f M^2$

- $\frac{dM}{dt}$, the growth rate of male population defined by three different terms.
- It is positively influenced by current male population size, as shown by the term $h_e m_p E$.
- It is negatively influenced by natural death rate of male, as shown by the term $-n_d M$.
- It is negatively influenced by death rate of male due to transmission and fall off, as shown by the term $-t_f M^2$.

The model $\frac{dE}{dt} = g_e F - e_m E - h_e E$

- It is positively influenced by eggs population size, as shown by the term g_e.
- It is negatively influenced by death rate of eggs, as shown by the term $-e_m E$.
- It is negatively influenced by death rate of hatching of eggs, as shown by the term $-h_e E$.

V. ESTIMATION OF PARAMETER

The parameters are determined assuming equal distribution of the lice and egg population over various phases and ages through the life cycle.

- The rate constant for hatching of eggs is given by $h_e + e_m = \frac{1}{8}$ in an equal distribution as it takes 8 days for an egg to hatch. It has been found that 88% is the maximum hatch rate i.e. 12% of the eggs turn out to be unproductive. So the rate of egg mortality is 88% of the rate of generation of eggs.

  $\therefore h_e = 0.88 \times \frac{1}{8} = 0.11$ and $e_m = 0.12 \times \frac{1}{8} = 0.015$.

- A female in its lifetime of 27 days (excluding 8 days of egg stage) lays eggs for the last 17days. So the ratio of number of egg laying female to the total number of females is $\frac{17}{27}$.

  $g_e * N$ = rate of generation of eggs=6*f per day

  $\therefore g_e = \frac{6*f}{N} = \frac{102}{27} = 3.778.$

- Since the population is distributed equally over different ages the death rate constant n_d is given by $\frac{1}{27} = 0.037$.
- The fraction of male population is found to be 0.4 and hence $m_p = 0.4$.
- The rate constant for transmission and fall off varies from person to person depending on the length and density of hair, surface area as it determines the number of lice per unit area. Depending on t_f the severity of the infestation also varies. So different values of t_f were tried out to find the t_f for the case of severe infestation where the total number of lice usually saturates at about 100.

Therefore, the differential system (1) is,

$\frac{dF}{dt} = 0.66E - 0.037F - t_f F^2,$

$\frac{dM}{dt} = 0.044E - 0.037M - t_f M^2,$

$\frac{dE}{dt} = 3.778F - 0.125E$

VI. EQUILIBRIUM POINTS

These are showed by the following computation. Let

$X = \frac{dF}{dt} = 0.66E - 0.037F - t_f F^2,$

$Y = \frac{dM}{dt} = 0.044E - 0.037M - t_f M^2,$

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$Z = \frac{dE}{dt} = 3.778F - 0.125E.$

To compute the equilibrium points we solve $X = 0, Y = 0$ and $Z = 0$.

Here, the equilibrium point is (0, 0, 0). i.e. $F = 0, M = 0$ and $E = 0$.

Now, to study the stability of the equilibrium points we first need to find the Jacobian matrix which is:

$$J(F, M, E) = \begin{bmatrix} \frac{\partial X}{\partial F} & \frac{\partial X}{\partial M} & \frac{\partial X}{\partial E} \\ \frac{\partial y}{\partial F} & \frac{\partial y}{\partial M} & \frac{\partial y}{\partial E} \\ \frac{\partial Z}{\partial F} & \frac{\partial Z}{\partial M} & \frac{\partial Z}{\partial E} \end{bmatrix} = \begin{bmatrix} -0.037 - 2Ft_f & 0 & 0.66 \\ 0 & -0.037 - 2Mt_f & 0.044 \\ 3.778 & 0 & -0.125 \end{bmatrix}$$

To study the stability of (0, 0, 0)

$$\therefore J(0,0,0) = \begin{vmatrix} -0.037 - \lambda & 0 & 0.66 \\ 0 & -0.037 - \lambda & 0.044 \\ 3.778 & 0 & -0.125 - \lambda \end{vmatrix}$$

The characteristic polynomial is,

$\lambda^3 + 0.19900000000000007\lambda^2 - 2.4828609999999993\lambda - 0.09208763499999956$

And the eigenvalues of the equation is

$\{-1.6606885769036883,$

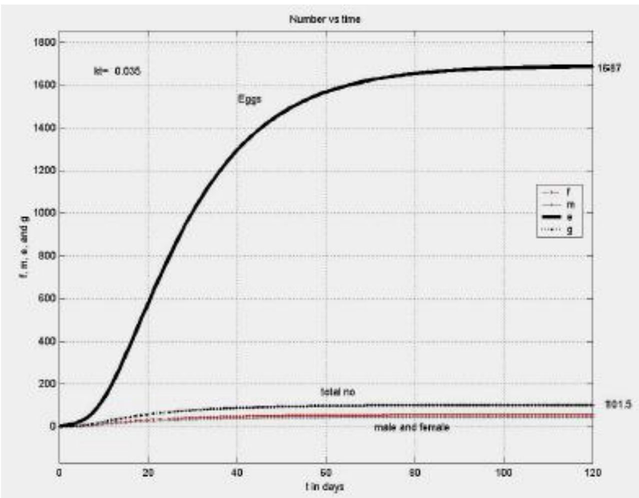
$-0.03699999999999985,$

$1.4986885769036882\}$

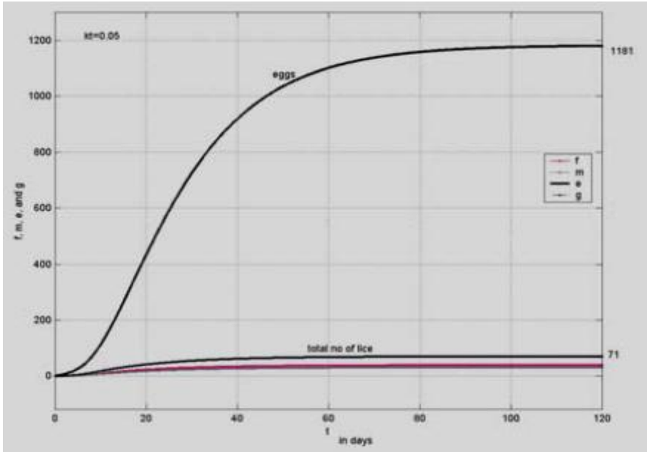
Here two eigenvalues negative and one eigenvalues positive.

Hence, the system is unstable.

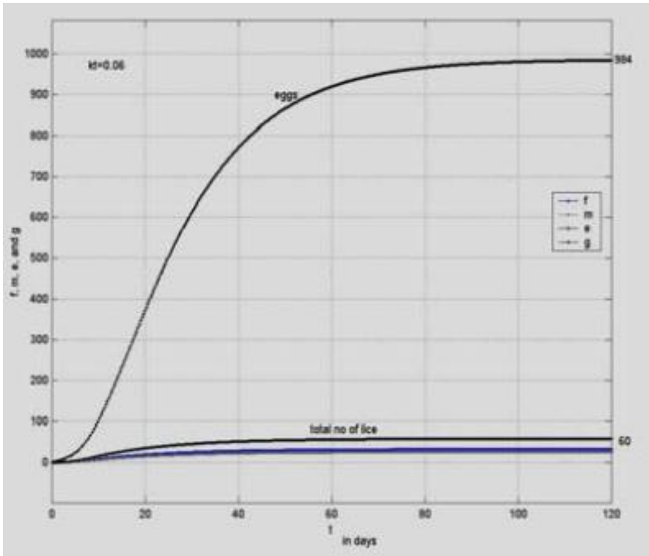
VII. SOLUTION OF DIFFERENTIAL EQUATIONS USING MATLAB



The above graph for $t_f = 0.035$ gives the total no of lice that saturates at about 100 which is the real situation in cases of severe infestation. The following graphs show the variation of total no of lice and eggs with respect to time for different values of t_f (varying levels of infestation).A saturation of value of number of lice at about 20 is considered to be moderate infestation.



$t_f = 0.05, E = 1181, L = 71$



$t_f = 0.06, E = 984, L = 60$

VIII. CONCLUSION

By the above discussion we say that lice do not increase in number indefinitely but are limited due to crowding effects and lack of area. So the rate of transmission or fall off is significant. Here, we find that the louse is unstable in human heads by eigenvalues of system. The claim of antilice treatment to ensure freedom from lice in 4 weeks is quite authentic.

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Nomenclature:

h_e	:	Hatching of eggs
n_d	:	Natural death

t_f	:	Transmission and fall off
g_e	:	Egg generation or laying of eggs
e_m	:	Egg mortality
m_p	:	Fraction of male population
E	:	number of eggs
F	:	female lice
M	:	male lice
G	:	total number of live lice

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